Nonlinear Evolution of Spatio-Temporal Structures in Dissipative Continuous Systems

Edited by

F. H. Busse and L. Kramer

University of Bayreuth
Bayreuth, Federal Republic of Germany

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ON THE SCALE-INVARIANT THEORY OF DEVELOPED HYDRODYNAMIC TURBULENCE KOLMOGOROV TYPE

Victor S. L'vov

Institute of Automation and Electrometry
Siberian Branch of Academy of Science
630090 Novosibirsk, U.S.S.R

The modern statistical theory of hydrodynamic turbulence goes back to the papers by Kraichnan and Wyld [1,2], who suggested to simulate excitation of stationary space-homogeneous developed hydrodynamic turbulence with the help of a space-distributed variable force $\vec{f}(\vec{r},t)$. According to the Kolmogorov-Obukhov's universality hypothesis [3,4] one can believe that in the limit of a large Reynolds number, the properties of fine-scale part of the turbulence (in the inertial range) will not depend on the way of turbulence excitation, i.e. of the type of the boundary conditions for the liquid flow or characteristics of the exciting force $\vec{f}(\vec{r},t)$. Therefore, one can suppose that the force $\vec{f}$ is a random force with a Gaussian statistics, it does not excite the mean flow: $\langle\vec{f}(\vec{r},t)\rangle = 0$, and its pair correlator D depends only on the coordinates and time difference:

$$\langle f_i(\vec{r}_1,t_1) f_j(\vec{r}_2,t_2)\rangle = D_{ij}(\vec{r}_1-\vec{r}_2, t_1-t_2) \quad (1)$$

This formula is the condition for the turbulence excitation to be stationary and homogeneous. On the other hand, one should warn against arbitrary assumptions on the properties of $D(R,\tau)$: if follows from physical consideration that there is no forced excitation of the turbulence in the inertial range. Therefore, the correlator $D(R,\tau)$ must be concentrated in the region $R > L, \tau > L/V$. There L, that is the external turbulence scale, is a characteristic scale of energy-containing vortices, $V_T = \langle|v(\vec{r},t)|^2\rangle^{1/2}$ is the mean square turbulence velocity. Provided that (as it is in a number of papers [5-7]) D is the power function of R proportional to $\delta(\tau)$ (which means that it is not localized in the frequency domain at all), we come to a curious problem of behavior of the velocity field with that exciting force. The reference of this problem to "natural" hydrodynamic turbulence is not, however, clear.

The next principal step, conventional for researchers (see monograph [4]), is to come over to $\vec{k}$-representation, that is to expande the turbulent liquid velocity field in

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plane waves. Such expansion is a long way from intuitive knowledge of the liquid turbulence as a system of interacting well-localized vortices. On the other hand, it will enable one to use detailed and rather powerful techniques for the diagram analysis of the perturbation theory serieses in the $\vec{k}$-space.

In the $\vec{k}$,t-representation the Navier-Stokes equations for incompressible liquid with an exciting force $\vec{f}$ in the right-hand side can be rendered in the form [1,2,4]:

$$\partial v_i(t,\vec{k})/\partial t+\nu k^2 v_i(t,\vec{k})=(i/2)\int \gamma_{ijl}(\vec{k},\vec{1},\vec{2})\, *$$
$$*v_j(t,\vec{k}_1)v_l(t,\vec{k}_2)\,d\vec{k}_1\,d\vec{k}_2+f_i(t,\vec{k}).$$
$$\gamma_{ijl}(\vec{k},\vec{1},\vec{2})=\Delta_{im}(\vec{k})\gamma_{mjl}(\vec{k})\,\delta(\vec{k}-\vec{k}_1-\vec{k}_2), \qquad (2)$$
$$\gamma_{ijl}(\vec{k})=(k_j\,\delta_{il}+k_l\,\delta_{ij}),\ \Delta_{ij}(k)=\delta_{ij}-k_i k_j/k^2.$$

Here $\gamma_{ijl}(k,1,2)$ is the <u>complete Euler vortex</u>, $\Delta_{ij}(k)$ is the transverse projector. For statistical description of the turbulence, we define in a usual manner the <u>different time pair correlator</u> (PC) of the velocity $\hat{F}(\vec{k},\tau)$ and its Fourier image $F(\vec{k},\omega)$:

$$2\pi F_{ij}(\vec{k},\omega)\,\delta(\omega-\omega')\,\delta(\vec{k}-\vec{k}')=\langle v_i(\vec{k},\omega)v_j^*(\vec{k}',\omega')\rangle,$$
$$\hat{F}(\vec{k},\tau)=\int\hat{F}(\vec{k},\omega)\exp(-i\omega\tau)d\omega/2\pi, \qquad (3)$$
$$\hat{F}(\vec{k})=\int\hat{F}(\vec{k},\omega)d\omega/2\pi.$$

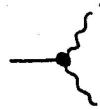
Note that in the theory of weak wave turbulence the socalled kinetic equation can be constructed. It describes time evolution of simultaneous PCs of wave field amplitudes and is closed. Kraichnan and Wyld [1,2] has demonstrated that this approach is impossible in the theory of developed hydrodynamic turbulence: $F(\vec{k})$ proves to be a functional not of simultaneous PC $F(\vec{k}')$, but of the different time velocity PC $\hat{F}(\vec{k}',\omega')$ and the <u>Green functions</u> (GF) $\hat{G}(\vec{k}',\omega')$:

$$\hat{G}_{ij}(\vec{k},\omega)\,\delta(\vec{k}-\vec{k}')\delta(\omega-\omega')=\langle\delta v_i(\vec{k},\omega)/\delta f_j(\vec{k}',\omega')\rangle. \qquad (4)$$

These values satisfy the Dyson-Wyld equation [2]:

$$\hat{G}(q)=\hat{G}_0(q)+\hat{G}_0(q)\hat{\Sigma}(q)\hat{G}(q),$$
$$\hat{G}_{0,ij}(q)=\Delta_{ij}(\vec{k})[\omega+i\nu k^2]^{-1}, \qquad (5)$$
$$\hat{F}(q)=\hat{G}(q)[\hat{D}(q)+\Phi(q)]G^*(q),\quad q=\vec{k},\omega.$$

where $\hat{G}_0$ is the bare GF of noninteracting field, $\hat{D}(q)$ is the Fourier image of the random force correlator (1), the <u>mass operators</u> $\Sigma(q)$, $\Phi(q)$ are expressed through $G(q)$ and $F(q)$ according to the Feynman's rules [4] with the vertex of a single type:

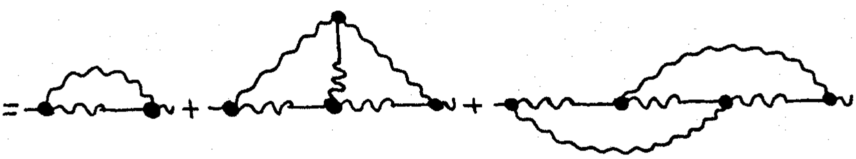
$$\gamma_{ijl}(q_1 q_2 q_3) = iq_1$$  iq_2, lq_3 $$= \gamma_{ijl}(\vec{k})\,\delta(q-q_1-q_2), \qquad (6)$$

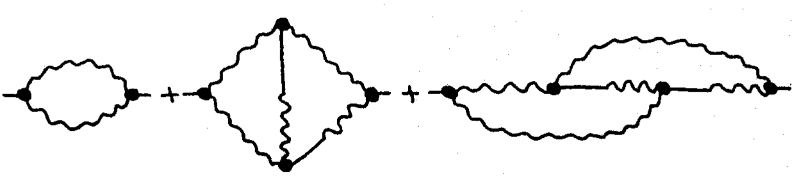
provided that we take "natural" graphic notation for $\hat{G}$ and $\hat{F}$ [5,6]:

$G(q) =$  , $G^{*}(q) = G(-q) =$  ,

$F(q) = F^{*}(+q) = F(-q) =$. (7)

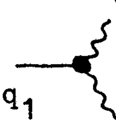
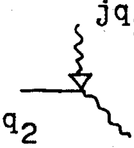
Let us write first diagrams for $\Sigma(q)$ and $\Phi(q)$:

$\Sigma(q) =$  $+ \ldots$

$\Phi(q) = \frac{1}{2}$  $+ \ldots$

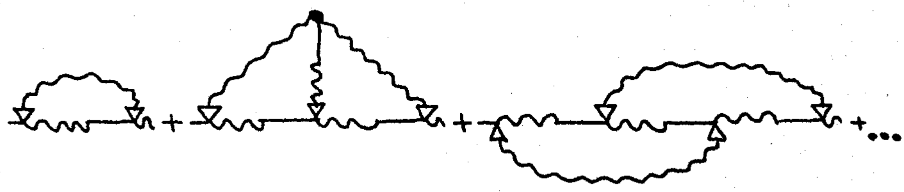
(8)

As Kraichnan [1] and Wyld [2] mentioned, the integrals in the momenta q of the velocity PC in the above diagrams diverge in the region of small q. They related these divergences to the effect of small k-vortices transfer (i.e. the vortices with characteristic size 1/k) by almost homogeneous velocity field of large L-vortices with the energy-containing size L. In 1977 the author suggested the homogeneous transfer approximation [8], that enables one to extract the most diverging part from each diagram (8). Given this approximation, in the diagrams for $\Sigma(q)$ one should neglect the PC momenta q_j compared to the outer momentum q in the arguments of vetices and backbone GF (i.e. in the way along GF from the entry to the exit of the diagram). Formally, the homogeneous transfer approximation can be obtained if the complete Euler vertices (2) and (6) in the backbone are replaced by the so-called kinematic vertices:

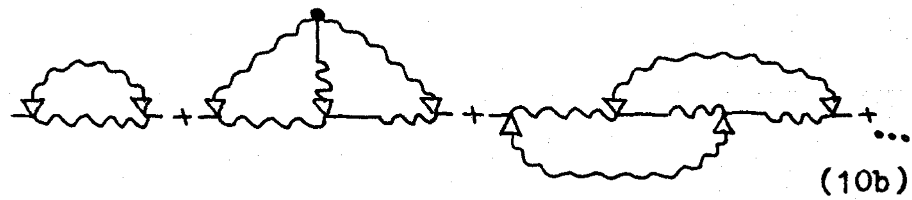
$\gamma_{ijl}(qq_1q_2) = iq_1$  jq_2, lq_3 $\rightarrow$ iq_2  jq_2, lq_3 $=$ (9)

$$= \tilde{\gamma}_{ijl}(qq_1q_2) = k_j\,\delta_{il}\,\delta(q-q_2)\ .$$

The diagram for $\Sigma(q)$ and $\Phi(q)$ in the homogeneous transfer approximation have the form:

$\Sigma_t(q)=$ 

(10a)

$\Phi_t(q)=$ 

(10b)

By summing up the whole diagram series (10a) the following expression for GF is derived [8]:

$$G_{t,ij}(q)=\Delta_{ij}(\vec{k})\langle[\omega-\vec{k}\vec{v}(\vec{r},t)+i\nu k^2]^{-1}\rangle . \qquad (11)$$

Its physical meaning is fairly simple: G_t represents the GF of noninteracting k-vortices (9b), which takes the Doppler frequency shift $\omega \Rightarrow \omega-\vec{k}\vec{v}(\vec{r},t)$ into account . This shift accounts for the homogeneous transfer of these vortices at constant velocity v(r,t). The averaging is done over the turbulence velocity ensemble at a fixed (arbitrary) point $\vec{r}$ and at a fixed (arbitrary) time t. It follows from (1) that in the inertial range (where $kv \gg \nu k^2$) the GF has the form:

$$G_{t,ij}(q)=\frac{\Delta_{ij}(\vec{k})}{kV_T}\, q\left(\frac{\omega}{kV_T}\right). \qquad (12)$$

Expressions (11), (12) in the theory of turbulence for the inertial scale range should be considered as final: $G_t(q)$ is expressed through external (for this theory) characteristics of turbulence in the energy-containing range: $V_T \simeq \varepsilon^{1/3}L^{1/3}$, where ε is the energy flux along the spectrum, and statistical properties of the velocity field determining the form of the structural function q respectively.

Also, the author has managed [8] to analyse the series (10b) for Φ_t and to demonstrate that in the approximation of the homogeneous transfer, the structure of $F(k,\omega)$ in ω is completely determined by energy-containing vortices:

$$\hat{F}_t(\vec{k},\omega)=\hat{F}(\vec{k})\langle\delta(\omega-\vec{k}\vec{v}(\vec{r},t))\rangle . \qquad (13)$$

At the same time, the simultaneous velocity PC F(k) remains arbitrary. It is not surprising because the homogeneous part of the transfer extracted by us has a kinematic character. It results in no energy exchange between the vortices and, thus, has no influence on the energy distribution over scales, that is on the form of simultaneous velocity PC F(k).

PC F(k).

It is clear that the velocity field of large vortices is indeed inhomogeneous. This leads to some deformation of k-vortices while these are being transfered. The deformation is the more essential the less is the contrast in the scales of interacting vortices. It is this dynamic interaction (but not the kinematic effect of the homogeneous transfer) that must determine the energy scale distribution. Thus the problem is to distinguish the relatively low dynamic interaction against the masking homogeneous transfer in the formal technique of the theory. A natural way to solve this problem is to use the variables for which the homogeneous transfer is absent. For this propouse Kraichnan [9] suggested to use Lagrange variables $u(r_i,t_o,t)$, i.e. the velocity at the time t of a liquid particle which was at the initial point r_i at the time t_o. The particle trajectory can easily be found from the Lagrange velocity

$$\vec{r}(t)=\vec{r}_i+\int_{t_0}^{t} \vec{u}(\vec{r}_i,t_0,\tau)d\tau . \qquad (14)$$

It is evident, that the Lagrange velocity $\vec{u}(\vec{r}_i,t_o,t)$ coincides with the Euler velocity $\vec{v}(\vec{r},t)$, if $\vec{r}(t)$ is taken for r on the trajectory (14):

$$\vec{u}(\vec{r}_i,t_0,t)=\vec{v}\left[\vec{r}_i+\int_{t_0}^{t} \vec{u}(\vec{r}_i,t_0,\tau)d\tau ,t\right]. \qquad (15)$$

Unfortunately, the diagram technique (DT) for the Lagrange velocity proved to be very sophisticated, and Kraichnan had advanced the theory until the approximation of direct interactions only [9]. The fact is that Lagrange variables exclude the transfer in the whole volume of turbulent liquid, which is unnecessary for physical consideration of the problem. Indeed, if one whishes to describe the evolution of a certain k-vortex placed whithin a certain volume of the order 1/k, it is sufficient to exclude the homogeneous part of the transfer exactly from that volume. However, to have something good out of it in the formal technique of the theory, one have to give up the purpose of describing the turbulence in the whole $\vec{r}$-space. To do so, the functions G(q) and F(q) (which describe global properties of the turbulence) must be replaced in the new theory by the local characteristics $G(\vec{r},q)$ and $F(\vec{r},q)$ describing the properties of the turbulence at a specific point $\vec{r}$. If the new theory should be adequate to physical consideration of the problem then the functions $G(\vec{r},q)$ and $F(\vec{r},q)$ in its formalism will be functionals of $G(\vec{r}',q')$ and $F(r',q')$ defined in the bounded domains $k|\vec{r}-\vec{r}'| \simeq 1$ and $|q-q'|\simeq q$. In 1986, Belinicher and L'vov [10] constructed the theory of turbulence, which possessed those properties. The theory used a quasi-Lagrange velocity $\vec{u}(\vec{r}_o,\vec{r},t_o,t)$. The expression for this velocity can be derived from (15). For this purpose, one should replace in the integrand the velocities $\vec{u}(\vec{r}_i,\vec{r}_o,t)$ of the particles being contained in the volume of interest by the velocity of one of them. As such one should take, for example, the velocity $u(r_o,r_o,t)$:

$$\vec{v}(\vec{r},t)=\vec{u}\left[\vec{r}_0,\vec{r}-\int_{t_0}^{t}\vec{u}(\vec{r}_0,\vec{r}_0,\tau)d\tau\ ,t_0,t\right]. \qquad (16)$$

The argument r_o is the coordinate of the above-mentioned point $\vec{r}_o$. It should be emphasized that formula (16) containes no approximations. It is an exact relation between the Euler and quasi-Lagrange velocities. Also note that the quasi-Lagrange velocity is a physical reality just like Euler or Lagrange velocities. It can be experimentally measured, successfully used for numerical simulation of turbulence, and the theory of turbulence can be adequately constructed in its terms [10].

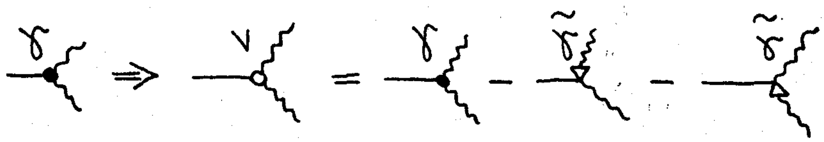
The equations of motion for the quasi-Lagrange velocity coincide in form with the Navier-Stokes equation, but with a different vertex V:

$$[\partial/\partial t+\nu k^2]u_i(\vec{r}_0,\vec{k},t)=\int V_{ijl}(\vec{r}_0,\vec{k},\vec{1},\vec{2})u_j(\vec{r}_0,\vec{k}_1,t)*$$

$$*u_l(\vec{r}_0,\vec{k}_2,t)d\vec{k}_1 d\vec{k}_2+f_i(\vec{r}_0,\vec{k},t), \qquad (17a)$$

$$V_{ijl}(\vec{r}_0,\vec{k},\vec{1},\vec{2})=\{k_{2j}\,\delta_{il}[\delta(\vec{k}-\vec{k}_1-\vec{k}_2)-\delta(\vec{k}-\vec{k}_2)\exp(i\vec{k}_1\vec{r}_0)]+$$

$$+k_{il}\,\delta_{ij}[\delta(\vec{k}-\vec{k}_1-\vec{k}_2)-\delta(\vec{k}-\vec{k}_1)\exp(i\vec{k}_2\vec{r}_0)]\}, \qquad (17b)$$



(17c)

The expression for the vertex V is the difference between complete Euler vortex γ (2) and two kinematic vertices (9) describing the homogeneous transfer. Therefore, it is clear that V describes the dynamic vertices interaction <u>only</u>. So, the vertex V can be naturally referred to as <u>dynamic</u>. The principal difference between the quasi-Lagrange equations (17) and Navier-Stokes one (2) is that the momentum $\vec{k}$ in the dynamic vertex V is not conserved. Needless to say, this accounts for to the fact that the point $\vec{r}_o$, where transfer is excluded, is fixed while coming over to quasi-Lagrange variables. As a result, the GF and PC in quasi-Lagrange variables are nondiagonal in $\vec{k}$. The diagonal form in ω, however, remaines unchanged:

$$F_{ij}(\vec{r}_0,\vec{k}_1,\vec{k}_2,\omega_1)\,\delta(\omega_1-\omega_2)=\langle u_i(\vec{r}_0,\vec{k}_1,\omega_1)u_j^*(r_0,k_2,\omega_2)\rangle,$$

$$(18)$$

$$G_{ij}(\vec{r}_0,\vec{k}_1,\vec{k}_2,\omega_1)\,\delta(\omega_1-\omega_2)=\langle\delta u_i(\vec{r}_0,\vec{k}_1,\omega_1)/\delta f_j(\vec{r}_0,\vec{k}_2,\omega_2)\rangle.$$

To physically interpret the theory, the transition to a mixed $(\vec{k},\vec{r})$-representation is extremly useful:

$$L(\vec{r}_0-\vec{r},\vec{k},\omega)=\int \exp(i\vec{s}\vec{r})L(\vec{r}_0,\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega)d\vec{s}\ , \qquad (19)$$

Here L is any of the functions G and F . In this way of coming over to the $\vec{r}$-representation (dictated by the quasi-classical approach), the functions $G(\vec{r},\vec{k},\omega)$ and $F(\vec{r},\vec{k},\omega)$ have the physical meaning mentioned above, that is, the GF and PC near the point r, respectively. Note that G and F in formula (19) do not depend on $\vec{r}$ and $\vec{r}_o$ separately, but on their difference $\vec{r}-\vec{r}_o$. The reason for that is that the homogeneous turbulence is translationally symmetric.

In discussion on the theory, it is of principal importance to study the dependence of the velocity PC and the GF on their arguments k_1, k_2 and ω . One can naturally suppose that they contain term diagonal in (k_1-k_2), that is, then can be represented in the form:

$$\hat{F}(\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega)=\hat{F}_1(\vec{k},\omega)\,\delta(\vec{s})+F_2(\vec{k}-\vec{s}/2,\vec{k}-\vec{s}/2,\omega),$$

$$\hat{G}(\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega)=G_1(\vec{k},\omega)\,\delta(\vec{s})+G_2(\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega). \qquad (20)$$

Here the functions F_2 and G_2 are either regular at $s\to 0$, or have an integrable singularity. Belinicher and L'vov[10] had assumed with practically no discussion that $F_1=G_2=0$, and then they found the functions F_2 and G_2. It was stated in [10] that F_2 and G_2 do not depend on the external turbulence scale L^2 and they describe the Kolmogorov type of turbulence, i.e. the one for which the simultaneous velocity PC $F(k)\propto k^{-11/3}$. Later on, in Ref. 11, this point of view was questioned and a quite different statement was suggested: the Kolmogorov turbulence is described by the functions F_1 and G_1, independent of L and having the form:

$$\hat{G}_1(\vec{k},\omega)=\hat{g}_1\ (\omega/\,\varepsilon^{1/3}k^{2/3})/\,\varepsilon^{1/3}k^{2/3},$$

$$\hat{F}_1(\vec{k},\omega)=\ \varepsilon^{1/3}f_1(\omega/\,\varepsilon^{1/3}k^{2/3})/k^{2/3},\ F\ (\vec{k})=\ \varepsilon^{2/3}k^{-11/3}\ , \qquad (21)$$

where ε is the energy spectrum flux, with dimensionless numerical coefficients ommitted. The authors of Ref. 11 assumed the functions F_2 and G_2 to describe a non-Kolmogorovian part of the turbulence associated with intemetency.

I do not agree with the statements proposed in Ref.11, and suppose them to be erroneous. First of all, the dependence of $\hat{G}(\vec{r},\vec{k},\omega)$ and $\hat{F}(\vec{r},\vec{k},\omega)$ on $\vec{r}$ (or, which is the same, the nondiagonality of $G(k_1,k_2,\omega)$ and $F(k_1,k_2,\omega)$ in $(\vec{k}_1-\vec{k}_2)$) is a formal consequence of the fact that a certain $\vec{r}$-coordinate is fixed in the quasi-Lagrange formulation of the problem. Likewize, the procedure of "zero momenta subtraction" suggested in Ref. 12, 13 and used in Ref. 11 is equivalent to the just mentioned one. This fact is somehow disquised in Refs. 11, 13 by absence of the letter r_o in the there presented formulae, because from the very beginning r_o was set to zero. So, as opposed to the statements in Ref. 11, we believe the nondiagonality in $(\vec{k_1}-\vec{k_2})$ to bear no relationship to physical problems, such as fluctuations of the energy spectrum flux, intermetency, etc.

It is accepted to understand the intermetency as the following property of hydrodynamic turbulence: at $Re \to \infty$ the fraction

$$K_n = \langle (\partial V/\partial r)^n \rangle / \langle (\partial V/\partial r)^2 \rangle^{n/2} \qquad (22)$$

tends to infinity proportionaly to a certain power of Re. This property we shall term a strong intermetency. This phenomenon if exists at all, can not be found in a superficial insight into the turbulent theory which uses the diagram technique of both for the cannonical Klebsh variables [14, 8, 11-13] and the Euler velocity [2, 5-7, 10]. The intermetency can be caused by the onset of a Kolmogorov's solution instability with respect to perturbations which violate the spatial homogeneity of the turbulence. The new solution with the intermetency will then be concerned with a nonperturbative (i.e. giving zero in serieses of the perturbation theory) contribution to values to be measured in the experiment. Note that it might be that the strong intermetency does not exist in hydrodynamic turbulence. If so, the experiments [4] could be explained by a weak intermetency, when the value K (see Eq. (22)) at $Re \to \infty$ is finite, although numerically high.

Now again we consider the problem on the diagonal parts of GF and PC. Substituting functions (20) in the diagram serieses for mass operators of the quasi-Lagrange DT (they differ from (8) by replacement of $\vec{v}$ by V), one can easily see that the terms G_1 and F_1 form the system of equation, which does not contain G_2 and F_2. In these serieses, the diagrams diverge in the infra-red region exactly in the same way as they do in DT for the Euler velocity. All integrals in these diagrams one can regularize by putting $F(k_j)=0$ at $Lk<1$. The serieses obtained can be summed up in the framework of homogeneous transfer approximation.The method for this summing was developed in [8] and described in detail in [10]. As a result, we have (V.I.Belinicher and V.S.Lvov, to be published):

$$\hat{G}_1(\vec{k},\omega) = \langle [\omega - \vec{k}(\vec{V}_1-\vec{V}_2) + i\nu k^2]^{-1} \rangle_{V_1V_2} \, ,$$
$$\hat{F}_1(k,\omega) = \hat{F}_1(\vec{k}) \langle \delta(\omega - \vec{k}(\vec{V}_1-\vec{V}_2)) \rangle_{V_1V_2} \, . \qquad (23)$$

Here $\langle\ \rangle_{V_1V_2}$ - stands for avergaging over two statistically independent ensembles of the velocities V_1 and V_1, where $V_j^n = \langle v(r,t)^n \rangle$, j=1,2. To understand the physical meaning of these formulae we note that according to (19) and (20) $\hat{G}_1(\vec{k},\omega)$ and $\hat{F}_2(\vec{k},\omega)$ are the limits at $r\to\infty$ of the values $G(\vec{r}-\vec{r}_0,\vec{k},\omega)$ and $F(\vec{r}-\vec{r}_0,\vec{k},\omega)$, respectively. Also note that the transfer is excluded at the point $\vec{r}_0$, that is, the values G_1 and F_1 have been calculated in the reference system moving at the velocity $\vec{v}(\vec{r}_0,t)$. It is clear now, that in formula (23) $\vec{V}_1=\vec{v}(\vec{r}_0,t)$, $\vec{V}_2=\vec{v}(\vec{r},t)$. These velocities do not correlate, because the distance between the points r and r_0 exceeds the scale L of largest vortices. The structure of expression (23) corresponds to expression (11) and (13) for the GF and PC velocities in the homogeneous transfer approximation in the DT for the Euler velocity. Particularly, the following formulae, similar to (12), follow from Eq.(23):

$$\hat{G}_1(\vec{k},\omega) = \frac{1}{k\tilde{V}_T}\, g_1\left(\frac{\omega}{k\tilde{V}_T}\right),\quad \hat{F}_1(\vec{k},\omega) = \frac{F_1(k)}{k\tilde{V}_T}\, f_1\left(\frac{\omega}{k\tilde{V}_T}\right).$$

$$\tilde{V}_T^2 = 2V_T^2,\quad V_T^2 = \langle [v(r,t)]^2 \rangle \simeq \varepsilon^{2/3}L^{2/3}. \tag{24}$$

This expression comprises the external scale L. Which is a fundamental distinction from the Eq. (21) of the Ref.11.

The general structure of expressions (20) for the GF and velocity PC can be fairly well understood in the $(\vec{k},\vec{r})$-representation from the results given in Ref. 20 and formulae (22), (23). Namely:

$$\hat{G}(\vec{r},\vec{k},\omega) = \hat{g}\,[\omega/\Omega(k,r)]/\Omega(k,r),$$
$$\hat{F}(r,k,\omega) = \varepsilon^{2/3}k^{11/3}\hat{f}\,[\omega/\Omega(k,r)]\,\Omega(k,r). \tag{25}$$

At $r \gg L$ according to (24), $\Omega = \sqrt{2}\,kV_T$. At $L > r > 1/k$ the formulae for G and F have the form (24), but the expression for V_T has no contribution of vortices with $l > r$. Besides, $V_T(r) \simeq r^{1/3}\varepsilon^{1/3}$ [20]. Thus in this region, $\Omega = (\varepsilon r)^{1/3}k$.

Finally, at $kr \ll 1$ the transfer is completely excluded and Ω has the Kolmogorov value $\Omega = \varepsilon^{1/3}k^{2/3}$. For illustration, an interpolation formula for Ω can be suggested, which provides a qualitively valid representation of all asymptotics:

$$\Omega(k,r,L) = \varepsilon^{1/3}k^{2/3}\,\frac{1+(kr)^{1/3}}{1+(r/L)^{1/3}}. \tag{26}$$

Formulae (25), (26) give an idea of how the external turbulence scale L occurs in the structural function g and f for GF and velocity PC, respectively. At $L\to\infty$ the expression for Ω depends no longer on L. And expression (25) for G and F approaches zero, if $r\to\infty$. This means that $G_1=F_1=0$ in the limit $L\to\infty$. It is exactly that limit that was derived in Ref. 20. If L is finite, then $G_1=0$ and $F_1=0$ are given by Eqs. (22), (23). The expressions for G_2 and F_2 can be taken from Eqs. (25), (26), if one considers momentum representation in the variable $\vec{r}$. The above expressions can easily be analyzed, so we shall not do all that. Still note, that the integrals over ω in (25) for $G(\vec{r},\vec{k},\omega)$ and $F(\vec{r},\vec{k},\omega)$ are independent of $\vec{r}$, and thus

$$\int d\omega F_2(\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega)=F_2(\vec{k})\delta(\vec{s}),$$
$$\int d\omega G_2(\vec{k}+\vec{s}/2,\vec{k}-\vec{s}/2,\omega)=G_2(\vec{k})\delta(\vec{s}). \quad (27)$$

This fact is a consequence of the general statement that the simultaneous correlators of Euler and quasi-Lagrange velocities coincide.

Later on, we shall give detailed analysis of how the scale L influences upon some Kolmogorov turbulence properties in the inertial range.

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