Comment on "Universal Properties of the Two-Dimensional Kuramoto-Sivashinsky Equation"

Jayaprakash, Hayot, and Pandit reported a numerical coarse-graining procedure applied to the Kuramoto-Sivashinsky (KS) equation in 2+1 dimensions, aimed at extracting its long wavelength scale invariant properties [1]. Their Letter has three parts. The first details the numerical investigation, the second offers an analysis of this investigation, and the third discusses previous results [2,3] on the same issue. The first part contains valuable new information, which we shall reinterpret below. However, we believe that the second part is wrong, and that the third part contains misrepresentations and misunderstandings.

The essence of the first part of Ref. [1] is a numerical coarse-graining procedure [4] in which shells in **k** space are eliminated for $k > \Lambda$. This procedure yields the "renormalized" viscosity $\nu_\Lambda(k)$ which in principle can depend on the cutoff length Λ and on k. The main finding of Ref. [1] is that $\nu_\Lambda(k)$ *does not depend on either k or* Λ, and numerically $\nu_\Lambda(k) \approx 11|\nu_0|$ where ν_0 is the bare negative viscosity of the KS equation (taken as $\nu_0 = -1$ in Ref. [1]). This result was used to claim that the KS equation is in the same universality class in 2+1 dimensions as the Kardar-Parisi-Zhang (KPZ) equation, in contradiction with the theory of Refs. [2,3].

We argue that this result in fact supports the conclusion of Refs. [2,3]. The renormalized viscosity $\nu_\Lambda(k)$ is the coefficient of the contribution to the damping of fluctuations of wave vector k because of their interaction with other fluctuations having wave vector $k' > \Lambda$. In symbols, we write this contribution as $\gamma(k) = \nu_\Lambda(k) k^2$. The dynamical exponent z is related to $\gamma(k)$ via the relation $\gamma(k) \sim k^z$. Therefore the observed independence of $\nu_\Lambda(k)$ on k is a direct confirmation of our conclusion that $z = 2$ for KS in 2+1 dimensions. Stated differently, one expects [4] $\nu_\Lambda(k)$ to depend on Λ whenever $z \neq 2$, as $\nu_\Lambda(k) \sim \Lambda^{2-z}$. The observation that $\nu_\Lambda(k)$ is independent of Λ strengthens the same conclusion. We remind the reader that for KPZ in 2+1 dimensions one expects $z \approx 1.6$ in the strong coupling regime.

It should be stressed that we are dealing here with a strong coupling regime of the KS equation; the renormalized value of $\nu_\Lambda(k)$ corrects the linear viscosity significantly, and even has an opposite sign. In contrast, the result $z = 2$ for the KPZ equation is only available in the weak coupling regime. The physics of these two models for $z = 2$ is totally different. Finally we need to reiterate that the result of Ref. [1] that $\nu_\Lambda(k)$ is independent of Λ confirms convincingly our statement that the dressing of the KS problem is dominated by nonlocal interactions (meaning that the main effect on small k fluctuations comes from far away wave vector with $k' \gg \Lambda$). The independence of $\nu_\Lambda(k)$ on Λ means that interactions with fluctuation of $k' \approx \Lambda$ are negligible with respect to far away contributions of fluctuations with $k' \gg \Lambda$. Recall that in local solutions the choice of the cutoff Λ makes the renormalized viscosity $\nu_\Lambda(k)$ dependent on Λ like a power law, since the main contribution comes from k' values which are of the same order as k.

Next, the authors of Ref. [1] offered an estimate of the "effective" coupling constant $g \approx 0.4$ in terms of the renormalized viscosity and noise. Using the alleged KPZ effective equation, they estimated a crossover scale as if the dynamical equation were really KPZ. This crossover scale is $L_c \sim \exp(8\pi/g)$. For the real KPZ equation this scale is where logarithmic corrections to the free field theory begin to dominate. We should stress that this estimate is totally irrelevant, since the nonlocal solution of KS [2,3], which is confirmed in Ref. [1], has logarithmic corrections anyway. Theoretically the issue of logarithmic corrections is a thorny issue which has not been fully resolved. Reference [1] contributes nothing toward this issue. At best, if there were a crossover scale for KS, it would connect two *strong coupling* regimes (nonlocal for $k_c < k < k_{\max}$ and local for $k < k_c$) but not a crossover from a free field theory as in KPZ. It is not known whether for the KS equation a crossover exists, and if it does at which scale. There is no reason to expect that k_c is of $O(1/L_c)$ as suggested in Ref. [1].

Finally, we would like to stress that the third part of Ref. [1] in which comments on Refs. [2,3] are made is very inaccurate in not distinguishing between the nonlocal strong coupling solution with scaling exponent $z = 2$ and free field behavior. All the solutions discussed in Refs. [2,3] are strong coupling solutions which have nothing to do with free field behavior. Also, the lack of distinction between one-loop approximation and renormalized theory to all orders [3] is depressing, and it leads to misrepresentation and misunderstanding of the results of Refs. [2,3].

Victor L'vov and Itamar Procaccia
Departments of Nuclear and Chemical Physics
The Weizmann Institute of Science
Rehovot 76100, Israel

Received 19 July 1993
PACS numbers: 05.40.+j, 47.27.Sd, 47.52.+j, 68.35.−p

[1] C. Jayaprakash, F. Hayot, and R. Pandit, Phys. Rev. Lett. **71**, 12 (1993).
[2] I. Procaccia, M. H. Jensen, V. S. L'vov, K. Sneppen, and R. Zeitak, Phys. Rev. A **46**, 3220 (1992).
[3] V. S. L'vov and I. Procaccia, Phys. Rev. Lett. **69**, 3543 (1992).
[4] S. Zaleski, Physica (Amsterdam) **34D**, 427 (1989).