

# UNIVERSAL DYNAMICAL CONTROL OF DECAY AND DECOHERENCE FOR WEAK AND STRONG SYSTEM-BATH COUPLING

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A unified theory is given of dynamically modified decay and decoherence in driven two-level and multilevel quantum systems that are *weakly coupled* to arbitrary finite-temperature reservoirs and undergo random phase fluctuations. Criteria for the optimization of decoherence suppression and the limitations of this approach are obtained. For a driven qubit that is *strongly coupled* to the continuum edge of reservoir’s spectrum, we demonstrate that only an appropriately ordered sequence of abrupt changes of the resonance frequency, near the continuum edge, can effectively protect the qubit state from decoherence.

*Keywords:* Decoherence control; dynamical control; quantum information

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## 1 Introduction

Decay to a continuum or a thermal bath characterizes decoherence processes in many quantum systems, e.g., spontaneous emission of photons by excited atoms [1], vibrational and collisional relaxation of trapped ions [2] and cold atoms in optical lattices [3] and the relaxation of current-biased Josephson junctions [4]. Another source of decoherence via collisions is proper dephasing [5], which does not affect the populations of quantum states.

Perhaps the most promising approach suggested for the suppression of decoherence is the “dynamical decoupling” of the system from the bath [6, 7, 8, 9, 10, 11, 12, 13, 14]. In particular, “bang-bang” (BB) pulses have been proposed for the suppression of proper dephasing:  $\pi$ -phase flips of the coupling via strong and sufficiently fast resonant pulses applied to the system.[9, 10, 11] The identification of decoherence-free subspaces, wherein symmetrically degenerate states are decoupled from the bath, constitutes a complementary approach [14, 15, 16].

Here we purport to substantially expand the arsenal of decay and decoherence control. We present a detailed derivation of a *universal form of the decay rate* of unstable states into *any* reservoir (continuum), dynamically modified by perturbations with arbitrary time dependence. An analogous form is obtained for the dynamically modified rate of proper dephasing. The present account expands on and extends our previous publications [17, 18, 19, 20, 21, 22, 23, 24, 25]. Our unified, optimized approach reduces to the BB method in the particular case of proper dephasing or decay via coupling to *spectrally symmetric* (e.g.,

Lorentzian or Gaussian) noise baths with limited spectral width (see below). The type of phase modulation advocated for the suppression of coupling to *asymmetric* baths (e.g., phonon baths with frequency cutoff) is, however, drastically different from the BB method. Other situations to which our approach applies, but not the BB method, include *amplitude modulation* of the coupling to the continuum, as in the case of decay from quasibound states of periodically tilted washboard potential [17, 18, 19, 20]: such modulation has been experimentally shown [26] to give rise to either slowdown of the decay (Zeno-like behavior) or its speedup (anti-Zeno-like behavior), depending on the modulation rate. Our analysis can thus serve as a general recipe for *optimized* decay and decoherence suppression for quantum logic operations [27, 28] or decay enhancement for the chaos or chemical reactions [29].

Our general approach [19] to dynamical control of states coupled to an arbitrary *zero-temperature* “bath” or continuum has reaffirmed the intuitive anticipation that, in order to suppress their decay, we must modulate the system-bath coupling at a rate exceeding the spectral interval over which the coupling is significant. The spectra of baths (continua) corresponding to vibrational or collisional decay or decoherence typically allow *dynamical suppression*, using realistic rates of modulation [19]. These results have left several basic questions open: Would *the two-level system (TLS) model hold* at all for modulation rates, that are comparable to the TLS transition frequency  $\omega_a$  (between its states  $|e\rangle$  and  $|g\rangle$ ) which may invalidate the standard rotating-wave approximation (RWA) [1, 5]? Would *temperature effects*, which are known to incur *upward*  $|g\rangle \rightarrow |e\rangle$  transitions [30], further complicate the dynamics and perhaps hinder the suppression of decay? How to dynamically control decay and decoherence in an efficient, *optimal* fashion? We address these questions within a general theory of a TLS that is coupled to an *arbitrary bath* at zero-temperature [17, 18, 19, 20, 22] (Section 2). The theory is then generalized to finite temperatures and to a qubit driven by an *arbitrary* time-dependent field, which may cause the failure of the RWA [21] (Section 3). In Section 4 we extend, for the first time, the analysis to *multi-level systems*, where quantum interference between the levels may either inhibit or accelerate the decay.

Schemes of quantum information processing that are based on *optically manipulated atoms* face the challenge of protecting the quantum states of the system from decoherence, or fidelity loss, due to atomic spontaneous emission (SE) [2, 31, 32]. SE becomes the dominant source of decoherence at low temperatures, as nonradiative (phonon) relaxation becomes weak [33, 34]. SE suppression *cannot be achieved* by frequent modulations or perturbations of the decaying state, because of the extremely broad spectrum of the radiative continuum (“bath”) [1, 17]. A promising means of protection from SE is to embed the atoms in photonic crystals (three-dimensionally periodic dielectrics) that possess spectrally-wide, omnidirectional photonic bandgaps (PBGs) [35]: atomic SE would then be blocked at frequencies within the PBG [35]–[40]. Thus far, studies of coherent optical processes in a PBG have assumed *fixed* values of the atomic transition frequency [41]. However, in order to operate quantum logic gates, based on pairwise entanglement of atoms by field-induced dipole-dipole interactions [42, 43, 44], one should be able to switch the interaction on and off, most conveniently by AC Stark-shifts of the transition frequency of one atom relative to the other, thereby changing its detuning from the PBG edge. The question then arises: should such frequency shifts be performed adiabatically, in order to minimize the decoherence and maximize the quantum-gate fidelity? The answer is expected to be affirmative, based on existing treatments of adiabatic

entanglement and protection from decoherence [45, 46, 47] and on the tendency of nonadiabatic evolution to spoil fidelity and promote transitions to the continuum [48]. Surprisingly, our analysis (Section 5) demonstrates that only an *appropriately phased sequence of "sudden"* (strongly nonadiabatic) *changes* of the detuning from the PBG edge may yield *higher fidelity* of qubit and quantum gate operations than their adiabatic counterparts. This unconventional nonadiabatic protection from decoherence is valid for qubits that are *strongly coupled to the continuum edge* [49, 50], as opposed to the weak coupling approach in Sections 2-4.

In Section 6 we summarize our main findings. The merits of our comprehensive approach are brought out upon examining the cases where our approach coincides with the BB limit mentioned above, as opposed to cases where the BB method is inadequate. The applicability of the method of Section 5 to *strong coupling* of the qubit to the bath is contrasted with the limitations of other methods.

## 2 Qubit Modulation-Affected Decay Into Zero-Temperature Baths

### 2.1 General theory

Consider the decay of a state  $|e\rangle$  via its coupling to a bath, described by the orthonormal basis  $\{|j\rangle\}$ , which forms either a discrete or a continuous spectrum (or a mixture thereof). The total Hamiltonian is

$$H = H_S(t) + H_B(t) + H_I(t) \quad (1)$$

Here

$$H_S(t) = \hbar(\omega_a + \delta_a(t))|e\rangle\langle e| \quad (2)$$

is the dynamically modulated Hamiltonian of the system,  $\hbar\omega_a$  being the energy of  $|e\rangle$ . The time-dependent frequency  $\delta_a(t)$  can be attributed to the controllable dynamically imposed Stark shift, or to proper dephasing (uncontrolled, random fluctuation). The term

$$H_B(t) = \hbar \sum_j (\omega_j + \delta_f(t))|j\rangle\langle j| \quad (3)$$

is the time-dependent Hamiltonian of the bath,  $\hbar\omega_j$  being the energies of  $|j\rangle$ . The time-dependent frequencies  $\delta_f(t)$ , like  $\delta_a(t)$ , may arise from proper dephasing or dynamical Stark shifts. Finally

$$H_I(t) = \tilde{\epsilon}(t)\mu_{je}|e\rangle\langle j| + \text{h.c.}, \quad (4)$$

denotes the off-diagonal coupling of  $|e\rangle$  with the continuum/bath,  $\tilde{\epsilon}(t)$  being the dynamical modulation function (Fig. 1 – inset) and  $\mu_{je}$  are the system-bath coupling matrix elements.

We write the wave function of the system as

$$|\Psi(t)\rangle = \alpha(t)e^{-i\omega_a t - i \int_0^t \delta_a(t') dt'} |e\rangle + \sum_j \beta_j(t)e^{-i\omega_j t - i \int_0^t \delta_f(t') dt'} |j\rangle, \quad (5)$$

the initial condition being

$$|\Psi(0)\rangle = |e\rangle. \quad (6)$$

A one-level system  $|e\rangle$  which can exchange its population with the bath states  $\{|j\rangle\}$  represents the case of autoionization or photoionization. However, the above Hamiltonian describes also a qubit, which can undergo transitions between the excited and ground states  $|e\rangle$  and  $|g\rangle$ ,

respectively, due to its off-diagonal coupling to the bath. The bath may consist of quantum oscillators (modes) or two-level systems (spins) with different eigenfrequencies. Typical examples are spontaneous emission into photon or phonon continua. In the rotating-wave approximation (RWA), which is alleviated in Section 3, the present formalism applies to a relaxing qubit, under the substitutions

$$|e\rangle \rightarrow |e\rangle|\text{vac}\rangle, \quad |j\rangle \rightarrow |g\rangle|j\rangle \quad (7)$$

in Eqs. (4)-(6). Here  $|j\rangle$  denotes the first excited state with the energy  $\hbar\omega_j$  of the quantum system  $j$ , whereas  $|\text{vac}\rangle$  is the ground state of the reservoir, the energies of  $|g\rangle$  and  $|\text{vac}\rangle$  being zero.

One then obtains from the Schrödinger equation that the amplitude  $\alpha(t)$  obeys the *exact* integro-differential equation [51, 52, 53]

$$\dot{\alpha} = - \int_0^t dt' \epsilon^*(t) \epsilon(t') \Phi(t-t') e^{i\omega_a(t-t')} \alpha(t'). \quad (8)$$

The memory kernel in (8) is

$$\Phi(t) = \frac{1}{\hbar^2} \sum_j |\mu_{ej}|^2 e^{-i\omega_j t} \quad (9)$$

whose Fourier transform

$$G(\omega) = \int dt \Phi(t) e^{i\omega t} = \frac{1}{\hbar^2} \sum_j |\mu_{ej}|^2 \delta(\omega - \omega_j) \quad (10)$$

is the coupling spectrum, i.e., the density of states weighted by the strength of the coupling to the continuum or reservoir. The modulation function  $\epsilon(t)$  is related to  $\tilde{\epsilon}(t)$  in (4) via

$$\epsilon(t) = \tilde{\epsilon}(t) \exp \left[ -i \int_0^t \delta_{af}(t_1) dt_1 \right], \quad (11)$$

with

$$\delta_{af}(t) = \delta_a(t) - \delta_f(t). \quad (12)$$

The function (11) accounts for the dynamical modulation of *either diagonal or off-diagonal elements* of the unperturbed Hamiltonian. In what follows we shall separately treat coherent phase modulations (Section 2.3) and amplitude modulations (Section 2.4) which may also account for *random*  $\delta_{af}(t)$  (Section 2.5 and Section 3), arising either from measurements or from proper dephasing.

The assumption that the coupling (4) is a weak perturbation of (2) implies that  $\alpha(t)$  *varies sufficiently slowly* with respect to the kernel of Eq. (8). One can thus make the approximation  $\alpha(t') \approx \alpha(t)$  on the right-hand side (rhs) of Eq. (8) and obtain

$$\alpha(t) = \exp[-J(t)]. \quad (13)$$

Here

$$J(t) = \int_0^t dt' q(t') \quad (14)$$

with

$$q(t) = \int_0^t dt' \epsilon^*(t) \epsilon(t') \Phi(t-t') e^{i\omega_a(t-t')}. \quad (15)$$

Then one obtains the population of the level  $|e\rangle$ ,

$$P(t) = |\alpha(t)|^2, \quad (16)$$

in the form

$$P(t) = \exp \left[ - \int_0^t dt' r(t') \right], \quad (17)$$

with the *instantaneous decay rate*

$$r(t) = 2\text{Re}q(t). \quad (18)$$

The real and imaginary parts of (13) can be separated [22], on introducing the Fourier transform of the modulation function  $\epsilon(t)$  in the “window”  $(0, t)$ ,

$$\epsilon_t(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^t \epsilon(t_1) e^{i\omega t_1} dt_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \epsilon(t, t_1) e^{i\omega t_1} dt_1. \quad (19)$$

Here

$$\epsilon(t, t_1) = \epsilon(t_1) \theta(t-t_1) \theta(t_1), \quad (20)$$

$\theta(t)$  being the Heaviside step function. The inverse Fourier transform of the second equation (19) yields

$$\epsilon(t, t_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \epsilon_t(\omega) e^{-i\omega t_1}. \quad (21)$$

Inserting (15) into (14), taking account of (20) and (21), results in

$$\begin{aligned} J(t) &= \int_0^t dt' \epsilon^*(t') \int_{-\infty}^{t'} dt'' \epsilon(t, t'') \Phi(t'-t'') e^{i\omega_a(t'-t'')} \\ &= \frac{1}{\sqrt{2\pi}} \int_0^t dt' \epsilon^*(t') \int_{-\infty}^{t'} dt'' \int_{-\infty}^{\infty} d\omega \epsilon_t(\omega) e^{-i\omega t''} \\ &\quad \times \Phi(t'-t'') e^{i\omega_a(t'-t'')}. \end{aligned} \quad (22)$$

On introducing the variable  $\tau = t' - t''$  and changing the order of integration, (22) becomes

$$\begin{aligned} J(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \epsilon_t(\omega) \int_0^t dt' \epsilon^*(t') e^{-i\omega t'} \\ &\quad \times \int_0^{\infty} d\tau \Phi(\tau) e^{i(\omega+\omega_a)\tau} \\ &= \int_{-\infty}^{\infty} d\omega |\epsilon_t(\omega)|^2 \int_0^{\infty} d\tau \Phi(\tau) e^{i(\omega+\omega_a)\tau}. \end{aligned} \quad (23)$$

Using (9) and (10), one can separate the spectral representation of  $\Phi(t)$  into the real and imaginary parts

$$\tilde{\Phi}(\omega) = \int_0^{\infty} dt \Phi(t) e^{i\omega t} = \pi G(\omega) - i\chi(\omega), \quad (24)$$

which satisfy the Kramers-Kronig relations

$$\chi(\omega) = \mathcal{P} \int \frac{G(\omega') d\omega'}{\omega' - \omega}, \quad (25)$$

$\mathcal{P}$  denoting the principal value. Finally, inserting (24) into (23), we may write the amplitude (13) in the form

$$\alpha(t) = \exp\{-[R(t)/2 + i\Delta_a(t)]Q(t)\}. \quad (26)$$

Here we have factored the fluence

$$Q(t) = \int_0^t d\tau |\epsilon(\tau)|^2 \quad (27)$$

out of our universal formulae for the *average* dynamically modified decay rate

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega G(\omega + \omega_a) F_t(\omega) \quad (28)$$

and the dynamically-modified energy shift

$$\Delta_a(t) = - \int_{-\infty}^{\infty} d\omega \chi(\omega + \omega_a) F_t(\omega), \quad (29)$$

with

$$F_t(\omega) = \frac{|\epsilon_t(\omega)|^2}{Q(t)} \quad (30)$$

being the (normalized to unity) spectrum of the modulation function  $\epsilon(t)$  in the “window”  $(0, t)$ .

The universal result (26)-(29) is *valid to all orders of  $t$* , i.e., it keeps intact the *interferences* between the decay channels and their non-Markovian effects. We stress that Eqs. (26), (28), (29) apply to the decay of *superposed* states  $\sum_m \alpha_m |e_m\rangle$ , provided all of them decay and are modulated identically.

Henceforth we concentrate on the decaying population  $P(t)$ . Inserting (26) into (16) yields

$$P(t) = \exp[-R(t)Q(t)]. \quad (31)$$

Let us consider some important consequences of the universal form (31), (28). The modulation spectrum  $F_t(\omega)$  is roughly characterized by its width  $\nu_t$  and the frequency shift

$$\Delta_t = \int d\omega \omega F_t(\omega). \quad (32)$$

*A modulation may strongly modify the decay rate whenever*

$$\nu_t + |\Delta_t| \gtrsim \xi(\omega_a), \quad (33)$$

where  $\xi(\omega_a)$  is the characteristic spectral interval over which the weighted density of states  $G(\omega)$  changes near  $\omega_a$ . In particular, if  $\omega_a$  is near the *edge of the continuum* (as for radiative decay in photonic crystals or vibrational decay in ion traps, molecules and solids), then  $\xi(\omega_a)$  is the distance between  $\omega_a$  and the edge [24, 17] (Fig. 1a). Only in the opposite limit,  $\nu_t + |\Delta_t| \ll \xi(\omega_a)$ , can one approximately set  $F_t(\omega) \approx \delta(\omega)$  in Eq. (28), yielding

$$P(t) \approx \exp[-R_{\text{GR}}Q(t)], \quad (34)$$

where

$$R_{\text{GR}} = 2\pi G(\omega_a) \quad (35)$$

is the *extension of the Golden-Rule (GR) rate to the case of time-dependent coupling*.

## 2.2 Quasiperiodic amplitude and phase modulation (APM)

The modulation function  $\epsilon(t)$  can be either random or regular (coherent) in time, as detailed below. Consider first the most general coherent *amplitude and phase* modulation (APM) of the quasiperiodic form,

$$\epsilon(t) = \sum_k \epsilon_k e^{-i\omega_k t}. \quad (36)$$

Here  $\omega_k$  ( $k = 0, \pm 1, \dots$ ) are arbitrary discrete frequencies with the minimum spectral distance  $\Omega$ . For a given function  $\epsilon(t)$  one can obtain  $-i\omega_k$  and  $\epsilon_k$  as the poles and residues, respectively, of the Laplace transform  $\hat{\epsilon}(s)$ . If  $\epsilon(t)$  is periodic with the period  $\Omega$ , then  $\omega_k = k\Omega$ , and  $\epsilon_k$  become the Fourier components of  $\epsilon(t)$ . For a general quasiperiodic  $\epsilon(t)$ , one obtains

$$Q(t) = \epsilon_c^2 t + \epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{e^{i(\omega_l - \omega_k)t} - 1}{i(\omega_l - \omega_k)}, \quad (37)$$

$$|\epsilon_t(\omega)|^2 = \epsilon_c^2 t \sum_k |\lambda_k|^2 S(\eta_k, t) + \epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{1 + e^{i(\eta_k - \eta_l)t} - e^{i\eta_k t} - e^{-i\eta_l t}}{2\pi\eta_k \eta_l}. \quad (38)$$

Here

$$\epsilon_c^2 = \sum_k |\epsilon_k|^2 \quad (39)$$

equals the average of  $|\epsilon(t)|^2$  over a period of the order of  $1/\Omega$ ,  $\lambda_k = \epsilon_k/\epsilon_c$ , and  $\eta_k = \omega - \omega_k$ , whereas

$$S(\eta_k, t) = \frac{2 \sin^2(\eta_k t/2)}{\pi t \eta_k^2} \quad (40)$$

is a bell-like function of  $\eta_k$  normalized to 1.

For

$$t \gg \Omega^{-1} \quad (41)$$

the first term on the rhs of (38) is a sum of peaks, whose spacings are much greater than their width  $2/t$ . The fast oscillating second term is also peaked at  $\omega = \omega_k$ , but we then find that the ratio of the first to the second terms, and that of their counterparts in (37), is  $\sim (\Omega t)^{-1} \ll 1$ . In the long-time limit, we then neglect these fast oscillating terms and obtain from Eqs. (31)-(38) that

$$P(t) = \exp[-R(t)\epsilon_c^2 t], \quad (42)$$

where  $R(t)$  in Eq. (28) now involves

$$F_t(\omega) \approx \sum_k |\lambda_k|^2 S(\eta_k, t). \quad (43)$$

For a sufficiently long time, the function  $S(\eta_k, t)$  becomes narrower than the respective characteristic width  $\xi(\omega_a + \omega_k)$  of  $G(\omega)$  around  $\omega_a + \omega_k$ , and one can set

$$S(\eta_k, t) \approx \delta(\eta_k) \quad (t \gg 1/\xi(\omega_a + \omega_k)). \quad (44)$$

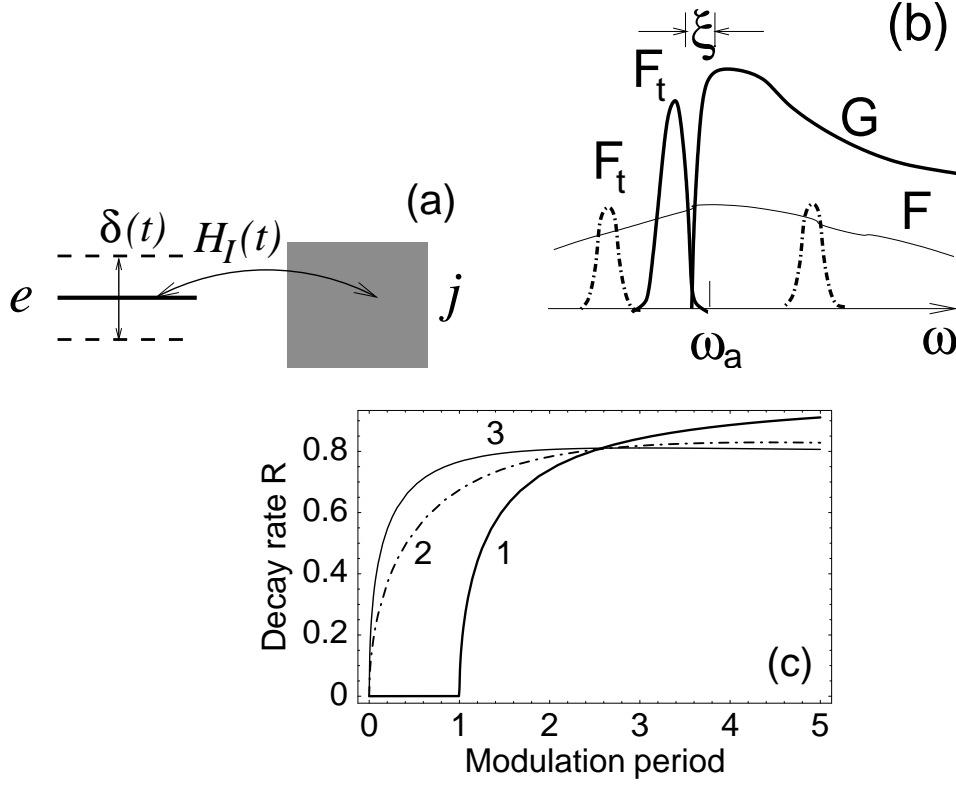


Fig. 1. Decay modification by modulation [Eqs. (31), (28)]. (a) Schematic view of the temporal modulation of the shift of level  $e$  and its coupling to a continuum. (b)  $\omega_a$  is near a band edge of  $G(\omega) = C\omega^{1/2}(\omega + \Gamma)^{-1}\theta(\omega)$ , where  $\theta(\omega)$  is the unit step function; then small phase shifts (solid peak) are more effective in reducing the decay rate  $R$  than large phase shifts  $\phi \simeq \pi$  (dash-dotted peaks) or frequent measurements/random  $\epsilon(t)$  (thin curve). (c) Decay rate  $R$  (in units of the Golden-Rule rate  $R_{\text{GR}}$ ) in case (b) with  $\omega_a = 0.1\Gamma$ : reduction by phase modulation (PM) [Eq. (46)] (curve 1 –  $\phi = 0.1$ , curve 2 –  $\phi = \pi$ ) and frequent impulsive measurements [17, 18, 23] (curve 3 – the quantum Zeno effect - QZE) as a function of perturbation period  $\tau$  (in units of  $\Gamma^{-1}$ ). Curve 1 gives the strongest reduction of  $R$  at a given  $\tau$ .

Thus, when both the inequalities (41) and

$$t \gg t_c \equiv \max_k \{1/\xi(\omega_a + \omega_k)\} \quad (45)$$

hold, where  $t_c$  is the effective *correlation (memory) time* of the reservoir, Eq. (28) is reduced to

$$R = 2\pi \sum_k |\lambda_k|^2 G(\omega_a + \omega_k). \quad (46)$$

For the validity of (46) it is also necessary that

$$\epsilon_c^2 R t_c \ll 1. \quad (47)$$

This condition is well satisfied in the regime of interest, i.e., weak coupling to essentially any reservoir, unless (for some harmonic  $k$ )  $\omega_a + \omega_k$  is extremely close to a sharp feature in

$G(\omega)$ , e.g., a band edge [50], a case covered by Section 5. Otherwise, the long-time limit of the general decay rate (28) under the APM is a sum of the GR rates, corresponding to the resonant frequencies shifted by  $\omega_k$ , with the weights  $|\lambda_k|^2$ .

Formula (46) provides a *simple general recipe* for manipulating the decay rate by APM. Its powerful generality allows for the *optimized* control of decay, not just for a single level, but also for a *band* characterized by a spectral distribution  $P(\omega_a)$  (e.g., inhomogeneous or vibrational spectrum). We can then choose  $\lambda_k$  and  $\omega_k$  in Eq. (46) so as to minimize the decay of (31) convoluted with  $P(\omega_a)$ . In what follows, various limits of (46) will be analyzed.

### 2.3 Coherent phase modulation (PM)

#### 2.3.1 Monochromatic perturbation

Let

$$\epsilon(t) = \epsilon_0 e^{-i\delta_{af}t}. \quad (48)$$

Then

$$R = 2\pi G(\omega_a + \Delta), \quad (49)$$

where  $\Delta = \delta_{af} = \text{const}$  is a frequency shift, induced by the ac Stark effect (in the case, e.g., of atoms) or by the Zeeman effect (in the case of spins). In principle, such a shift may drastically enhance or suppress  $R$  relative to  $R_{\text{GR}}$ . It provides the *maximal variation* of  $R$  achievable by an external perturbation, since it does not involve any averaging (smoothing) of  $G(\omega)$  incurred by the width of  $F_t(\omega)$ : the modified  $R$  can even *vanish*, if the shifted frequency  $\omega_a + \Delta$  is beyond the cutoff frequency of the coupling, where  $G(\omega) = 0$  (Fig. 1a,b). Conversely, the increase of  $R$  due to a shift can be much greater than that achievable by repeated measurements, i.e. the anti-Zeno effect [17, 18, 23, 24]. In practice, however, ac Stark shifts are usually small for (cw) monochromatic perturbations, whence pulsed perturbations should often be used, resulting in multiple  $\omega_k$  shifts as in (46).

#### 2.3.2 Impulsive phase modulation

Let the phase of the modulation function periodically jump by an amount  $\phi$  at times  $\tau, 2\tau, \dots$ . Such modulation can be achieved by a train of identical, equidistant, narrow pulses of non-resonant radiation, which produce pulsed frequency shifts  $\delta_{af}(t)$  in (11). Now

$$\epsilon(t) = e^{i[t/\tau]\phi}, \quad (50)$$

where  $[. . .]$  is the integer part. One then obtains that

$$Q(t) = t, \quad \epsilon_c = 1, \quad (51)$$

$$F_{n\tau}(\omega) = \frac{2 \sin^2(\omega\tau/2) \sin^2[n(\phi + \omega\tau)/2]}{\pi n\tau\omega^2 \sin^2[(\phi + \omega\tau)/2]}. \quad (52)$$

The decay, according to Eq. (31), has then the form (at  $t = n\tau$ )

$$P(n\tau) = \exp[-R(n\tau)n\tau], \quad (53)$$

where  $R(n\tau)$  is defined by Eqs. (28) and (52).

For sufficiently long times one can use Eq. (46). The poles and residues of

$$\hat{\epsilon}(s) = \frac{1 - e^{-s\tau}}{s(1 - e^{i\phi - s\tau})} \quad (54)$$

yield

$$\omega_k = \frac{2k\pi}{\tau} - \frac{\phi}{\tau}, \quad |\lambda_k|^2 = \frac{4 \sin^2(\phi/2)}{(2k\pi - \phi)^2} \quad (55)$$

For *small phase shifts*,  $\phi \ll 1$ , the  $k = 0$  peak dominates,

$$|\lambda_0|^2 \approx 1 - \frac{\phi^2}{12}, \quad (56)$$

whereas

$$|\lambda_k|^2 \approx \frac{\phi^2}{4\pi^2 k^2} \quad (k \neq 0). \quad (57)$$

In this case one can retain only the  $k = 0$  term in Eq. (46) [unless  $G(\omega)$  is changing very fast]. Then the modulation acts as a constant shift

$$\Delta = -\phi/\tau. \quad (58)$$

With the increase of  $|\phi|$ , the difference between the  $k = 0$  and  $k = 1$  peak heights diminishes, *vanishing* for  $\phi = \pm\pi$ . Then

$$|\lambda_0|^2 = |\lambda_1|^2 = 4/\pi^2, \quad (59)$$

i.e.,  $F_t(\omega)$  for  $\phi = \pm\pi$  contains *two identical peaks symmetrically shifted in opposite directions* (Fig. 1a) [the other peaks  $|\lambda_k|^2$  decrease with  $k$  as  $(2k - 1)^{-2}$ , totaling 0.19].

The above features allow one to adjust the modulation parameters for a given scenario to obtain an optimal decrease or increase of  $R$ . The phase-modulation (PM) scheme with a small  $\phi$  is preferable near the continuum edge (Fig. 1a,b), since it yields a spectral shift in the required direction (positive or negative). The adverse effect of  $k \neq 0$  peaks in  $F_t(\omega)$  then scales as  $\phi^2$  and hence can be significantly reduced by decreasing  $|\phi|$ . On the other hand, if  $\omega_a$  is near a *symmetric* peak of  $G(\omega)$ ,  $R$  is reduced more effectively for  $\phi \simeq \pi$ , as in Ref. [7, 8], since the main peaks of  $F_t(\omega)$  at  $\omega_0$  and  $\omega_1$  then shift stronger with  $\tau^{-1}$  than the peak at  $\omega_0 = -\phi/\tau$  for  $\phi \ll 1$ .

## 2.4 Amplitude modulation (AM)

### 2.4.1 On-off modulation

Amplitude modulation (AM) of the coupling arises, e.g., for radiative-decay modulation due to atomic motion through a high- $Q$  cavity or a photonic crystal [54, 55] or for atomic tunneling in optical lattices with time-varying lattice acceleration [26, 56]. Let the coupling be turned on and off periodically, for the time  $\tau_1$  and  $\tau_0 - \tau_1$ , respectively, i.e.,

$$\epsilon(t) = \begin{cases} 1 & \text{for } n\tau_0 < t < n\tau_0 + \tau_1, \\ 0 & \text{for } n\tau_0 + \tau_1 < t < (n+1)\tau_0 \end{cases} \quad (60)$$

( $n = 0, 1, \dots$ ). Now  $Q(t)$  in (31) is the total time during which the coupling is switched on, whereas [57]

$$F_{n\tau_0}(\omega) = \frac{2 \sin^2(\omega\tau_1/2) \sin^2(n\omega\tau_0/2)}{\pi n\tau_1\omega^2 \sin^2(\omega\tau_0/2)}, \quad (61)$$

so that

$$P(n\tau_0) = \exp[-R(n\tau_0)n\tau_1], \quad (62)$$

where  $R(n\tau_0)$  is given by Eqs. (28) and (61).

This case is also covered by (42) and (46), where the parameters are now found to be

$$\epsilon_c^2 = \frac{\tau_1}{\tau_0}, \quad \omega_k = \frac{2k\pi}{\tau_0}, \quad |\lambda_k|^2 = \frac{\tau_1}{\tau_0} \text{sinc}^2\left(\frac{k\pi\tau_1}{\tau_0}\right), \quad (63)$$

with

$$\text{sinc}(x) = \sin x/x, \quad \text{sinc}(0) = 1. \quad (64)$$

The above expressions for  $\omega_k$  and  $|\lambda_k|^2$  have been obtained taking into account that

$$\hat{\epsilon}(s) = \frac{1 - e^{-s\tau_1}}{s(1 - e^{-s\tau_0})}. \quad (65)$$

It is instructive to consider the limit wherein  $\tau_1 \ll \tau_0$  and  $\tau_0$  is much greater than the correlation time of the continuum, i.e.,  $G(\omega)$  does not change significantly over the spectral intervals  $(2\pi k/\tau_0, 2\pi(k+1)/\tau_0)$ . In this case one can approximate the sum (46) by the integral (28) with

$$F_t(\omega) \approx (\tau_1/2\pi) \text{sinc}^2(\omega\tau_1/2), \quad (66)$$

characterized by the spectral broadening  $\sim 1/\tau_1$  (Fig. 2 – inset). Then Eq. (28) for  $R$  reduces to that obtained when ideal projective measurements are performed at intervals  $\tau_1$  [17]. Thus the AM scheme *can imitate measurement-induced (dephasing) effects* on quantum dynamics, if the interruption intervals  $\tau_0$  exceed the *correlation time of the continuum*.

#### 2.4.2 Application to tunneling in optical lattices

The foregoing effects have indeed been experimentally observed [26] for atom tunneling in optical lattices whose tilt (acceleration) is periodically modulated, as above. For its analysis we have used the approximate expression [56] for  $\Phi(t)$ , which can be cast in the form

$$\Phi(t) = \frac{Ae^{-i\omega_a t}}{2T^2\sqrt{uv}^{7/2}} \exp\left\{-i\left\{\frac{\pi}{4} + \gamma[uv + \ln(u+v)]\right\}\right\}. \quad (67)$$

Here

$$u = t/T, \quad v = (1 + u^2)^{1/2}, \quad T = \frac{\omega_g}{k_L a},$$

$$A = \left(\frac{\pi a M^2}{2\hbar^2 k_L^2}\right)^{1/2}, \quad \gamma = \omega_g T/2, \quad (68)$$

where  $\hbar\omega_g$  is the band gap which equals approximately half the potential well depth,  $a$  the acceleration,  $k_L$  the wave number of the laser creating this potential, and  $M$  is the atomic mass. The reservoir spectrum  $G(\omega + \omega_a)$  obtained from (67) by (10) (Fig. 2 – inset) has one maximum at  $\omega \sim \omega_g$ , decreasing steeply (slowly) towards low (high) energies.

Since  $\Phi(t)$  is given by an explicit formula, it is convenient to solve numerically the differential equation

$$\dot{P} = -r(t)P, \quad (69)$$

which follows from (17). In (69) the quantity  $r(t)$  is given by (18) and (15). For a real  $\epsilon(t)$ , as in the present case,

$$r(t) = 2\epsilon(t) \int_0^t dt' \epsilon(t') \text{Re}\Phi(t-t') e^{i\omega_a(t-t')}. \quad (70)$$

The decay probability  $P(t)$ , calculated in Fig. 2 (curves 1-4) for parameters similar to Ref. [26], *completely coincides* with that obtained for ideal impulsive measurements at intervals  $\tau_1$  [17, 18, 23] and demonstrates either the quantum Zeno effect (QZE – curve 2) or the anti-Zeno effect (AZE – curve 3) behavior, depending on the rate of modulation.

Since the Hamiltonian for atoms in accelerated optical lattices is similar to the Leggett Hamiltonian for *current-biased Josephson junctions* [4], the present theory has been extended to describe effects of current modulations on the rate of macroscopic quantum tunneling in Josephson junctions in Ref. [20].

### 2.5 *Random modulation*

The universal Eq. (28), which is a result of *unitary* analysis, is valid also when  $\epsilon(t)$  is a *stationary random process*. If such a process is characterized by the correlation time  $\nu^{-1}$ , one can use a master equation to show that we have

$$P(t) \approx e^{-Rt} \quad (t \gg \nu^{-1}), \quad (71)$$

where the decay rate, provided that

$$R \ll \nu, \quad (72)$$

still has the general form (28), but with

$$F_t(\omega) \rightarrow F(\omega) = \pi^{-1} \epsilon_c^{-2} \text{Re} \int_0^\infty \overline{\epsilon^*(t)\epsilon(0)} e^{i\omega t} dt. \quad (73)$$

Here  $F(\omega)$  is the normalized spectrum of the random process and

$$\epsilon_c^2 = \overline{|\epsilon(t)|^2}, \quad (74)$$

where the overbar denotes ensemble averaging. Expression (28) with the substitution (73) is *completely analogous* to our universal formula describing *measurement effects* on quantum evolution [17, 18, 23]. This analogy between unitary and measurement effects stems from the ability to *emulate* projective measurements by the dephasing of the level evolution caused by classical random fields [17, 18, 23, 58].

There may, however, be a notable difference between projections and random-field dephasing. Projective measurements at an effective rate  $\nu$ , whether impulsive or continuous, usually result in a broadened (to a width  $\nu$ ) modulation function  $F(\omega)$ , without a shift of its center of gravity [59, 60, 17, 18, 23],

$$\Delta = \int d\omega \omega F(\omega) \approx 0. \quad (75)$$

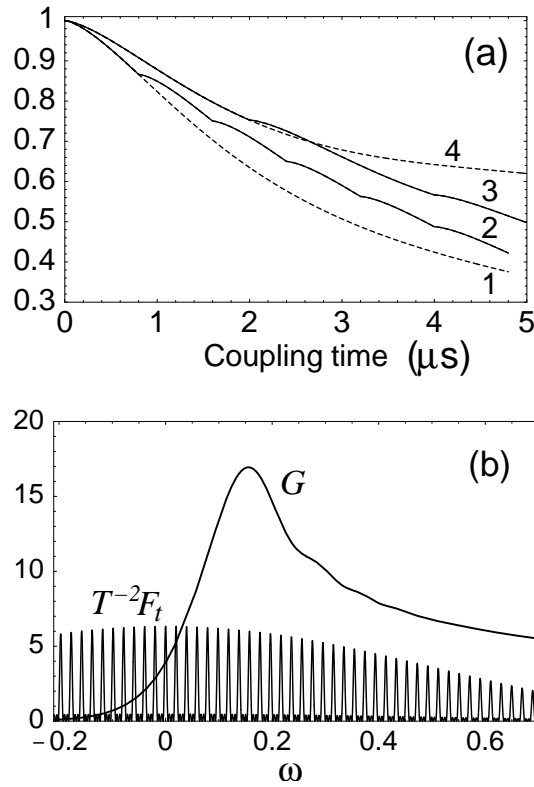


Fig. 2. Tunneling of sodium atoms in optical lattices perturbed by the AM scheme. (a) The decay probability  $P(t)$  as a function of the total coupling time. Curves 1, 4 – decay without modulation. Curve 2 – QZE (decay slowdown compared to curve 1) for  $\tau_1 = 0.8 \mu\text{s}$ ,  $\tau_0 = 50.8 \mu\text{s}$ . Curve 3 – AZE (decay speedup compared to curve 4) for  $\tau_1 = 2 \mu\text{s}$ ,  $\tau_0 = 52 \mu\text{s}$ . (b) The coupling spectrum  $G(\omega + \omega_a)$  and the scaled modulation function  $T^{-2}F_{4\tau_0}(\omega)$  for the conditions of curve 2. Vertical axis: kHz units. Horizontal axis: MHz units. Here  $T = \omega_g d / (\pi a)$ , where  $a = 15 \text{ km/s}^2$  is the lattice acceleration and  $d = 295 \text{ nm}$  is the lattice period.  $\omega_g = 91 \text{ kHz}$ ,  $\omega_g T = 2.05$  (for curves 1, 2);  $\omega_g = 116 \text{ kHz}$ ,  $\omega_g T = 3.32$  (for curves 3, 4).

This feature was shown in Ref. [17] to be responsible for either the standard quantum Zeno effect whereby  $R$  scales as,

$$R \propto 1/\nu, \quad (76)$$

or the anti-Zeno effect whereby  $R$  grows with  $\nu$ . In contrast, a weak and broadband chaotic field, such that

$$|\chi_a| \bar{I} \ll \nu_B, \quad (77)$$

where  $\bar{I}$  is the mean intensity,  $\nu_B$  is the bandwidth, and  $\chi_a$  is the effective polarizability (electric or magnetic, depending on the system), would give rise to a Lorentzian dephasing function  $F(\omega)$  in (73) with a substantial shift

$$\Delta = \chi_a \bar{I}. \quad (78)$$

This shift would have a much stronger effect on  $R$  than the QZE or AZE, which are associated

with the rate  $\nu$ , since

$$\nu \sim \chi_a^2 \bar{I}^2 / \nu_B \ll |\Delta|. \quad (79)$$

### 3 Dynamically Modified Decay and Decoherence in Qubits Coupled to Finite-Temperature Baths

#### 3.1 Derivation of generalized Bloch equations without RWA

The Hamiltonian in question is the sum of the system ( $S$ ), reservoir bath ( $B$ ) and system-bath interaction ( $I$ ) terms,

$$H = H_S(t) + H_B + H_I(t), \quad H_I(t) = S(t)B(t). \quad (80)$$

Here  $H_S(t)$  is the driven (and modulated) system Hamiltonian,  $S(t)$  is a system operator and  $B(t)$  is a bath operator, whose choice depends on the system-bath coupling (linear or quadratic, diagonal or off-diagonal). These operators vary with time due to the external fields. This general form of  $H_I(t)$ , unlike common treatments, *does not invoke the RWA* [1], which may fail for ultrafast modulation. The combined state of the system and the bath is described by the density matrix  $\rho_{S+B}(t)$ . Let  $\rho_{S+B}(0) = \rho(0) \otimes \rho_B$ , where  $\rho = \text{Tr}_B \rho_{S+B}$  is the density matrix of the system obtained by taking the partial trace over the bath and  $\rho_B = Z^{-1} \exp[-(\beta/\hbar)H_B]$  the density matrix of the bath in equilibrium, with  $Z$  as the normalization factor,  $\beta = \hbar/k_B T$  the inverse temperature (in frequency units), and  $k_B$  the Boltzmann constant. Using the projection-operator technique, we have derived, to *second order* in the coupling, the quantum ME in the following *non-Markovian differential* form

$$\dot{\rho} = -\frac{i}{\hbar}[H_S(t), \rho] + \int_0^t dt' \{ \Phi_T(t, t') [\tilde{S}(t', t) \rho, S(t)] + \text{H.c.} \}. \quad (81)$$

Here

$$\Phi_T(t, t') = \langle U_B^\dagger(t-t') B(t) U_B(t-t') B(t') \rangle \quad (82)$$

is the bath “memory” (correlation) function (CF),  $\langle \dots \rangle = \text{Tr}_B(\dots \rho_B)$  and

$$\tilde{S}(t', t) = U_S(t, t') S(t') U_S^\dagger(t, t'), \quad (83)$$

written using the evolution operators

$$U_B(t) = \exp \left[ -\frac{i}{\hbar} H_B t \right], \quad U_S(t, t') = T_+ \exp \left[ -\frac{i}{\hbar} \int_{t'}^t H_S(\tau) d\tau \right], \quad (84)$$

$T_+$  being the time-ordering operator. In the derivation of Eq. (81) we assumed that  $\langle B(t) \rangle = 0$ . It needs to be stressed that Eq. (81) *generalizes* previously known master equations [1] to *arbitrary* time-dependent hamiltonians,  $H_S(t)$  for the system and  $H_I(t)$  for system-bath coupling.

Henceforth, we explicitly consider a driven qubit undergoing decay, whose resonant frequency and *dipolar* coupling to the reservoir are dynamically modulated, whereas  $B(t) = B$  is constant, so that

$$H_S(t) = \hbar[\omega_a + \delta_a(t) + \delta_d(t)]|e\rangle\langle e| + V(t)\sigma_x, \quad H_I(t) = S(t)B = \tilde{\epsilon}(t)\sigma_x B. \quad (85)$$

Here  $\delta_a(t)$  is the dynamically imposed Stark shift of the qubit resonance frequency,  $\delta_d(t)$  is its *random* counterpart representing proper dephasing,

$$V(t) = V_0(t)e^{-i\omega_c t} + \text{c.c.} \quad (86)$$

is the control (flipping) field with the nominal frequency  $\omega_c$ ,  $V_0(t)$  being the Rabi frequency,  $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$  is the dipole-transition operator, whose time-modulated form is given by  $\mathcal{S}(t)$ , with the *real* amplitude  $\tilde{\epsilon}(t)$ . If the bath consists of oscillators, then

$$H_B = \sum_{\lambda} \hbar\omega_{\lambda} a_{\lambda}^{\dagger} a_{\lambda}, \quad B = \hbar \sum_{\lambda} (\kappa_{\lambda} a_{\lambda} + \kappa_{\lambda}^* a_{\lambda}^{\dagger}), \quad (87)$$

where  $\omega_{\lambda}$  and  $a_{\lambda}$  are the frequency and annihilation operator, respectively, of the mode  $\lambda$  and  $\kappa_{\lambda}$  is the coupling amplitude. Clearly, terms such as  $|e\rangle\langle g|\kappa_{\lambda}^* a_{\lambda}^{\dagger}$  or  $|g\rangle\langle e|\kappa_{\lambda} a_{\lambda}$  in the system-bath interaction  $H_I(t)$  are *anti-resonant*, in violation of the RWA.

Upon using Eq. (85) in (81), we obtain our generalized Bloch equations for the components of the qubit density matrix (compare with Ref. [1])

$$\dot{\rho}_{ee} = -\dot{\rho}_{gg} = iV(t)(\rho_{eg} - \rho_{ge}) - R_e(t)\rho_{ee} + R_g(t)\rho_{gg}, \quad (88)$$

$$\begin{aligned} \dot{\rho}_{eg} = \dot{\rho}_{ge}^* = & -\{R(t) + i[\tilde{\omega}_a(t) + \delta_a(t) + \delta_d(t)]\}\rho_{eg} \\ & + iV(t)(\rho_{ee} - \rho_{gg}) + [R(t) - i\Delta_a(t)]\rho_{ge}. \end{aligned} \quad (89)$$

Equations (88) and (89) account for the presence of *upward transitions*  $|g\rangle \rightarrow |e\rangle$  (caused by either temperature or anti-resonant effects – see below) at a rate  $R_g(t)$ , in addition to *downward* decay  $|e\rangle \rightarrow |g\rangle$  at a rate  $R_e(t)$ . Their half-sum  $R(t) = [R_e(t) + R_g(t)]/2$  contributes to the decoherence rate, which is further augmented by the random shift  $\delta_d(t)$  (see below). The resonance frequency is dynamically shifted by  $\tilde{\omega}_a(t) - \omega_a = \Delta_a(t) = \Delta_e(t) - \Delta_g(t)$ , where  $\hbar\Delta_{e(g)}(t)$  is the Lamb shift of  $|e\rangle$  ( $|g\rangle$ ), caused by the dynamically modified coupling to the bath. The last term on the right-hand side of Eq. (89) is known as “non-secular” [1]; though usually negligible, it can be important if the modulated resonant frequency  $\omega_a + \delta_a(t)$  can vanish or be comparable to  $R(t) + |\Delta_a(t)|$ .

The concurrent actions of the control field  $V(t)$  (rotations around the  $x$ -axis of the Bloch sphere) and the level modulation  $\delta_a(t)$  (rotations around the  $z$ -axis of the Bloch sphere) do not commute and thus complicate the dynamics. We shall therefore investigate Eqs. (88) and (89) separately during the *storage time*, when the control field is off [ $V(t) = 0$ ], and during *gate operations*, when the modulating (off-resonant) field is off [ $\delta_a(t) = 0$ ,  $\tilde{\epsilon}(t) = 1$ ].

Consider first situations wherein  $R_{e(g)}(t)$  and  $R(t)$ , *the rates of decay and decoherence are dominant compared to the proper-dephasing rate* [determined by  $\delta_d(t)$ ], so that the latter may be neglected in Eq. (89). The dynamically affected transition rates and shifts in (88) and (89) are then obtained from Eq. (81) during the storage time, when  $V = 0$ , and found to be the real and imaginary parts of the expression

$$R_{e(g)}(t)/2 + i\Delta_{e(g)}(t) = \int_0^t dt' K_{e(g)}(t, t') \Phi_T(t - t') \quad (90)$$

Here

$$K_i(t, t') = \langle i|\mathcal{S}(t)\tilde{\mathcal{S}}(t', t)|i\rangle \quad (i = e, g) \quad (91)$$

is the correlation function (CF) of the dipole moment in the state  $|i\rangle$ . One can show that

$$K_e(t, t') = K_g^*(t, t') = \epsilon(t)\epsilon^*(t'), \quad (92)$$

where  $\epsilon(t)$  is given in (11).

### 3.2 Dynamically modified decay rates

Since we are interested here in dynamical control of relaxation, we shall concentrate on the transition rates  $R_{e(g)}(t)$  rather than the level shifts. Using Eq. (90), one obtains that the average rate of the  $|e\rangle \rightarrow |g\rangle$  transition  $R_e(t)$  and its  $|g\rangle \rightarrow |e\rangle$  counterpart  $R_g(t)$  are given by

$$R_{e(g)}(t) = 2\pi \int_{-\infty}^{\infty} d\omega F_t(\omega) G_T(\pm\omega). \quad (93)$$

Here the upper (lower) sign corresponds to the subscript  $e$  ( $g$ ),  $Q(t)$  and  $F_t(\omega)$  are given in (27) and (30), respectively. The SD of the bath CF

$$G_T(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \Phi_T(t) e^{i\omega t} dt \quad (94)$$

can be shown [30] to be nonnegative, with  $G_T(-\omega) = e^{-\beta\omega} G_T(\omega)$ , and vanish for  $\omega < 0$  at  $T = 0$ :  $G_0(\omega) = 0$  ( $\omega < 0$ ). For the oscillator bath (87) one finds that

$$G_T(\omega) = [n(\omega) + 1]G_0(\omega) + n(-\omega)G_0(-\omega), \quad (95)$$

where  $G_0(\omega) = \sum_{\lambda} |\kappa_{\lambda}|^2 \delta(\omega - \omega_{\lambda})$  and  $n(\omega) = (e^{\beta\omega} - 1)^{-1}$  is the average number of quanta in the oscillator (bath mode) with frequency  $\omega$ .

We apply Eqs. (93) to the case of *coherent modulation of quasiperiodic* form, (see Eq. (36)). Without a limitation of the generality, we can assume that  $\sum_k |\epsilon_k|^2 = 1$ . We then find, using Eq. (93), that the rates  $R_{e(g)}(t)$  tend to the long-time limits

$$R_{e(g)} = 2\pi \int_{-\infty}^{\infty} d\omega F(\omega) G_T(\pm\omega), \quad (96)$$

where

$$F(\omega) = \lim_{t \rightarrow \infty} F_t(\omega) = \sum_k |\epsilon_k|^2 \delta(\omega - \omega_a - \omega_k), \quad (97)$$

or

$$R_{e(g)} = 2\pi \sum_k |\epsilon_k|^2 G_T(\pm(\omega_a + \omega_k)). \quad (98)$$

Equation (96) shows that  $R_e$  ( $R_g$ ) is given by the overlap of the modulation spectrum  $F(\omega)$  with the bath-CF spectrum  $G_T(\omega)$  [ $G_T(-\omega)$ ]. The limits (98) are approached when  $\Omega t \gg 1$  and  $t \gg t_c \equiv \max_k \{1/\xi(\pm(\omega_a + \omega_k))\}$ . Here  $t_c$  is the bath memory (correlation) time, defined as the inverse of  $\xi(\omega)$ , the spectral interval over which  $G_T(\omega)$  changes around the relevant frequencies.

Had we used the standard dipolar RWA hamiltonian in the case of an oscillator bath, dropping the antiresonant terms in  $H_I(t)$  [Eqs. (85) and (87)], we would have arrived at the transition rates

$$R_{e(g)}^{\text{RWA}} = 2\pi \int_0^{\infty} d\omega F(\omega) G_T(\pm\omega), \quad (99)$$

wherein the integration is performed from 0 to  $\infty$ , rather than from  $-\infty$  to  $\infty$ , as in (96). This means that the RWA transition rates hold for a slow modulation, when  $F(\omega) \simeq 0$  at  $\omega < 0$ , being peaked near  $\omega_a$ . However, whenever the suppression of  $R_{e(g)}$  requires modulation at a rate comparable to  $\omega_a$ , the RWA is inadequate. For instance, Eqs. (95) and (99) imply that, at  $T = 0$ , the rate  $R_g^{\text{RWA}}$  vanishes identically, irrespective of  $F(\omega)$ , in contrast to the true upward-transition rate  $R_g$  in Eq. (96), which may be comparable to  $R_e$  for ultrafast modulation. The difference between the RWA and non-RWA decay rates stems from the fact that the RWA implies that a downward (upward) transition is accompanied by emission (absorption) of a bath quantum, whereas the non-RWA (negative-frequency) contribution to  $R_{e(g)}$  in Eq. (96) allows for just the opposite: downward (upward) transitions that are accompanied by absorption (emission). The latter processes are possible since the modulation may cause *level  $|e\rangle$  to be shifted below  $|g\rangle$* .

The validity of the (decohering) qubit model in the presence of modulation at a rate  $\gtrsim \omega_a$  is now elucidated: it requires that  $R_{e(g)jt} \ll 1$ ,  $R_{e(g)j}$  being the effective transition rate from level  $e$  ( $g$ ) to *any other* level  $j$ , and, in particular,  $R_{e(g)}t \ll 1$ . If  $R_{e(g)}$  are strongly suppressed by the modulation, the TLS model holds for long times.

### 3.3 Dynamically modified proper dephasing

We turn now to proper dephasing when it dominates over decay. *The random frequency fluctuations*  $\delta_d(t)$  in (89) are typically characterized by a (single) correlation time  $t_d$ , with ensemble mean  $\bar{\delta}_d = 0$ . When the field  $V(t)$  is used only for gate operations, we assume that it does not affect proper dephasing. The ensemble average over  $\delta_d(t)$  results in Eqs. (88) and (89) with  $\delta_d(t) = 0$  and no decay modulation [i.e.,  $\delta_a(t) = 0$  in Eq. (89) and  $K_{e(g)}(t, t') = 1$  in (90)], whereas  $R(t) \rightarrow R(t) + R_d(t)$  with the dephasing rate

$$R_d(t) = \int_0^t dt' \Phi_d(t'), \quad \Phi_d(t) = \overline{\delta_d(t)\delta_d(0)} \quad (100)$$

The dephasing CF  $\Phi_d(t)$  is the counterpart of the bath CF  $\Phi_T(t)$ .

Assuming, for simplicity, that the decay is neglected and the control field  $V(t)$  is resonant ( $\omega_c = \omega_a$ ) with real envelope  $V_0(t)$ , we derive the ME for the qubit density matrix averaged over the random fluctuations  $\delta_d(t)$ . To this end, we transform the system to the rotating frame, write the pseudospin vector in spherical coordinates,

$$Q \equiv (Q_{-1}, Q_0, Q_1) = (\rho_{ge}, (\rho_{gg} - \rho_{ee})/\sqrt{2}, -\rho_{eg}), \quad (101)$$

and tilt the frame to diagonalize the Hamiltonian of the TLS-field coupling [the last term in  $H_S(t)$ , Eq. (85)] by the transformation

$$Q_m = \sum_{m'} Q'_{m'} d_{m'm}^{(1)} \left(-\frac{\pi}{2}\right), \quad (102)$$

where  $d_{m'm}^{(1)}(-\frac{\pi}{2})$  is the finite-rotation matrix for spin 1 [48]. In the tilted frame, the master equation is

$$\dot{Q}'_{\pm 1} = \{\pm i[V_0(t) + \Delta_d(t)] - R_d(t)/2\}Q'_{\pm 1}, \quad \dot{Q}'_0 = -R_d(t)Q'_0, \quad (103)$$

where the dynamically affected decoherence rate and shift are given by the real and imaginary parts of

$$R_d(t) + 2i\Delta_d(t) = \int_0^t dt' \Phi_d(t-t') \exp\left(i \int_{t'}^t V_0(t'') dt''\right). \quad (104)$$

At  $t \gg t_d$  the decoherence rate and shift approach their asymptotic values

$$R_d = \lim_{t \rightarrow \infty} R_d(t), \quad \Delta_d = \lim_{t \rightarrow \infty} \Delta_d(t). \quad (105)$$

For the validity of Eq. (103) it is necessary that

$$R_d, |\Delta_d| \ll 1/t_d. \quad (106)$$

In Eq. (103) we have made the secular approximation, which holds if

$$V_0(t) \gg R_d, |\Delta_d|. \quad (107)$$

Equation (104) reveals the *analogy of dynamically modified dephasing to dynamically modified decay* [Eqs. (90)], both inferred from our unified treatment. By analogy with Eqs. (90)-(93), one can obtain from Eq. (104) that

$$R_d(t) = \pi \int_{-\infty}^{\infty} d\omega F_t(\omega) G_d(\omega), \quad (108)$$

where  $F_t(\omega)$  is given by (30) with

$$\epsilon(t) = \exp\left[-i \int_0^t V(t') dt'\right], \quad Q(t) = t, \quad (109)$$

and

$$G_d(\omega) = \frac{1}{\pi} \int_0^{\infty} \Phi_d(t) \cos \omega t dt. \quad (110)$$

As follows from Eq. (110),  $G_d(\omega)$  is a symmetric function,

$$G_d(-\omega) = G_d(\omega). \quad (111)$$

The *proper dephasing* rate associated with

$$\Phi_d(t) = A e^{-t/t_d} \quad (112)$$

is

$$R_d = A t_d. \quad (113)$$

In the presence of a *constant*  $V_0$  [cw  $V(t)$ ], it is modified according to Eq. (104) into

$$R_d = \frac{A t_d}{V_0^2 t_d^2 + 1}. \quad (114)$$

For a sufficiently strong field, the dephasing rate  $R_d$  can be *suppressed* by the factor  $1/(V_0 t_d)^2 \ll 1$ . This suppression reflects the ability of strong, near-resonant Rabi splitting to shift the

system out of the randomly fluctuating bandwidth, or average its effects. Quantum gate operations may be performed by *slight modulations* of the control field, which can flip the qubit without affecting proper dephasing. By comparison, the “bang-bang” (BB) method involving  $\tau$ -periodic  $\pi$ -pulses [9, 11, 16] is an analog of the above “parity kicks”. Using the analog of Eq. (96), such pulses can be shown to suppress  $R_d$  approximately according to Eq. (114) with  $V_0 = \pi/\tau$ . This BB method requires pulsed fields with Rabi frequencies  $\gg 1/\tau$ , i.e., *much stronger fields than the cw field* in our Eq. (114). Using  $t_d \sim 10^{-7}$  s, cw Rabi frequencies exceeding 1 MHz achieve a significant dephasing suppression.

## 4 Dynamical Control of Zero-Temperature Decay in Multilevel Systems

### 4.1 General formalism

Here we discuss in detail a model for dynamical decay modifications in a multilevel system. The system with energies  $\hbar\omega_n$ ,  $1 \leq n \leq N$ , is coupled to a zero-temperature bath of harmonic oscillators with frequencies  $\omega_j$ . Using the factorized coupling defined in Section 2.1, the corresponding Hamiltonian is found to be as in (1), where:

$$H_S(t) = \hbar \sum_n \omega_n |n\rangle\langle n| + \sum_{nn'} O_{nn'}(t) |n\rangle\langle n'| \quad (115)$$

$$H_B(t) = \hbar \sum_j \omega_j |j\rangle\langle j| \quad (116)$$

$$H_I(t) = \sum_{n,j} \epsilon_n(t) \mu_{jn} |n\rangle\langle j| + \text{h.c.} \quad (117)$$

where now each level has a different modulation  $\epsilon_n(t)$  and a *different coupling to the bath*  $\mu_{jn}$ , and  $O_{nn'}(t)$  denotes a gate operation.

The system evolution is divided into two phases, one of storage without gate operations and a gate operation of finite duration:

$$O_{nn'}(t) = \begin{cases} g_{nn'}, & t' \leq t \leq t' + \tau \\ 0, & \text{elsewhere} \end{cases} \quad (118)$$

The full wave function is given by:

$$|\Psi(t)\rangle = \sum_n \alpha_n(t) e^{-i\omega_n t} |n\rangle + \sum_j \beta_j(t) e^{-i\omega_j t} |j\rangle \quad (119)$$

Similarly to what was said in Section 2.1, one can consider two types of situations. The above Eqs. (115)-(119) were written for an  $N$ -level system which can exchange its population with the reservoir. In addition, one can consider an  $(N+1)$ -level system, where transitions are possible between any level  $|n\rangle$  and a lower level  $|g\rangle$ , the reservoir consisting of quantum systems, as described in Section 2.1. The theory in Section 4 holds for the both situations, with the minor difference that one should substitute [cf. (7)]

$$|n\rangle \rightarrow |n\rangle|\text{vac}\rangle, \quad |j\rangle \rightarrow |g\rangle|j\rangle \quad (120)$$

in Eqs. (117) and (119) and perform a similar substitution in Eq. (123) below.

In order to find the solution, one has to diagonalize the system hamiltonian by introducing a matrix that rotates the amplitudes  $\alpha(t) = \{\alpha_n(t)\}$ :

$$\alpha(t) = \Theta(t) \tilde{\alpha}(t), \quad (121)$$

such that, by defining  $\Omega_{nn'}(t) = \omega_n \delta_{nn'} + O_{nn'}(t)$ , one gets

$$\tilde{\Omega}_{nn'}(t) = (\Theta^{-1}(t)\mathbf{\Omega}\Theta(t))_{nn'} = \tilde{\omega}_n(t)\delta_{nn'} \quad (122)$$

where  $\tilde{\omega}_n(t)$  are the eigenvalues of the new rotated system. Thus the transformed wave function becomes:

$$|\Psi(t)\rangle = \sum_{\tilde{n}} \tilde{\alpha}_n(t) e^{-i \int_0^t \tilde{\omega}_n(\tau) d\tau} |\tilde{n}\rangle + \sum_j \beta_j(t) e^{-i\omega_j t} |j\rangle \quad (123)$$

Using for these rotated state amplitudes, a procedure similar to that used for one level, one finds that they obey the following integro-differential equations, assuming slowly varying  $\alpha_n(t)$ :

$$\dot{\tilde{\alpha}}(t) = -[\mathbf{f}(t) + \mathbf{q}(t)]\tilde{\alpha}. \quad (124)$$

Here the  $\mathbf{f}(t)$  and  $\mathbf{q}(t)$  matrices are given by:

$$\begin{aligned} f_{nn''}(t) &= it\dot{\tilde{\omega}}_n(t)\delta_{nn''} + \sum_{n'} [\Theta^{-1}(t)]_{nn'} \dot{\Theta}_{n'n''}(t) e^{i \int_0^t [\tilde{\omega}_n(\tau) - \tilde{\omega}_{n''}(\tau)] d\tau}, \\ q_{nn''}(t) &= \sum_{n'} \int_0^t dt' K_{nn'}(t, t') \Phi_{nn'}(t - t') e^{i \int_0^t \tilde{\omega}_n(\tau) d\tau - i \int_0^{t'} \tilde{\omega}_{n''}(\tau) d\tau} \Theta_{n'n''}(t'), \end{aligned} \quad (125)$$

$\mathbf{K}(t, t')$  and  $\mathbf{\Phi}(t)$  being the modulation and reservoir-response matrices, respectively, given by:

$$K_{nn'}(t, t') = \epsilon_n^*(t) \epsilon_{n'}(t'), \quad (126)$$

$$\Phi_{nn'}(t) = \int d\omega G_{nn'}(\omega) e^{-i\omega t}, \quad (127)$$

where

$$G_{nn'}(\omega) = \hbar^{-2} \sum_j \mu_{nj}^* \mu_{n'j} \delta(\omega - \omega_j). \quad (128)$$

During the storage phase, one has  $\Theta_{nn'}(t) = \delta_{nn'}$ ,  $\tilde{\omega}_n(t) = \omega_n$  and  $\tilde{g}_{nn''}(t) = \delta_{nn''}$ , while during the gate-operation phase,  $\Theta_{nn'}(t) = \Theta_{nn'}$ ,  $\tilde{\omega}_n(t) = \tilde{\omega}_n$  and  $\tilde{g}_{nn''}(t) = g_{nn''}$ .

The solution to (124) is of the form:

$$\tilde{\alpha}(t) = T_+ e^{-\int_0^t [\mathbf{f}(t') + \mathbf{q}(t')] dt'} \tilde{\alpha}(0). \quad (129)$$

To simplify the analysis, one can define the fluence and the modulation spectral matrices, analogously to (27), (30)

$$Q_{nn'}(t) = \int_0^t d\tau \epsilon_n^*(\tau) \epsilon_{n'}(\tau), \quad (130)$$

$$F_{nn',t}(\omega) = \frac{\epsilon_{i,n}^*(\omega - \omega_n) \epsilon_{i,n'}(\omega - \omega_{n'})}{Q_{nn'}(t)}. \quad (131)$$

The relevant imaginary parts of the spectral response of the reservoir can be expressed, analogously to (24), (25), by the Kramers-Kronig relations

$$\tilde{\Phi}_{nn'}(\omega) = \pi G_{nn'}(\omega) - i\chi_{nn'}(\omega), \quad (132)$$

$$\chi_{nn'}(\omega) = \mathcal{P} \int \frac{G_{nn'}(\omega') d\omega'}{\omega' - \omega}. \quad (133)$$

Defining

$$\mathbf{J}(t) = \int_0^t dt' \mathbf{q}(t'), \quad (134)$$

we shall now represent  $J_{nn'}(t)$  in different regimes (phases):

(i) As a reference, it is important to consider the decoherence effects with no modulations at all, i.e.  $F_{nn',t}(\omega) = \delta(\omega - \omega_n)\delta(\omega_n - \omega_{n'})$ . In this case one obtains a *diagonal* decoherence matrix:

$$J_{nn'}(t) = t\delta_{nn'}\tilde{\Phi}_{nn'}(\omega_n). \quad (135)$$

This means that interference of decaying levels  $n$  and  $n'$  cancel out in the long time limit, and the decoherence is without cross-relaxation.

(ii) During the storage phase, (134) results in:

$$J_{nn'}(t) = Q_{nn'}(t) \int_{-\infty}^{\infty} d\omega F_{nn',t}(\omega) \tilde{\Phi}_{nn'}(\omega). \quad (136)$$

One can easily see that for the off-diagonal terms, a *simple separation into decay rates and energy shifts is inapplicable* in this formulation.

(iii) During gate operations, (134) assumes the form:

$$J_{nn'}(t) = \sum_{n''} Q_{nn''}(t) \Theta_{n''n'} \int_{-\infty}^{\infty} d\omega F_{nn'',t}(\omega) \tilde{\Phi}_{nn''}(\omega) \quad (137)$$

In a more compact and enlightening form, one can rewrite this equation as  $\mathbf{J}^{gate} = \mathbf{J}^{storage} \times \Theta$ , where  $\mathbf{J}^{storage}$  is given in Eq. 136

#### 4.2 Quasiperiodic local and global modulation

Using the quasiperiodic modulation, as in Section 2.2, but with possibly *different modulations for each level*, we define:

$$\epsilon_n(t) = \sum_k \epsilon_{n,k} e^{-i\nu_{n,k}t} \quad (138)$$

where  $\nu_{n,k}$  ( $k = 0, \pm 1, \dots$ ) are arbitrary discrete frequencies with minimal spectral interval  $\Omega_n$ . The fluence and modulation spectral matrices for this type of modulation are given by:

$$Q_{nn'}(t) = \epsilon_{c,nn'}^2 t + \epsilon_{c,nn'}^2 \sum_{\nu_{n,k} \neq \nu_{n',l}} \lambda_{n,k}^* \lambda_{n',l} \frac{e^{i(\nu_{n,k} - \nu_{n',l})t} - 1}{i(\nu_{n,k} - \nu_{n',l})} \quad (139)$$

$$F_{nn',t}(\omega) = \frac{\epsilon_{c,nn'}^2}{2\pi} \sum_{k,l} \lambda_{n,k}^* \lambda_{n',l} \left( \frac{1 - e^{-i\eta_{n,k}t}}{\eta_{n,k}} \right) \left( \frac{1 - e^{i\eta_{n',l}t}}{\eta_{n',l}} \right) \quad (140)$$

where  $\eta_{n,k} = \omega - \gamma_{n,k}$ ,  $\gamma_{n,k} = \nu_{n,k} + \omega_n$  and

$$\epsilon_{c,nn'}^2 = \sum_{kl} \delta_{\nu_{n,k}, \nu_{n',l}} \epsilon_{n,k}^* \epsilon_{n',l} \quad (141)$$

For long times,  $t \gg \Omega_n^{-1} \forall n$ , we find that the ratio between the fast oscillating terms of different modulation frequencies ( $\gamma_{n,k} \neq \gamma_{n',l}$ ) and for similar modulation frequencies is  $(\Omega_n t)^{-1} \ll 1$ . Also  $Q_{nn'}(t) = \delta_{\nu_{n,k}, \nu_{n',l}} \epsilon_{c,nn'}^2 t$ . Thus the modulation spectral matrix becomes:

$$F_{nn',t}(\omega) \approx \sum_{k,l} \delta_{\gamma_{n,k}, \gamma_{n',l}} \lambda_{n,k}^* \lambda_{n',l} S(\eta_{n,k}, t) \quad (142)$$

For a sufficiently long time, the storage-phase decoherence matrix becomes:

$$J_{nn'}(t) = \epsilon_{c,nn'}^2 t \sum_{k,l} \delta_{\gamma_{n,k}, \gamma_{n',l}} \lambda_{n,k}^* \lambda_{n',l} \tilde{\Phi}_{nn'}(\gamma_{n,k}) \quad (143)$$

Two special cases need to be considered. The first one is the case of *global modulations*, i.e.  $\gamma_{n,k} = \gamma_k$ . One can easily see that (143) becomes a single sum of  $k$ , but retains its off diagonal elements:

$$J_{nn'}^{global}(t) = \epsilon_{c,nn'}^2 t \sum_k \lambda_{n,k}^* \lambda_{n',k} \tilde{\Phi}_{nn'}(\gamma_k) \quad (144)$$

The second case is that of *local modulations*, where  $\gamma_{n,k} \neq \gamma_{n',l}$  ( $\forall n, n', k, l$ ). This results in a diagonalization of the decoherence matrix:

$$J_{nn'}^{local}(t) = \delta_{nn'} (R_n/2 + i\Delta_n(t)) \epsilon_c^2 t, \quad (145)$$

where

$$R_n = 2\pi \sum_k |\lambda_{n,k}|^2 G_{nn}(\nu_{n,k} + \omega_n), \quad (146)$$

$$\Delta_n = - \sum_k |\lambda_{n,k}|^2 \chi_{nn}(\nu_{n,k} + \omega_n). \quad (147)$$

Thus, by using different (local) modulations on each level, in the long-time limit, one eliminates the off-diagonal, or cross-level, decoherence.

Using local modulation, one can achieve another feat. By selecting specific local modulations, one can create *symmetrization of the levels*, which means selecting  $\epsilon_{n,k}, \nu_{n,k}$  such that

$$R_n = R, \quad \Delta_n = \Delta. \quad (148)$$

Thus state amplitudes decohere with the same decay rate and the same energy shift. By encoding the qubit information into the system levels, the relative decoherence can be reduced or even eliminated. This symmetrization is valuable when using amplitude error-correction codes. [27]

#### 4.2.1 *Example: Global and local impulsive phase modulations of systems coupled to Gaussian baths*

In order to compare global and local impulsive phase modulation, one should note (Section 2.3.2) that the main relevant parameter that governs the reduction of the decoherence is the ratio of the phase jump  $\phi$  to the interval between consecutive impulses  $\tau$ . Hence, one can conjecture that a global modulation with the optimal  $\phi/\tau$  will be the best modulation. However, as discussed in Section 4.2, different modulation for each level (e.g., choosing different  $\phi_n$  for a given  $\tau$ ) can *symmetrize* the decoherence, and thereby allow efficient application of amplitude-error correction [27].

In this example we use a Gaussian spectral response matrix of the form:

$$G_{nn'}(\omega) = c_{nn'} d_n^* d_{n'} e^{-\omega^2/2\Gamma^2} \quad (149)$$

where  $d_n = \cos \theta_n$  being the angle of transition dipole,  $c_{nn'}$  being a constant coupling matrix and  $\Gamma$  is the width of the Gaussian. Here, the system states are equidistant,  $\omega_n = \omega_1 + (n-1)\Delta$ .

In Fig.3 one can observe the symmetrization of  $J_{nn'}(t)$  as a function of time. The system has  $N = 4$  levels. By choosing  $\phi_n$  such that the long-time limit of  $J_{nn}$  is the same for all levels, one achieves the elimination of the off-diagonal terms by different modulations, and the symmetrization of the diagonal elements.

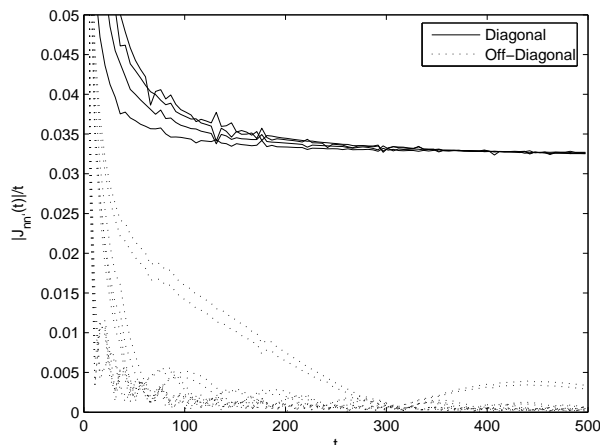


Fig. 3. Decoherence matrix elements as a function of time. Here  $\Gamma = 1.5$ ,  $\Delta = 0.1$ ,  $\theta_n = \pi(n-1)/10$  and  $c_{nn} = 1.0$ ,  $c_{nn'} = 0.5$ . The modulation interval time  $\tau = 1/\Gamma$ . The solid lines denote the 4 diagonal elements of  $J$ , while the dashed lines are the off-diagonal ones.

The fidelity, defined as

$$f(t) = \text{Tr} \left[ |\psi(t)\rangle\langle\psi(t)| |\psi(0)\rangle\langle\psi(0)| \right], \quad (150)$$

is shown in Fig. 4(a) as a function of the energy  $\hbar\phi/\tau$  invested in the impulse phase modulation. For global modulation,  $\phi_n = \pi$  is the optimized modulation phase for each level coupled to a Gaussian bath, whereas for local modulation, the phase modulation for each level  $\phi_n$  at a given  $\tau$  is found such that symmetry is achieved. The  $x$ -axis units are those of the mean  $\phi_n/\tau$ . As can be seen, one loses some fidelity by not using the optimized modulation for each level, but gains a symmetrization of the system, so that amplitude error-correction codes can now be used more efficiently.

Fig. 4(b) shows the normalized fidelity  $f/f_{\text{ref}}$ , where  $f_{\text{ref}}$  is calculated for decay without modulation. As can be seen, even for modulation with long periods  $\tau$ , a large increase in fidelity is achieved compared to the unmodulated system.

## 5 The strong-coupling regime: decoherence control near continuum edge by nonadiabatic interference

The analysis expounded thus far has been based on a *perturbative* treatment of the system-bath coupling. Here we address the regime of *strong system-bath coupling*, as in the case of a resonance frequency very near to the continuum edge, a situation that may be encountered in atomic excitation near the ionization energy, vibrational excitation frequency in a solid near the Debye cutoff, or an atomic excitation in a photonic crystal near a photonic bandgap. In the strong-coupling regime it is advantageous to work in the combined basis of the system

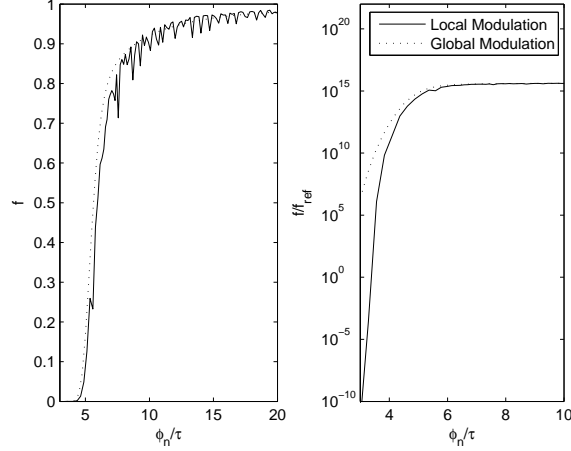


Fig. 4. Fidelity as a function of energy invested in impulse phase modulation. For global modulation  $\phi = \pi$  whereas for local modulation for each  $\tau$  the modulation phase  $\phi_n$  is chosen such that symmetry is achieved. Here  $t = 100$ . (a) Fidelity as a function of mean  $\phi_n/\tau$ . (b) Normalized fidelity, where  $f_{\text{ref}}(t)$  is calculated without modulation.

(qubit) and field (bath) states that incorporate the system-bath interaction. Dynamical control of the decay can then be analysed by exact solution of the Schrodinger equation in this basis. Analytical expressions are obtainable for alternating static evolutions with different parameters (e.g., resonant frequency), the dynamical control resulting from their *interference*. Specifically, we shall consider optical manipulations of atoms embedded in photonic crystals with atomic transition frequencies near a photonic bandgap (PBG), *i.e.* near the edge of the photonic mode continuum, where the qubit *is strongly coupled* to the continuum, and spontaneous emission (SE) is only partially blocked, because an initially excited atom then evolves into a *superposition of decaying and stable states*, the stable state representing photon-atom binding [49, 50]. In what follows we shall demonstrate the ability of appropriately alternating sudden changes of the detuning to *augment the interference* of the emitted and back-scattered photon amplitudes, thereby increasing the probability amplitude of the stable (photon-atom bound) state. As a result, phase-gate operations effected by dipole-dipole interactions can be performed with higher fidelity than in the case of adiabatic frequency change.

### 5.1 Hamiltonian and equations of motion

We consider a two-level atom with excited and ground states  $|e\rangle$  and  $|g\rangle$  coupled to the field of a discrete (or defect) mode and to the photonic band structure (PBS) in a photonic crystal. The hamiltonian of the system in the rotating-wave approximation assumes the form [50]

$$\begin{aligned}
 H = & \hbar\omega_{at}|e\rangle\langle e| + \hbar \int_0^{+\infty} \omega a_\omega^\dagger a_\omega \rho(\omega) d\omega + \hbar \left( \kappa_d^* a_d^\dagger |g\rangle\langle e| + h.c. \right) \\
 & + \hbar \int_0^{+\infty} [\kappa^*(\omega) a_\omega^\dagger |g\rangle\langle e| + h.c.] \rho(\omega) d\omega.
 \end{aligned} \tag{151}$$

Here  $\hbar\omega_{at}$  is the energy of the atomic transition frequency,  $a_\omega^\dagger$  and  $a_\omega$  are, respectively, the creation and annihilation operators of the field mode at frequency  $\omega$ ,  $\rho(\omega)$  is the mode density of the PBS,  $\kappa(\omega)$  and  $\kappa_d$  are the coupling rates to the atomic dipole of a mode from the continuum and the discrete mode, respectively.

Let us first consider the initial state obtained by absorbing a photon from the discrete mode:

$$|\Psi(0)\rangle = |e, \{0_\omega\}\rangle, \quad (152)$$

where  $|\{0_\omega\}\rangle$  is the vacuum state of the field. Then the evolution of the wavefunction  $|\Psi(t)\rangle$  has the general form

$$|\Psi(t)\rangle = \alpha(t) |e, \{0_\omega\}\rangle + \beta_d(t) |g, 1_d\rangle + \int_0^{+\infty} \beta_\omega(t) |g, 1_\omega\rangle \rho(\omega) d\omega \quad (153)$$

where we have denoted by  $|1_\omega\rangle$  and  $|1_d\rangle$  the single-photon state of the relevant modes. The Schrödinger equation  $i\dot{\Psi}(t) = H \Psi(t)$  then leads to the set of coupled differential equations

$$\begin{aligned} \dot{\alpha}(t) &= -i\omega_{at}\alpha(t) - i\kappa_d\beta_d(t) - i\int_0^{+\infty} \kappa(\omega)\beta_\omega(t)\rho(\omega) d\omega, \\ \dot{\beta}_d(t) &= -i\omega_d\beta_d(t) - i\kappa_d^*\alpha(t), \\ \dot{\beta}_\omega(t) &= -i\omega\beta_\omega(t) - i\kappa^*(\omega)\alpha(t). \end{aligned} \quad (154)$$

This evolution reflects the interplay between the off-resonant Rabi oscillations of  $|e, \{0_\omega\}\rangle$  and  $|g, 1_d\rangle$ , at the driving rate  $\kappa_d$ , and the *partly-inhibited oscillatory decay* from  $|e, \{0_\omega\}\rangle$  to  $|g, \{1_\omega\}\rangle$  via coupling to the continuum  $\rho(\omega)$ . This decay depends on the detuning of  $\omega_{at}$  from the continuum edge at  $\omega_U$  (the upper cutoff of the PBG - Fig.5). For a spectrally steep edge (see below), we are in the regime of *strong coupling* to the mode continuum (as in a high-Q cavity [40]) which allows for the existence of an oscillatory, non-decaying, component of  $\alpha(t)$ , associated with a photon-atom bound state [39, 50].

## 5.2 Periodic sudden changes of the detuning

Let us now introduce abrupt changes of  $\omega_{at}$ , *i.e.*, of the detuning  $\Delta_{at} = \omega_U - \omega_{at}$  from the upper cutoff,  $\omega_U$ , of the PBG (by fast AC-Stark modulations as discussed below), at intervals  $\tau$ . In the sudden-change approximation for  $\omega_{at}$ , the amplitudes  $(\alpha_{dyn}(t), \beta_{dyn}, \{\beta_{\omega_{dyn}}(t)\})$  of the excited state, the discrete mode and the continuum still evolve according to Eqs. (154), except that from  $t = 0$  to  $t = \tau$  the atomic transition frequency is  $\omega_{at} = \omega_A$ , *i.e.*, the detuning  $\Delta_{at} = \omega_U - \omega_A = \Delta_A$ , while for  $t > \tau$ , we have  $\omega_{at} = \omega_B$ , *i.e.*,  $\Delta_{at} = \Delta_B$ . This dynamics leads to the relation

$$\begin{aligned} \alpha_{dyn}(t) &= \alpha_A(t), \quad \beta_{dyn}(t) = \beta_{d,A}(t), \\ \beta_{\omega_{dyn}}(t) &= \beta_{\omega,A}(t), \quad (t \leq \tau); \\ \alpha_{dyn}(t) &= \alpha_B^{(s)}(t), \quad \beta_{dyn}(t) = \beta_{d,B}^{(s)}(t), \\ \beta_{\omega_{dyn}}(t) &= \beta_{\omega,B}^{(s)}(t), \quad (t > \tau). \end{aligned} \quad (155)$$

Here both  $(\alpha_A(t), \beta_{d,A}(t), \{\beta_{\omega,A}(t)\})$  and  $(\alpha_B^{(s)}(t), \beta_{d,B}^{(s)}(t), \{\beta_{\omega,B}^{(s)}(t)\})$  are solutions of Eqs. (154) with a static (fixed) atomic transition frequency,  $\omega_A$  or  $\omega_B$ . However, the initial condition at the instant  $t = \tau$  of the frequency change from  $\Delta_A$  to  $\Delta_B$  is no longer the excited state (152) but the superposition:

$$|\Psi(\tau)\rangle = \alpha_A(\tau) |e, \{0_\omega\}\rangle + \beta_{d,A}(\tau) |g, 1_d\rangle + \int_0^{+\infty} \beta_{\omega,A}(\tau) |g, 1_\omega\rangle \rho(\omega) d\omega. \quad (156)$$

In other words, the dynamics is equivalent to two successive static evolutions, the second one starting from initial conditions  $(\alpha_A(\tau), \beta_{d,A}(\tau), \{\beta_{\omega,A}(\tau)\})$ .

Using the Laplace transform of the system (154) with the initial condition (156), it is possible to express the dynamic amplitude of the excited state after the sudden change as

$$\alpha_{dyn}(t) = \alpha_A(\tau)\alpha_B(t-\tau) + \beta_{d,A}(\tau)\beta_{d,B}(t-\tau) + \int_0^{+\infty} \beta_{\omega,A}(\tau)\beta_{\omega,B}(t-\tau)\rho(\omega)d\omega, \quad (t > \tau), \quad (157)$$

where we have used the initial conditions  $(\alpha_A(\tau), \beta_{d,A}(\tau), \{\beta_{\omega,A}(\tau)\})$  and the solution  $(\alpha_B(t), \beta_{d,B}(t), \{\beta_{\omega,B}(t)\})$  of Eqs. (154) for the initial condition (152).

There is an advantageous feature to the sudden change: since the time dependence of  $\alpha_{dyn}(t)$  in (157) arises from the static amplitudes  $\alpha_B, \beta_{d,B}$  and  $\beta_{\omega,B}$  at the *shifted* time  $t-\tau$ , a consequence of the sudden change is to revive the excited-state population oscillations, which tend to disappear at long times in the static case. Hence, by applying several *successive* sudden changes, we should be able to maintain large-amplitude oscillations of the *coherence* between  $|e\rangle$  and  $|g\rangle$ . The scenario leading to the largest amplitude consists in *periodic* shifts of the energy detuning from  $\Delta_A$  to  $\Delta_B$ . When the initial detuning  $\Delta_A$  is large and we first reduce it to  $\Delta_B$  before it increases to  $\Delta_A$ , the *dynamic population and the  $|e\rangle - |g\rangle$  coherence*, thanks to the revival of oscillations, are *periodically larger* than the static ones (!) (Fig. 5). This remarkable result comes unexpectedly: it implies that successive abrupt changes can *reverse* the decay to the continuum, even though they *cannot be associated with the Zeno effect*: they occur at intervals much longer than the correlation (Zeno) time of the radiative continuum, which is utterly negligible ( $10^{-18}$  s) [17], or even longer than the static-oscillation half-period. The fact that this happens only for the rather "counter-intuitive" ordering of detuning values (from large to small then back again) is a manifestation of interference between successive static evolutions: their relative phases determine the beating between the emitted and reabsorbed (back-scattered) photon amplitudes and thereby the oscillation of  $\alpha_{dyn}(t)$ .

Let us now consider the initial superposition

$$|\Psi(0)\rangle = \alpha(0) |e, \{0_\omega\}\rangle + \beta_d(0) |g, 1_{\omega_d}\rangle \quad (158)$$

and a non-negligible coupling constant  $\kappa_d$ . In this case, the periodic dynamic population of the excited state also strongly exceeds the static one. Most importantly, the *instantaneous dynamic fidelity*  $|\langle\Psi(0)|\Psi(t)\rangle|^2$  is periodically enhanced as compared to the static one, as demonstrated numerically (dot-dashed line in Fig. 6).

In order to use these results for quantum logic gates, let us consider the example of the dipole-dipole induced control-phase gate, which consists in shifting the phase of the target-qubit excited state by  $\pi$  via interaction with the control qubit [42, 43, 44]. The phase shift must be accumulated gradually, to preserve the coherence of the system. We have found that ten or twenty sudden shifts of  $\pi/10$  or  $\pi/20$ , respectively, alternating with appropriate detuning changes, can keep the fidelity high, with little decoherence. The system begins to evolve following the "counter-intuitive" detuning sequence discussed above (not to be confused with the adiabatic STIRAP method [45, 46, 47]!). As soon as two sudden changes of the detuning have been performed, the conditional phase shift of  $\pi/10$  or  $\pi/20$  takes place and the process is further repeated. The total gate operation is completed within the time-interval of maximum fidelity. Figure 6, showing the fidelity of the system relative to its initial state

during the realization of a control phase gate, with alternating detunings, is perhaps our most impressive finding. We can see that the fidelity is increased using the "counterintuitive" sequence of detunings (solid line) as compared to the static (fixed) choice of maximal detuning (long-dashed line), or compared to the dynamically enhanced fidelity  $|\langle \Psi(0) | \Psi(t) \rangle|^2$  obtained without gate operations (dot-dashed line).

### 5.3 Approximations and numerical solutions

We proceed to discuss our numerical solutions and the approximations therein.

A) In order to solve the system (154) the continuum is discretized, the coupling  $\kappa(\omega)$  is approximated to be constant (independent of  $\omega$ ) and real, and an analytic expression for the density of modes is used, based on the scalar, isotropic approximation which is known to be adequate [50] in photonic crystals (see below):

$$\rho(\omega) \propto \frac{(\omega - \omega_U)^{\frac{1}{2}} \theta(\omega - \omega_U)}{\omega - \omega_U + \epsilon} \quad (159)$$

where  $\theta$  is the step function and the value of  $\epsilon$  determines the edge steepness.

B) All parameters and variables in Fig. 5, 6 are scaled to the effective coupling  $\gamma_c = \kappa^2 / \sqrt{\epsilon}$ , which again depends on the edge steepness  $\epsilon$ . The modification of these results to allow for the *anisotropy* of the density of modes in a realistic photonic crystal has also been undertaken by us, yielding *qualitatively similar* results [61, 62].

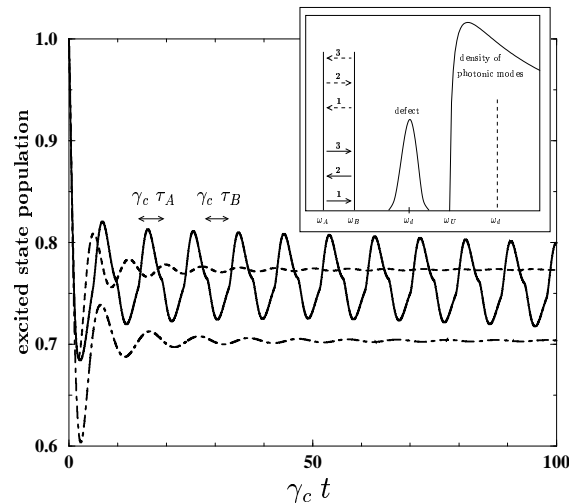


Fig. 5. Excited state population as a function of dimensionless time  $\gamma_c t$ . Dashed line: static detuning  $\Delta_A/\gamma_c = 0.5$ . Dot-dashed line: static detuning  $\Delta_B/\gamma_c = 0.25$ . Solid line: periodic sudden shifts of the detuning between  $\Delta_A$  and  $\Delta_B$ , starting with detuning  $\Delta_A$  (solid arrows in inset). Frequencies  $\omega_A$  and  $\omega_B$  are in the vicinity of a discrete mode  $\omega_d$  and of a PBG edge  $\omega_U$  (inset)

C) Since the sudden change approximation is not realizable experimentally, we have considered the effects of finite transition times between  $\Delta_A$  and  $\Delta_B$ , by using a sum of pulses that vary as  $\exp[-(t - \tau_n)^8/L^8]$ , *i.e.*, are centered on  $\tau_n$  and have half-width  $L$  of the order of

$\tau_A/4 \sim \tau_B/4$ . The excited state population is only slightly modified by such finite rise- and falloff-times.

#### 5.4 Comparison with the weak-coupling regime

We have compared the results of this method, which allows for *possibly strong coupling* of  $|e\rangle$  with the continuum edge, with those of the universal formula of Section 2 [Eq. (28)], which expresses the decay rate of  $\alpha(t)$  by the convolution of the modulation spectrum and the PBS coupling spectrum. We find good agreement with this formula only in the regime of *weak coupling* to the PBG edge, when the dimensionless detuning parameter  $\Delta_{at}/\gamma_c > 5$ , as expected from the limitations of the theory in Section 2.

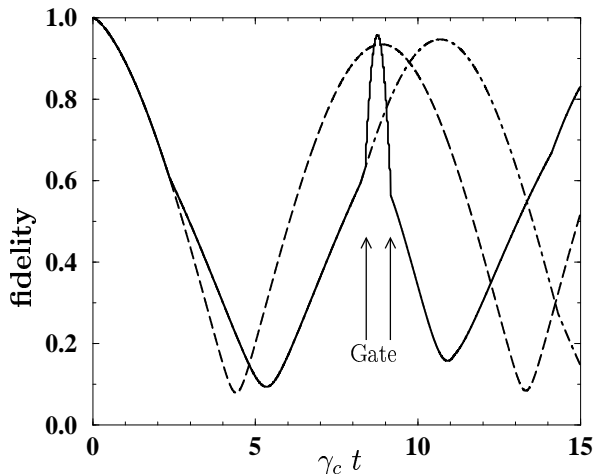


Fig. 6. Fidelity of a superposition state (158) as a function of the dimensionless time  $\gamma_c t$ . Long-dashed line: static detuning  $\Delta_A/\gamma_c = 0.5$ . Dot-dashed line: periodic sudden shifts of the detuning from  $\Delta_A/\gamma_c = 0.5$  to  $\Delta_B/\gamma_c = 0.25$ . Solid line: periodic shifts alternating with control phase shifts. Arrows mark the start and the end of the control-phase operation.

#### 5.5 Experimental scenario

The following experimental scenario may be envisioned for demonstrating the proposed effect: pairs of qubits are realizable by two species of active rare-earth dopants [63, 64] or quantum dots in a photonic crystal. The transition frequency of one species is initially detuned by  $\Delta_{at} \sim 1 \text{ MHz}$  from the PBG edge with coupling constant  $\gamma_c \sim 10 \text{ MHz}$ , and by  $\sim 3 \text{ MHz}$  from the resonance of the other species. This  $\Delta_{at}$  is abruptly modulated by *fsec* non-resonant laser pulses which exert  $\sim 3 \text{ MHz}$  AC Stark shifts. Between successive shifts, the qubits are near-resonant with their neighbours and therefore become dipole-dipole coupled, thus effecting the high-fidelity phase-control gate operation [42, 43, 44], as in Fig. 6. The required pulse rate is  $\gamma_c/10 \sim 1 \text{ MHz}$ , much lower than the pulse rate stipulated under similar conditions by previously proposed strategies [8, 19, 6].

## 6 Conclusions

In this paper we have expounded our comprehensive approach to the dynamical control of decay and decoherence. Our analysis of dynamically modified coupling between a qubit and a bath has resulted in the universal formula (28) for the dynamically modified decay rate into a zero-temperature bath, as well as its counterparts (96) for excited- and ground-state dynamical decay into finite-temperature baths. This ground-state dynamically induced decay results from RWA violation by ultrafast modulation. For multilevel systems with partly interfering decaying levels (Section 4), the merits of *global* (common to all levels) versus *local* (specific to each level) modulations have been compared.

The previous ground-breaking works on dynamical decoupling [6, 7, 8, 9, 10, 11, 12, 13, 14] are more restrictive than the universal approach detailed in Sections 2-4 of this paper:

(i) In most works [9, 10, 11, 12, 13, 14] the coupling between the system and the bath had only proper dephasing elements, i.e.,  $H_I = \hbar\sigma_z \sum_\lambda (\kappa_\lambda a_\lambda + \kappa_\lambda^* a_\lambda^\dagger)$ . This coupling means that there is no population decay to the bath and the decoherence is only via the phase. In other pioneering works [6, 7, 8], off-diagonal coupling to a Lorentzian bath was considered within the rotating-wave approximation (RWA). The Hamiltonians in all of these works should be contrasted with the more general Hamiltonians, (1) and (80), which account for both dephasing and decay into arbitrary baths, without the RWA.

(ii) The “parity kicks” [6, 7] or “bang-bang” (BB) pulses have usually been considered [8, 9, 10, 11, 12, 13, 14] to be so strong and fast that during the pulses there is no coupling to the bath, while between the pulses there is natural evolution of the system coupled to the bath. By contrast, the approach outlined in the present paper allows for modulations that are effected *during* the coupling to the bath.

(iii) The BB pulses [9] that have been previously proposed flip the phase of the system state by  $\pi$ , so that to achieve suppression of the decoherence. In particular in Ref. [13] it was shown that the use of BB pulses results in suppression of the low-frequency modes of the spectral density, thus effectively suppressing the decoherence due to  $1/f$ -type spectral densities. The BB pulses formalism then yields for  $R_d(t_{2N})t_{2N}$  the same expression [Ref. [10], Eq. (4)] as our universal approach, if one chooses in our Eq. (108) the specific form [10]  $G_d(\omega) = \coth(\beta\omega/2)G(\omega)$  and adopts

$$F_{t_{2N}}(\omega) = 2 \tan^2(\omega\tau/2) \frac{1 - \cos \omega t_{2N}}{\pi\omega^2 t_{2N}}, \quad (160)$$

which coincides with our result in Eq. (52) upon setting  $\phi = \pi$  and  $n = 2N$ ,  $t_{2N} = 2N\tau$ .

(iv) As shown in Section 3.3, a cw field may be more effective (much less energy consuming) in suppressing proper dephasing as BB pulses.

(v) The pioneering results [6, 7, 8] on dynamical suppression of decay into a Lorentzian or Gaussian bath coincide with ours, advocating  $\phi = \pi$  phase flips as the most appropriate [Eq. (59)]. However, our more general results (55) and (98) identify small periodic phase shifts  $\phi \ll 1$  as being far more suitable near the Debye cutoff frequency of a phonon bath.

(vi) Our innovative treatment of *multilevel decay via coupling to partly correlated* baths in Section 4 gives a flavor for the pros and cons of decay suppression via local versus *global* phase modulations.

(vii) The ground-breaking theory and experiment [26] concerning decay to the continuum in a periodically tilted washboard potential coincide with our results on amplitude modulation

(Section 2.4), but our treatment also covers other parameter ranges, suitable for modulated Josephson junctions [20].

In Section 5, we have outlined a method complementary to the one in Sections 2-4. We have demonstrated that a "counterintuitively" ordered sequence of abrupt changes of the detuning between an atomic transition and a continuum edge (the photonic band cutoff), is able to protect the atomic state from spontaneous emission more effectively than the intuitively obvious alternative, which is to fix the resonance frequency well within the forbidden bandgap, as far as possible from the continuum edge. This method is effective even under conditions of *strong coupling to the continuum*, since it is based on phase-dependent changes of the atomic state that is "dressed" by the continuum, rather than on modulation of the "bare" state that is weakly perturbed by the bath [6, 8, 19], as opposed to all previously proposed dynamical modulation strategies [17, 6, 8, 19], including that of Sections 2-4. The ability to maintain high fidelity of quantum states and quantum-logic operations in the presence of decoherence by nonadiabatic interference, contrary to the prevailing adiabatic approach to quantum-state control [32, 2, 45, 46, 47], may pave the way to new methods of controlling decay and decoherence in spectrally structured continua [28, 12, 16, 14].

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