Flatland optics. III. Achromatic diffraction

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Received October 23, 2000; revised manuscript received February 14, 2001; accepted February 27, 2001

In the previous two sections of "Flatland optics" [J. Opt. Soc. Am. A **17**, 1755 (2000); **18**, 1056 (2001)] we described the basic principles of two-dimensional (2D) optics and showed that a wavelength λ in three-dimensional (3D) space (*x*, *y*, *z*) may appear in Flatland (*x*, *z*) as a wave with another wavelength $\Lambda = \lambda/\cos\alpha$. The tilt angle α can be modified by a 3D-Spaceland individual, who then is able to influence the 2D optics in a way that must appear to be magical to 2D-Flatland individuals—in the spirit of E. A. Abbott's science fiction story of 1884 [*Flatland, a Romance of Many Dimensions*, 6th ed. (Dover, New York, 1952)]. Here we show how the light from a white source can be perceived in Flatland as perfectly monochromatic, so diffraction with white light will be free of color blurring and the contrast of interference fringes can be 100%. The basic considerations for perfectly achromatic diffraction are presented, along with experimental illustration of Talbot self-imaging performed with broadband illumination.

OCIS codes: 050.1960, 030.1640.

1. INTRODUCTION

In this series on "Flatland optics" we try to describe optics as it would appear to a two-dimensional (2D) physicist, limited to live (and observe light) on an infinite plane. This is in the spirit of E. A. Abbott's science fiction story *Flatland* of 1884.[1] Flatland optics takes place in free space, or more specifically in a plane ($y = 0$) with coordinates (x, z). The third dimension (y) can provide us, the three-dimensional (3D) creatures, with the ability to manipulate the (x, z) optics of Flatland.

The fundamentals of Flatland optics, especially its basic wave equation, were presented in part I[2] of our three-part series on Flatland, where we showed that a 2D physicist will measure a 2D wavelength Λ that may be different from the 3D wavelength λ. The connection between these two wavelengths is

$$\Lambda = \frac{\lambda}{\cos\alpha}, \tag{1}$$

where α is the tilt angle of the beam in the y–z plane. Intuitively, this means that the k vector of the light for the 2D physicist in Flatland is the projection of the 3D k vector onto the plane of Flatland. The reality, or the objectivity, of Flatland optics was demonstrated in part II,[3] where the Flatland-equivalent basic experiments of wave optics, i.e., the Young biprism fringes, the Talbot effect, Fresnel diffraction on a zone plate, and Fraunhofer diffraction from a double slit and a grating, were performed. The aim of this part III of "Flatland optics" is to demonstrate how a 3D physicist can manipulate the illumination spectrum in Flatland in a lossless manner. In particular, we demonstrate how such manipulations can be exploited to obtain perfectly achromatic diffraction in white light.

2. ACHROMATIC DIFFRACTION

Achromatic Fresnel diffraction systems have been investigated for some time.[4–9] In some cases, certain approximations had to be incorporated to achieve the achromatization. In others, either complicated diffractive components were required or the system was specialized for periodic objects. Here we demonstrate how one can achieve perfectly achromatic diffraction in Flatland by using an arbitrary broadband 3D light source.

By perfectly achromatic we mean that the intensity distribution $I(x, y, z)$ in the half-space ($z \geqslant 0$) behind the object is the same for all wavelengths λ, whereby

$$|V(x, y, z; n, \lambda)|^2 = I(x, y, z), \tag{2}$$

where $V(x, y, z; n, \lambda)$ is the distribution of the complex amplitude at wavelength λ and refractive index n. Such perfect achromatization is possible theoretically, if the refractive index $n(\lambda)$ of the medium in ($z \geqslant 0$) were to be linearly proportional to the wavelength, as

$$n(\lambda) = n_1\lambda. \tag{3}$$

As a consequence of Eq. (3) the length k of the wave vector would be the same for all wavelengths, so

$$k(\lambda) = \frac{2\pi n}{\lambda} = 2\pi n_1. \tag{4}$$

We are not aware of any material whose refractive index would obey Eq. (3). However, in accordance with our Flatland optics,[2,3] it is possible to readily control the refractive index in Flatland by tilting the illumination as in the generic setup shown in Fig. 1. Specifically, in Flatland the effective 2D wavelength Λ depends on the 3D wavelength λ as

$$\Lambda = \frac{\lambda}{\cos\alpha} = \Lambda(\lambda, \alpha). \tag{5}$$

According to Eq. (5), the $\cos\alpha$ takes the place of the refractive index, so perfect achromaticity would be achieved if the $\cos\alpha(\lambda)$ varied as

$$\cos\alpha(\lambda) = \lambda \times \text{const.} \tag{6}$$

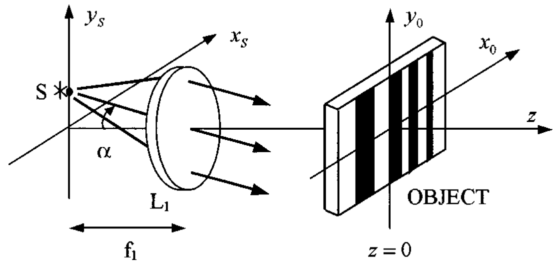

Fig. 1. Optical setup in which a 1D object at $z = 0$ is illuminated by a plane wave, tilted by an angle α in the y direction.

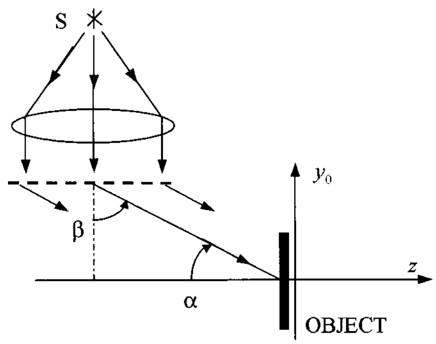

Fig. 2. Optical setup for tuning the tilt angle $\alpha(\lambda)$ by means of a diffraction grating oriented orthogonally to the object.

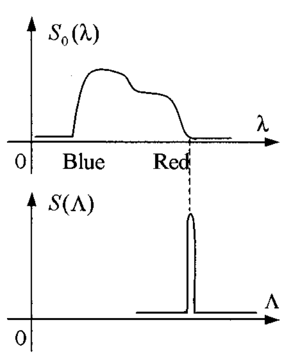

Fig. 3. Spectrum of a source in 3D Spaceland and in 2D Flatland obtained with the setup of Fig. 2.

Grating diffraction obeys a similar law:

$$\sin \beta(\lambda) = \lambda / D, \tag{7}$$

where $\beta(\lambda)$ is the angle of diffraction and D is the grating period. Thus what we need is a setup in which the grating angle β is connected to the Flatland angle α by

$$\beta(\lambda) + \alpha(\lambda) = \pi/2. \tag{8}$$

That is possible by bending the optical axis by 90°, yielding

$$\cos \alpha(\lambda) = \sin \beta(\lambda) = \lambda / D. \tag{9}$$

When comparing Eq. (9) with Eq. (5) we realize that the Flatland 2D wavelength Λ is identical to the grating period D, which is valid for every illumination 3D wavelength λ. The basic optical setup for achieving achromatization is presented in Fig. 2. There a polychromatic source is collimated to form a plane wave normally incident on a grating that is oriented at 90° to the object. Plane waves are diffracted from the grating at an angle β with respect to the normal to the grating and impinge on the object at an angle α with respect to the normal to the object. With such a setup it should be possible to transform an arbitrary 3D spectrum $S_0(\lambda)$ into the monochromatic 2D spectrum $S(\Lambda)$ in Flatland without any loss, as shown schematically in Fig. 3.

3. EXPERIMENTAL PROCEDURE AND RESULTS

We performed a Talbot self-imaging experiment with the setup shown in Fig. 2. The diffraction grating had a period $D = (1/1200)$ mm, the white light source was a white discharge lamp imaged onto a pinhole, and the object was a Ronchi grating (200 lines/in.). The resulting Talbot self-image is shown in Fig. 4. This image was taken at the first Talbot x–y plane (at a distance of $z = 3.9$ cm from the object) with a commercial color CCD camera. Note that a Flatland scientist, who lives in a particular $y =$ constant plane, would perceive only a one-dimensional (1D) slice of what is recorded in Fig. 4. The quality of the self-imagery remains essentially the same even at a larger distance z. That would not be the case with a "normal" Talbot self-image experiment using polychromatic illumination, where the longitudinal period $2D^2/\Lambda$ is wavelength dependent. However, in our Flatland experiment the spectrum $S(\Lambda)$ effectively became monochromatic, although the spectrum $S_0(\lambda)$ of the original source was essentially polychromatic.

4. OTHER POSSIBLE SPECTRAL MANIPULATIONS

In the preceding section we showed how a 3D broadband spectrum can be manipulated so it will be perceived as monochromatic in Flatland. Alternatively, it is possible to transform a 3D monochromatic spectrum into a 2D broadband spectrum. Figure 5 illustrates the generic setup for performing such a transformation. Here, a linear grating $M(y)$ diffracts the light from a single source into three beams, so the illumination appears virtually as an array of three sources. These three sources are coherent in three dimensions (and in two dimensions) so that interference fringes will appear along the longitudinal direction in the Flatland plane. A Flatland physicist would interpret these interferences as "beats," as a result of coherent superposition of three different 2D wavelengths.

For a more detailed illustration, let us assume (for simplicity) a grating $M(y)$ with only two diffraction orders. These two diffraction orders have two different 2D wave vectors K_1 and K_2 but the same temporal frequency ω. The two waves will propagate in the x–z plane as

$$\begin{aligned} V(z, t) &= \exp[i(K_1 z - \omega t)] + \exp[i(K_2 z - \omega t)] \\ &= \exp[i(\bar{K} z - \omega t)] 2 \cos(\Delta K z), \end{aligned} \tag{10}$$

where $\bar{K} = (K_1 + K_2)/2$ and $\Delta K = (K_1 - K_2)/2$. The perceived intensity in Flatland contains longitudinal beats, of the form

$$|V(z, t)|^2 = 2[1 + \cos(2\Delta K z)]. \tag{11}$$

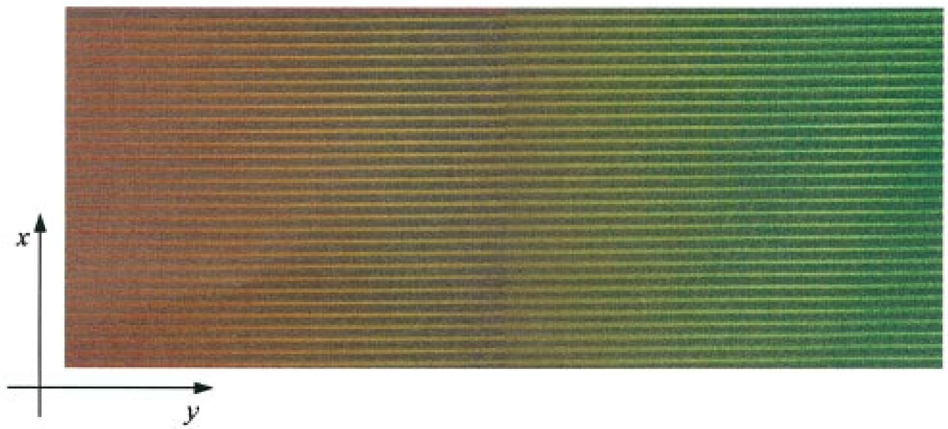

Fig. 4. Experimental result of an achromatic Talbot self-image in Flatland. The illumination spectrum is spread along the y axis. The image is composed of two CCD captures taken at the same longitudinal distance z, one at the red–yellow spectrum and the other at the yellow–green spectrum.

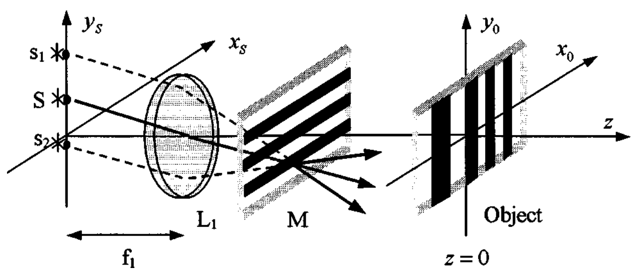

Fig. 5. Optical setup for transforming monochromatic light in 3D Spaceland into polychromatic light in 2D Flatland.

We can let the two 2D wavelengths be mutually incoherent by moving the grating M in the y direction, so that the beats phase would change fast enough to average out during the detection time of a Flatland detector. In other words, there would be two slightly different frequencies ω_1 and ω_2, and if the time constant τ of the detector were such that $(\omega_1 - \omega_2)\tau \gg 2\pi$, then the beats would disappear. For the Flatland physicist, the illumination is composed essentially of two sharp spectral lines.

It is important to note that the speed of light in Flatland is not constant. It may happen that two waves of the same 2D wavelength will propagate in Flatland with different speeds (nontilted 3D red light propagates faster in Flatland than tilted 3D blue light). For this reason, our 3D speed of light c is just an upper limit for the speed of light in Flatland. Thus a dispersion relation between temporal frequency and wavelength does not exist in Flatland, so beats in Flatland can be temporally stationary if the two 2D wavelengths have the same temporal frequency. In three dimensions the dispersion relation prohibits such a situation, so 3D beats always appear in both space and time.

More complicated spectral transformations are possible by considering different gratings M with more complicated grating functions along the y direction. The design of such gratings is equivalent to the design of 1D holograms.

5. CONCLUDING REMARKS

We have shown that the 2D spectrum in Flatland can be readily manipulated so that polychromatic light in the 3D world is perceived in the 2D Flatland world as if it were perfectly monochromatic. This makes it possible to obtain perfectly achromatic diffraction, even with white light.

ACKNOWLEDGMENT

This work was supported in part by the Albert Einstein Minerva Center for Theoretical Physics.

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