# Safed Workshop on Quantum Dissipation
# Open Problems in Open Quantum Systems

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## Open Quantum Systems

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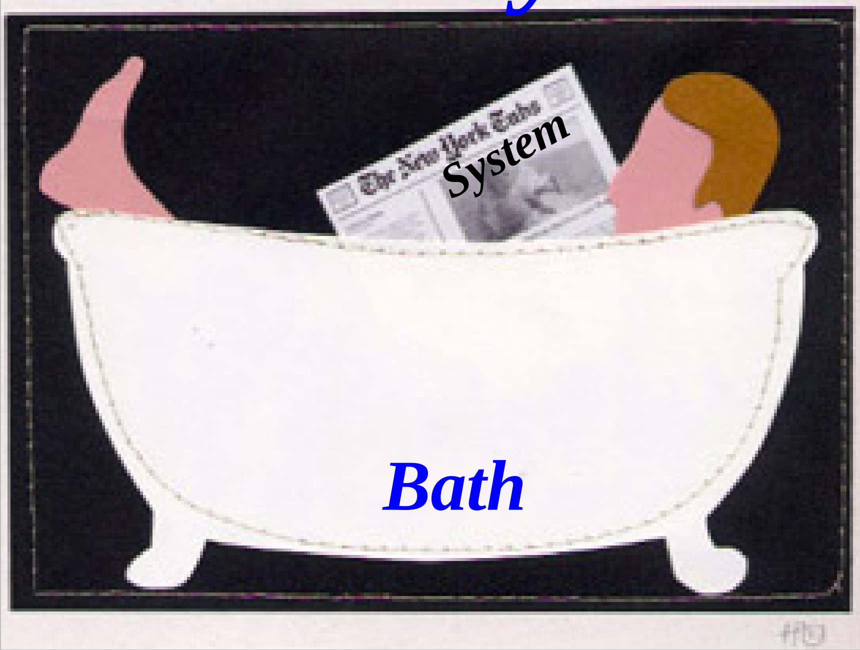
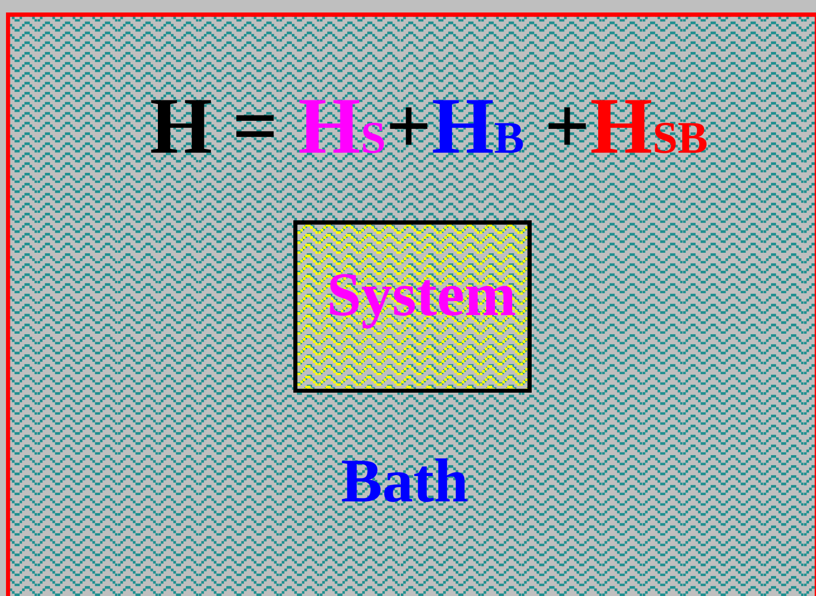
# *Reduced dynamics*




$$H = H_S + H_B + H_{SB}$$




# Some quantum definitions:

**Relation between the state the operator and the observable**

***Statics:*** $\langle \hat{A} \rangle = (\hat{A}^{+} \bullet \hat{\rho}) = Tr\{\hat{A}\,\hat{\rho}\}$

***Dynamics:*** $\hbar = 1$

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}(\hat{\rho}) \qquad \mathcal{L}_{H}(\hat{\rho}) = -i\,[H, \hat{\rho}]$$

$$\frac{d\hat{A}}{dt} = \mathcal{L}^{*}(\hat{A}) + \frac{\partial \hat{A}}{\partial t} \qquad \mathcal{L}^{*}_{H}(\hat{A}) = i\,[\hat{H}, \hat{A}]$$

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^{\dagger}(t) \qquad \hat{A}(t) = \hat{U}^{\dagger}(t)\hat{A}(0)\hat{U}(t)$$

$$\frac{dU}{dt} = -i\,HU \qquad U(t) = e^{-i\,Ht}$$

# *Reduced dynamics*

PHYSICAL REVIEW A **70**, 024301 (2004)

**Completely positive non-Markovian decoherence**

Jinhyoung Lee

$$\hat{H}_{\text{int}}(\tau) = \sum_{\alpha} \hat{S}_{\alpha} \otimes \hat{D}_{\alpha}(\tau), \qquad (1)$$

$$\hat{\rho}_T(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_R(0). \qquad (2)$$

$$\frac{d}{d\tau}\hat{\rho}_T(\tau) = -i[\hat{H}_{\text{int}}(\tau), \hat{\rho}_T(\tau)]. \qquad (3)$$

$$\hat{U}_{\text{int}}(\tau) = \mathbf{T}\,\exp\left(-i\int_0^{\tau} d\tau' \hat{H}_{\text{int}}(\tau')\right) \qquad (4)$$

$$\hat{\rho}_T(\tau) = \hat{U}_{\text{int}}(\tau)\hat{\rho}_T(0)\hat{U}^{\dagger}_{\text{int}}(\tau), \qquad (5)$$

$$\hat{\rho}_S(\tau) = \text{Tr}_R\, \hat{\rho}_T(\tau).$$

**partial trace**

(6) if: $\hat{\rho}_R(0) = \Sigma_n p_n |n\rangle_R\langle n|$,

$$\hat{\rho}_S(\tau) = \mathcal{S}(\tau)\hat{\rho}_S(0) = \sum_{n,m} \hat{K}_{nm}(\tau)\hat{\rho}_S(0)\hat{K}^{\dagger}_{nm}(\tau),$$

**Kraus representation**

$$\hat{K}_{nm}(\tau) = {}_R\langle n|\hat{U}_{\text{int}}(\tau)|m\rangle_R\sqrt{p_m}.$$

# *Reduced dynamics*

$$\hat{\rho}(0)=\hat{\rho}_S\otimes\hat{\rho}_B \;\Rightarrow\; \hat{\rho}(t)=\hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)$$

$$\hat{\rho}_S=\mathrm{tr}_B\{\hat{\rho}(0)\} \;\Rightarrow\; \hat{\rho}_S(t)=\Gamma(\hat{\rho}_S(0))=\mathrm{tr}_B\{\hat{\rho}(t)\}$$

**Quantum operations**

$$\Gamma(\hat{\rho}_S(0))=\sum_j \hat{K}_j\hat{\rho}_S\hat{K}_j^\dagger$$

$$K_j=\langle e_j|U|e_0\rangle$$

operator sum representation or Kraus operator

K. Kraus. Ann. Phys. **64** 311 (1971)

where $|e_j\rangle$ is a complete set in the bath and $|e_0\rangle$ is a purified initial state

## Completely positive map

$$\Gamma(\hat{\rho}_s(0)) = \sum \hat{K}_j \hat{\rho}_s \hat{K}_j^{\dagger}$$

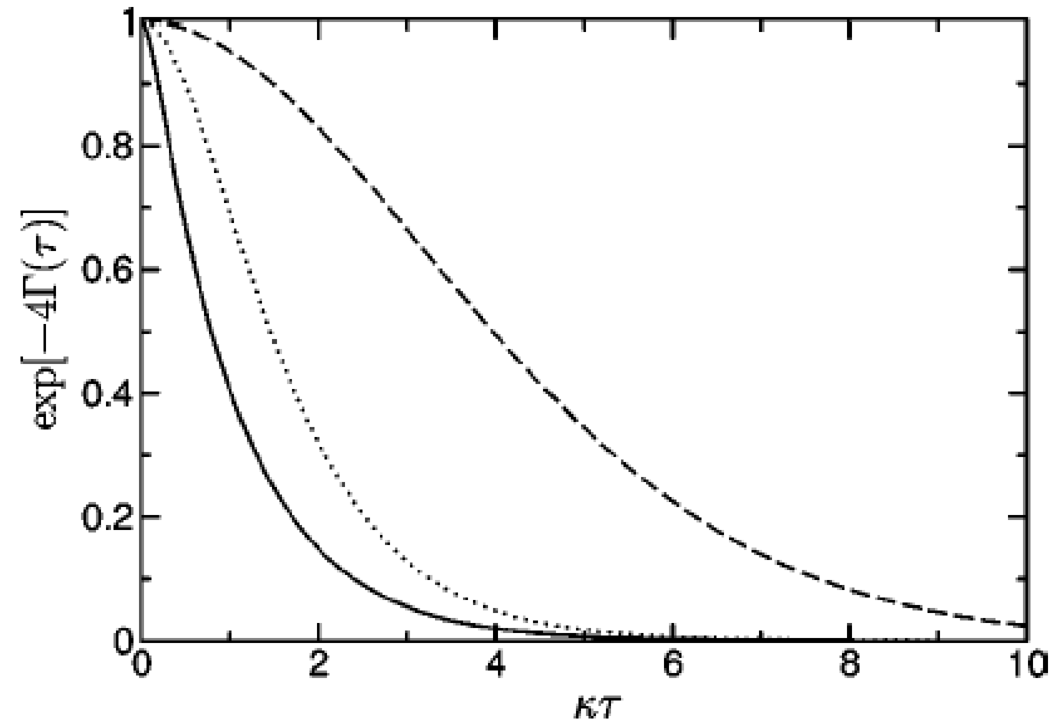
$\Gamma$ maps density operators into density operatos since the eigenvalus of $\hat{\rho}$ are positive the map has to preserve the positivity of the eigenvalues.

Can a positive map be always made out of a unitary dynamics of a larger system+bath?

# *Reduced dynamics*

$$\hat{H}_{\text{int}}(\tau) = \lambda(\tau)\hat{\sigma}_z \otimes \hat{\sigma}_z,$$

$$\lambda(\tau) = \frac{2\gamma(\tau)\exp[-4\Gamma(\tau)]}{\sqrt{1-\exp[-8\Gamma(\tau)]}},$$




$$\hat{\rho}_S(0) = \frac{1}{2}(\hat{\mathbb{1}} + \mathbf{s}\cdot\hat{\boldsymbol{\sigma}}),$$

$$\hat{\rho}_R(0) = \frac{1}{2}(\hat{\mathbb{1}} + \mathbf{r}\cdot\hat{\boldsymbol{\sigma}}), \qquad (12)$$

$\mathbf{s}=(s_x, s_y, s_z)$ a Bloch vector $\hat{\boldsymbol{\sigma}}=(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$. $\mathbf{r}=(r_x, r_y, 0)$, is

$$\mathbf{s}(\tau) = \{s_x \exp[-4\Gamma(\tau)], s_y \exp[-4\Gamma(\tau)], s_z\}. \qquad (13)$$

$$\hat{H}_{\text{int}}(\tau) = \lambda(\tau) \sum_{\alpha,\beta=x,y,z} g_{\alpha\beta}\hat{\sigma}_{\alpha} \otimes \hat{\sigma}_{\beta}, \qquad (15)$$

TABLE I. The coupling matrix $g_{\alpha\beta}$ and the initial state $\mathbf{r}$ of the effective environment for dephasing, depolarizing, and amplitude damping processes.

| | $g_{\alpha\beta}$ | $\mathbf{r}$ |
|---|---|---|
| Dephasing | $\delta_{\alpha z}\delta_{\beta z}$ | $(r_x, r_y, 0)$ |
| Depolarizing | $\delta_{\alpha\beta}$ | $(0, 0, 0)$ |
| Amplitude damping | $\delta_{\alpha\beta}(\delta_{\beta x}+\delta_{\beta y})$ | $(0, 0, r_z)$ |

$$\mathbf{s}_{\text{pol}}(\tau) = \mathbf{s}\ \exp[-8\Gamma(\tau)]. \qquad (16)$$

For the amplitude damping process, it is given by

$$[\mathrm{s}_{\text{amp}}(\tau)]_x = s_x\ \exp[-4\Gamma(\tau)],$$

$$[\mathbf{s}_{\text{amp}}(\tau)]_y = s_y\ \exp[-4\Gamma(\tau)], \qquad (17)$$

$$[\mathbf{s}_{\text{amp}}(\tau)]_z = r_z + (s_z - r_z)\exp[-8\Gamma(\tau)].$$

# Quantum dynamical semigroup.

G. Lindbald, Commun. Math. Phys. **48,** 119 (1976)

V. Gorini, A. Kossakowski, and E.C.G. Sudarshan, J. Math. Phys. **17** 821 (1976)

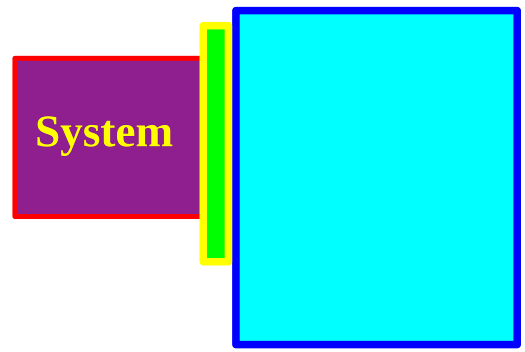
$$\Gamma(\hat{\rho}_s(0)) = \sum \hat{K}_j \, \hat{\rho}_s \, \hat{K}_j$$

**Markovian property**

$$\Gamma(t)\Gamma(s) = \Gamma(t+s)$$

$$\Gamma(t) = e^{\mathcal{L} t}$$

$$\mathcal{L} = \mathcal{L}_H + \mathcal{L}_D .$$




$$\mathcal{L}_H^*(\hat{\mathbf{A}}) = i[\hat{\mathbf{H}}, \hat{\mathbf{A}}].$$

$$\mathcal{L}_D^*(\hat{\mathbf{A}}) = \sum_j \left[ \hat{\mathbf{F}}_j \hat{\mathbf{A}} \hat{\mathbf{F}}_j^\dagger - \frac{1}{2} (\hat{\mathbf{F}}_j \hat{\mathbf{F}}_j^\dagger \hat{\mathbf{A}} + \hat{\mathbf{A}} \hat{\mathbf{F}}_j \hat{\mathbf{F}}_j^\dagger) \right],$$

Generator of a completely positive map

J. Phys. A: Math. Gen. **29** (1996) 4197–4207.

# On the existence of quantum subdynamics

Göran Lindblad

**Abstract.** It is shown, using only elementary operator algebra, that an open quantum system coupled to its environment will have a subdynamics (reduced dynamics) as an exact consequence of the reversible dynamics of the composite system only when the states of system and environment are uncorrelated. Furthermore, it is proved that for a finite temperature the KMS condition for the lowest-order correlation function cannot be reproduced by any type of linear subdynamics except the reversible Hamiltonian one of a closed system. The first statement can be seen as a particular case of a more general theorem of Takesaki on the properties of conditional expectations in von Neumann algebras. The concept of subdynamics used here allows for memory effects, no assumption is made of a Markov property. For dynamical systems based on commutative algebras of observables the subdynamics always exists as a stochastic process in the random variable defining the open subsystem.

## The quantum structure of system bath coupling

**A mixed system can always be purified**

$$\hat{\rho}_s = \mathrm{tr}_B\{|\Psi\rangle\langle\Psi|\}$$

Due to quantum entanglement there is a difference between positivity and complete positivity

$$\Gamma(\hat{\rho}_s(0)) = \sum \hat{K}_j \hat{\rho}_s \hat{K}_j$$

KMS condition