Non-Equilibrium Continuum Physics **HW set #6**

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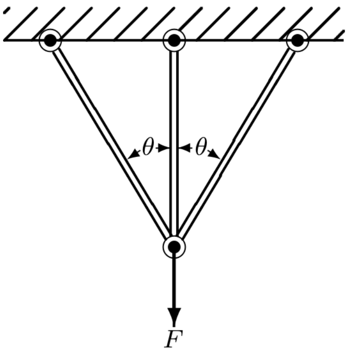
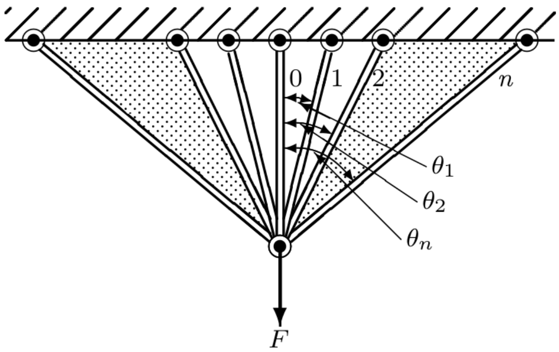
Plasticity

1. An incompressible elastic-perfect-plastic cylindrical rod, of Young's modulus E, yield stress $\sigma_Y \ll E$, length L and cross section A is compressed/pulled under uniaxial stress along its axis until its length is multiplied by a factor λ. How much work did the external loading preform? How much of it was dissipated? Work in the regime that $|\lambda - 1| \ll 1$, but plastic deformation does occur.

2. Consider the setting shown in Fig 1a: three elastic-perfect-plastic rods with cross sectional area A are connected with pins that can transfer only axial forces but no torques, and a vertical force F is exerted on them. The top pins are held at fixed positions to the ceiling (but not at a fixed angle). All rods have Young's modulus E and yield stress $\sigma_Y \ll E$. When $F = 0$ the system is stress-free. Assume small deformations.



(a) Three rod setup



(b) Bonus question setup

Figure 1: n-rods setup.

   (a) Denote the vertical displacement of the loading point by Δ. Calculate and plot $\Delta(F)$ (choose some values for the parameters you need). What is the maximal force F_E for which the response is elastic? What is the maximal force F_U that can be applied?

   (b) Calculate the residual strains and stresses if the force is removed after the displacement was Δ.

   (c) Suppose no force is applied, but the temperature is increased (or decreased) by ΔT. Calculate the minimal temperature difference ΔT_E that causes plastic deformation (assume α_T, σ_Y, E are T-independent).

   (d) Bonus: repeat (a) for the case where there are 5 bars, or better yet, $2n + 1$. The setup is shown in Figure 1b. Assume the system is symmetric with respect to horizontal reflection.

3. In class, we've found the elasto-plastic solution for a spherical shell. We now look at some interesting aspects of the results.

(a) Examine numerically Eq. (11.38) from the lecture notes. For the case that $b = 10a$, plot c as a function of p. Can you analytically explain what happens when $p \to p_U$? (hint: yes you can).

(b) For the case that $p = \sigma_Y$, plot c/a as a function of b/a. What is the asymptotic value of c when $b/a \to \infty$?

(c) Find the displacement field $u_r(r)$ (from symmetry, $\vec{u}$ is a function of r only and other components vanish). Is the stress/strain/displacement field continuous/differentiable across the elasto-plastic boundary?

Guidance: In the elastic region, there's a particularly simple relation between u_r and some of the strain components. In the plastic region, the volumetric part of the deformation is still elastic - we still have $\text{tr}\,\boldsymbol{\sigma} = K\,\text{tr}\,\boldsymbol{\epsilon}$, where K is the bulk modulus.

4. Continuing our TA session, consider an elastic-perfect-plastic 2D annulus with internal and external radii a, b, subject to internal pressure p and zero outer pressure, under *plane-stress* conditions. Use the Tresca yield criterion, and preform the analysis that was done in class for the case of a spherical shell:

(a) Find the stress field $\sigma_{ij}(r)$, the minimal internal pressure that induces plastic flow (p_E), the ultimate pressure for which the entire annulus is plastic p_U, and give an equation that determines the radius of the elasto-plastic boundary c.

(b) Show that your solution is valid only if

$$1 + \frac{c^2}{b^2} - \log\frac{c^2}{a^2} \geq 0 \; . \tag{1}$$

What happens if this criterion is not satisfied? Why is this problem not present in plane strain conditions?

(c) Considering this, what is the condition on a/b that ensures that p_U exists? Give an equation that describes, for a given value of a/b, the maximal possible value of c. What is this value when $b/a \to \infty$?