Teachers' views on the relevance of advanced mathematics studies to secondary school teaching*

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*Based on Even (2011)

Overarching research question

- What might be the relevance of advanced mathematics courses taught by research mathematicians to developing expertise in secondary school mathematics instruction?

Specific research question

- What are the views on the above matter of Israeli secondary school mathematics teachers who participate in a program where such courses comprise a sizeable share?

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Specific research question

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The Rothschild-Weizmann Program

- Unique master's program, designed for excellent Israeli secondary school teachers of mathematics, physics, chemistry, or biology.
- A major component of the mathematics strand of the program comprises mathematical studies at an advanced level.
- Prominent research mathematicians, who usually teach only mathematics master and doctoral students, design and teach these courses.

The RW advanced mathematics courses

- Courses dealing with central topics in the school curriculum at a graduate level:
  - Algebra
  - Analysis
  - Geometry
  - Probability and Statistics
- Courses dealing with the use and application of mathematics in other domains:
  - in computer science
  - in the natural sciences (applied mathematics)
  - in the social sciences/everyday life
- Course on the history and philosophy of mathematics

Methodology

Participants

- 15 teachers of the first two cohorts.

Data sources and analysis

- Individual position papers on the relevance and contribution of the mathematics courses in the program to the work of the math teacher.
- Group interview on this issue.
- Analysis aimed at identifying and characterizing different perspectives.

Teachers' views on the relevance of advanced mathematics courses

Advanced mathematics courses:

1. as a resource for teaching secondary school mathematics.
2. for advancing teachers' understanding about what mathematics is.
3. for reminding teachers what learning mathematics feels like.

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Resource for teaching secondary school mathematics

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- Regular teaching.
- Enrichment and extracurricular activities.

Resource for regular teaching

Using some of the math problems, ideas, and topics presented in courses:

- Adding geometric meaning to the definition of a concave function.
- Better understanding of how to teach analysis:

  "It helped me understand what to focus on in my teaching of these topics and what to emphasize. Additionally, it simplified the terms for me so that it was easier for me to explain to the students these difficult and abstract concepts."

- …

Resource for enrichment

The courses on the use of mathematics in the natural sciences and in the social sciences

Using examples taught in the courses:

- Population dynamics in life sciences.
- The Black Swan Phenomenon and its implications.
- …

The Black Swan Phenomenon

“In parallel to my preparation for my presentation on the topic of the Black Swan in the course, I taught the topic of normal distribution. This time, in addition to the emphasis on the normal distribution, I added an introduction to the Black Swan Phenomenon and why we are not aware of it.

I presented a graph of employees’ average income in deciles that does not satisfy the characteristics of the normal distribution and from this I moved on to talk about the “long tail.” We talked about the economical and social influences of the “long tail” in Internet commerce and additional applications of it.

I felt that the students received a comprehensive picture and they reached by themselves the conclusion that social phenomena, which depend on people’s behavior, are not distributed normally according to a Gaussian curve and that the normal distribution is mainly expected for human physical measures.”

Resource for enrichment

The course on the history and philosophy of mathematics

Using stories about important figures and events from the course:

- Vieta
- Euclid
- Pythagoras
- Thales
- the discovery of complex numbers
- the discovery of non-Euclidean geometries
- …

Stories from the history of math

“When I studied for my Bachelor degree, we had an instructor that whenever he mentioned a theorem named after a mathematician, he used to add brief information about that mathematician. I always wanted to add this information for my students, but time constraints prevented me from doing so.

The course on the history of mathematics gave me the motivation to devote time for that in class.

I see importance in bringing the history of mathematics to pupils’ attention, in addition to mere curiosity. It teaches “how” mathematics is done. Even great mathematicians made mistakes (although not many) and they corrected them, and they tried again, until they achieved their mathematical discovery.”

Resource for enrichment

The geometry course

Introducing non-Euclidean geometries presented in the course.

Non-Euclidean geometries

“Students in school are usually exposed only to Euclidean geometry. The concept of an axiom or a system of axioms is not easy for many students to understand. An axiom is perceived as a claim that is always true.

When I bring to class a balloon or a ball and we check what the sum of the angles of a triangle is, the students are very surprised when they find out that the sum of the angles of a triangle is greater than 180 degrees... Students also better understand the connection between the parallel postulate and the sum of the angles of a triangle.”

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Advancing teachers' understanding about what mathematics is

Advancing teachers' understanding about what mathematics is

- Doing mathematics as problem solving.
- The power of visual tools for doing mathematics.
- Intuitive proofs in mathematics.
- Mathematics as a "living and kicking" domain.

Doing mathematics as problem solving

The courses on the use of mathematics in the natural sciences and in the social sciences

- Demonstrating the use of mathematics (e.g., differential equations) to solve problems in other domains (physics, biology, medicine, economics).
  - Addressing students' common complaint: "Why do we need to study mathematics? It doesn't help in real life!"
- Adopting the approach of first framing a problem and then inspecting what mathematical tools are suitable for solving it.
  - In contrast to common practice.

The power of visual tools for doing math

"The biggest change in my view was the way the instructor presented the different topics. Almost all topics were presented algebraically and geometrically. Often, the algebraic approach was complicated and required extensive knowledge of algebra. In contrast, the geometric approach was visual and brought me to a higher level of understanding.

For me it was something new… For the first time in studying mathematics I felt the power of geometry and I enjoyed it."

Intuitive proofs in mathematics

“From [the analysis instructor] I personally took that intuitive proofs are legitimate and are welcomed by mathematicians. This is something that perhaps I should expose my students to, and I should encourage them to provide intuitive proofs before moving on to formal proofs.”

Mathematics as a “living and kicking” domain

- Mathematics is not a finite closed domain where everything is known.
- It is alive and keeps developing.
  - Examples to illustrate this to students.

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Reminding teachers what learning mathematics feels like

Reminding teachers what learning mathematics feels like

- Becoming more aware of, and sensitive to, students' emotional difficulties.
- Better understanding students' need for concrete examples.

Sensitivity to students' emotional difficulties

- "The situation where I sit in front of an exercise and I don't know how to solve it – this is something that I have not experienced for a long time. It is important to preserve this feeling and be conscious of it when we teach… Identifying with the difficult experience of studying math makes me a better teacher."
- "The mere feeling, sometimes, of incapability when dealing with complex exercises in mathematics reminds us, as teachers, of the weak students in class that cannot solve exercises by themselves. It makes the teacher, instead of expressing impatience and dissatisfaction, express empathy towards the student and to identify with his situation."

Understanding students' need for concrete examples

"My struggle with difficulties that keep emerging throughout the learning process helps me better understand my students, their need for concrete examples, just as I found myself needing them when I encounter abstract concepts and unfamiliar mathematical symbols."

Conclusion

SMK dimensions

Reference to 2 critical dimensions of SMK for teaching mathematics:

1. Knowledge of mathematics (i.e., knowledge of specific topics, procedures, concepts, and the relationships among them).
2. Knowledge about the nature of mathematics.

Advanced mathematics courses – as resource for classroom teaching

Development of a more robust teaching repertoire:

- Powerful examples that illustrate and help develop insights into, and a deeper understanding of, important ideas, principles, properties, theorems, etc.

Advanced mathematics courses – for advancing understanding about what mathematics is

- Enabled teachers to –
  - better represent the discipline of mathematics,
  - incorporate changes into traditional teaching,
  - address students' common complaint about the need to study mathematics.
- Not needed in other professions that use mathematics, yet is not *Specialized Content Knowledge* (in Ball et al.'s term) –
  - crucial aspect of mathematicians' professional knowledge.

Advanced mathematics courses – practicing vs. prospective teachers (1)

The role of teaching experience in transforming new advanced math knowledge into math knowledge useful for teaching:

> "Personally, my attitude toward mathematical studies is different now from when I studied towards the first degree. After acquiring experience in working in the field and in teaching students, I find that already during lectures, I start to think whether it is possible to connect what is learned to the content required for my students, and in what way."

Advanced mathematics courses – practicing vs. prospective teachers (2)

Improved sensitivity to students' difficulties might be related to the length of time since teachers last experienced difficulties when studying unfamiliar mathematics.

Young prospective teachers may not feel this way.

Final note

- Advanced mathematics courses taught by research mathematicians could play a significant role in the professional development of secondary teachers.