Contents

0. Introduction 1

1. Equations of Motion and the Hamiltonian Formalism 9
1.1 The Hamiltonian Formalism for Waves in Continuous Media 9
1.1.1 The Hamiltonian in Normal Variables 9
1.1.2 Interaction Hamiltonian for Weak Nonlinearity 15
1.1.3 Dynamic Perturbation Theory. Elimination of Nonresonant Terms 18
1.1.4 Dimensional Analysis of the Hamiltonian Coefficients 21
1.2 The Hamiltonian Formalism in Hydrodynamics 25
1.2.1 Clebsh Variables for Ideal Hydrodynamics 25
1.2.2 Vortex Motion in Incompressible Fluids 29
1.2.3 Sound in Continuous Media 29
1.2.4 Interaction of Vortex and Potential Motions in Compressible Fluids 31
1.2.5 Waves on Fluid Surfaces 33
1.3 Hydrodynamic-Type Systems 37
1.3.1 Langmuir and Ion-Sound Waves in Plasma 37
1.3.2 Atmospheric Rossby Waves and Drift Waves in Inhomogeneous Magnetized Plasmas 43
1.4 Spin Waves 51
1.4.1 Magnetic Order, Energy and Equations of Motion 51
1.4.2 Canonical Variables 53
1.4.3 The Hamiltonian of a Heisenberg Ferromagnet 54
1.4.4 The Hamiltonian of Antiferromagnets 56
1.5 Universal Models 58
1.5.1 Nonlinear Schrödinger Equation for Envelopes 59
1.5.2 Kadomtsev-Petviashvili Equation for Weakly Dispersive Waves 60
1.5.3 Interaction of Three Wave Packets 61

2. Statistical Description of Weak Wave Turbulence 63
2.1 Kinetic Wave Equation 63
2.1.1 Equations of Motion 63

2.1.2 Transition to the Statistical Description 64
2.1.3 The Three-Wave Kinetic Equation 66
2.1.4 Applicability Criterion of the Three-Wave Kinetic Equation (KE) 67
2.1.5 The Four-Wave Kinetic Equation 70
2.1.6 The Quantum Kinetic Equation 72
2.2 General Properties of Kinetic Wave Equations 75
2.2.1 Conservation Laws 75
2.2.2 Boltzmann's H-Theorem and Thermodynamic Equilibrium 78
2.2.3 Stationary Nonequilibrium Distributions 80

3. Stationary Spectra of Weak Wave Turbulence 83
3.1 Kolmogorov Spectra of Weak Turbulence in Scale-Invariant Isotropic Media 83
3.1.1 Dimensional Estimations and Self-Similarity Analysis 84
3.1.2 Exact Stationary Solutions of the Three-Wave Kinetic Equation 86
3.1.3 Exact Stationary Solutions for the Four-Wave Kinetic Equations 93
3.1.4 Exact Power Solutions of the Boltzmann Equation . 101
3.2 Kolmogorov Spectra of Weak Turbulence in Nearly Scale-Invariant Media 102
3.2.1 Weak Acoustic Turbulence 102
3.2.2 Media with Two Types of Interacting Waves 108
3.3 Kolmogorov Spectra of Weak Turbulence in Anisotropic Media 117
3.3.1 Stationary Power Solutions 117
3.3.2 Fluxes of Integrals of Motion and Families of Anisotropic Power Solutions 120
3.4 Matching Kolmogorov Distributions with Pumping and Damping Regions 123
3.4.1 Matching with the Wave Source 124
3.4.2 Influence of Dissipation 135

4. The Stability Problem and Kolmogorov Spectra 145
4.1 The Linearized Kinetic Equation and Neutrally Stable Modes 145
4.1.1 The Linearized Collision Term 145
4.1.2 General Stationary Solutions and Neutrally Stable Modes 147
4.2 Stability Problem for Kolmogorov Spectra of Weak Turbulence 156
4.2.1 Perturbation of the Kolmogorov Spectrum 160

4.2.2 Behavior of Kolmogorov-Like Turbulent Distributions. Stability Criterion 173
4.2.3 Physical Examples 184
4.3 Nonstationary Processes and the Formation of Kolmogorov Spectra 190
4.3.1 Analysis of Self-Similar Substitutions 191
4.3.2 Method of Moments 197
4.3.3 Numerical Simulations 200

5. Physical Applications 207
5.1 Weak Acoustic Turbulence 207
5.1.1 Three-Dimensional Acoustics with Positive Dispersion: Magnetic Sound and Phonons in Helium 209
5.1.2 Two-Dimensional Acoustics with Positive Dispersion: Gravity-Capillary Waves on Shallow Water and Waves in Flaky Media 218
5.1.3 Nondecay Acoustic Turbulence: Ion Sound, Gravity Waves on Shallow Water and Inertio-Gravity Waves 227
5.2 Wave Turbulence on Water Surfaces 229
5.2.1 Capillary Waves on Deep Water 229
5.2.2 Gravity Waves on Deep Water 230
5.2.3 Capillary Waves on Shallow Fluids 232
5.3 Turbulence Spectra in Plasmas, Solids, and the Atmosphere 233
5.3.1 Langmuir Turbulence in Isotropic Plasmas 233
5.3.2 Optical Turbulence in Nonlinear Dielectrics and Turbulence of Envelopes 236
5.3.3 Spin Wave Turbulence in Magnetic Dielectrics 237
5.3.4 Anisotropic Spectra in Plasmas 239
5.3.5 Rossby Waves 242

6. Conclusion 245

A. Appendix 249
A.1 Variational Derivatives 249
A.2 Canonicity Conditions of Transformations 250
A.3 Elimination of Nonresonant Terms from the Interaction Hamiltonian 252

References 257

Subject Index 263