Color correction in planar optics configurations

L. Eisen, A. A. Friesem, M. Meyklyar, and M. Golub

Department of Physics of Complex Systems, Weizmann Institute of Science, P.O. Box 26, Rehovot 76100, Israel

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Color correction in planar optics configurations can be achieved by resorting to gradient-index rather than uniform-refractive-index substrates. The basic configuration, principle of correction, and calculated and experimental results are presented. The results reveal that, with an appropriate refractive index distribution along the thickness of the substrates, the color can be corrected over a wavelength range up to 155 nm depending on incidence angles. © 2006 Optical Society of America

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During the past two decades planar optics configurations, with several diffractive elements recorded on one substrate, have been incorporated into heads-up and visor displays,[1,2] optical interconnects,[3,4] beam shapers, splitters, and deflectors,[5,6] and beam expanders.[7] In general, monochromatic illumination and substrates of uniform refractive index are used, so chromatic dispersions that occur with the diffractive elements do not play a role. However, with polychromatic illumination, the chromatic dispersion results in excessive color displacement.

In this Letter we investigate planar optics configurations in which color displacement is overcome by resorting to substrates with a gradient refractive index (GRIN). We determine, both theoretically and experimentally, the main conditions and parameters for such planar optics configurations. These include the refractive index distribution in the substrate, the range of incidence angles and corresponding diffraction angles, the periods of diffractive elements, and the corresponding illumination wavelengths.

Consider the basic planar optics configuration that comprises two laterally displaced diffraction gratings arranged on a substrate of uniform refractive index, as shown schematically in Fig. 1. One input grating serves to couple the incident polychromatic beam, comprised of three wavelengths, into a substrate, where it is trapped by total internal reflection and propagates toward the other output grating that decouples light from the substrate. As is evident, the light of each wavelength, λ_1, λ_2, and λ_3, propagates along a different path inside the substrate and arrives to a different location at the output grating, thereby resulting in color displacement.

The corresponding planar optics configuration with the gratings arranged on a GRIN substrate is schematically shown in Fig. 2. The substrate thickness is d and refractive index $n_{\text{sub}}(x,\lambda)$ is a function of wavelength λ and distance x from the substrate surface ($x=0$). The refractive index gradually decreases from a maximum value, denoted $n_{\text{sub}}(0,\lambda)$ at $x=0$, to a minimum value, denoted $n_{\text{sub}}(d,\lambda)$ at $x=d$; accordingly, $n_{\text{sub}}(0,\lambda)>n_{\text{sub}}(d,\lambda)$. Also, the GRIN substrate is surrounded with a nondispersive material (typically air) of refractive index n_{sup}, where $n_{\text{sub}}(x,\lambda)>n_{\text{sup}}$. As shown, the light of each wavelength λ_1, λ_2, and λ_3 propagates along a different trajectory inside the substrate and arrives at one desired location at the output grating. The incidence and diffraction angles at the input grating are denoted θ_{inc} and θ_{diff}, and the location of the light emerging from the output grating is z_p, which corresponds to a distance between two successive bounces. We use a convention where the signs of θ_{inc} and θ_{diff} are the same, for the geometry shown in Fig. 2.

We start from the basic differential equation that describes the propagation of a monochromatic beam of wavelength λ in an optically inhomogeneous medium,[8] written as

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(n_{\text{sub}}(\mathbf{r},\lambda)\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s}\right)=\nabla n_{\text{sub}}(\mathbf{r},\lambda), \tag{1}$$

where s is the distance along the beam measured from some fixed point and $\mathbf{r}$ the position vector at s. We now consider an ideal refractive index profile for the GRIN substrate[9,10] as

$$n_{\text{sub}}^2(x,\lambda)=n_{\text{sub}}^2(0,\lambda)\left[1-(g_\lambda x)^2+\frac{2}{3}(g_\lambda x)^4-\frac{17}{45}(g_\lambda x)^6\right], \tag{2}$$

with $x\leqslant d$ and g_λ the gradient constant. Then, following the conventional ray-tracing method for analyzing meridianal ray trajectories in a GRIN rod lens,[11] the paraxial solution of Eq. (1) for the trajectory of the beam propagating in the $x-z$ plane is

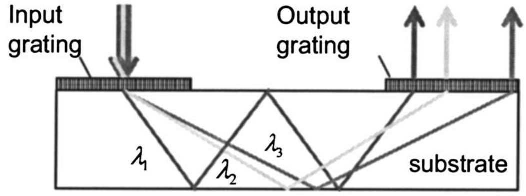

Fig. 1. Basic planar optics configuration with two laterally displaced diffraction gratings arranged on a substrate of uniform refractive index. The incident light, comprising three different wavelengths, λ_1, λ_2, and λ_3, is dispersed and trapped inside the substrate by total internal reflection, and the light of each wavelength propagates along a different path toward the output grating, so each emerges at a different location.

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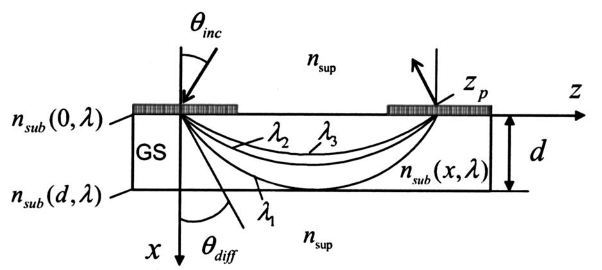

Fig. 2. Basic planar optics configuration with two laterally displaced diffraction gratings arranged on a GRIN substrate. The incident light is diffracted by the input grating and propagates toward the output grating along a different trajectory for each wavelength, but emerges at one location.

$$x(z) = \frac{\cos(\theta_{\text{diff}})}{g_\lambda}\sin(g_\lambda z). \tag{3}$$

In accordance with Eq. (3), the beams inside the GRIN substrate propagate in a sinusoidal trajectory with the amplitude determined by the diffraction angle θ_{diff}, and the distance between successive bounces, namely a half-period of the sinusoidal trajectory, of

$$z_p = \frac{1}{2}\left(\frac{2\pi}{g_\lambda}\right). \tag{4}$$

Equation (4) indicates that the distance z_p is determined solely by the gradient constant g_λ regardless of incidence and diffraction angles.[12] The amplitudes of the sinusoidal trajectories, $\cos(\theta_{\text{diff}})/g_\lambda$, provide the boundaries or limits for θ_{diff}, as

$$0 \leqslant \cos(\theta_{\text{diff}}) \leqslant g_\lambda d. \tag{5}$$

Incorporating these boundaries of θ_{diff} into the basic diffraction grating equation of

$$n_{\text{sub}}(0,\lambda)\sin(\theta_{\text{diff}}) = -n_{\text{sup}}\sin(\theta_{\text{inc}}) + \lambda/\Lambda_x \tag{6}$$

leads to maximum allowable incidence angle $\theta_{\text{inc,max}}$ in air ($n_{\text{sup}}=1$), as

$$\sin(\theta_{\text{inc,max}}) = \frac{\lambda}{\Lambda_x} - n_{\text{sub}}(0,\lambda)\sqrt{1-(g_\lambda d)^2}. \tag{7}$$

Relations (5) and (6) were also used to determine the bounds for the range of allowable wavelengths $\lambda_{\min}$ and $\lambda_{\max}$ to yield, for $n_{\text{sup}}=1$,

$$\lambda_{\min} - \Lambda_x[\sin(\theta_{\text{inc,max}}) + n_{\text{sub}}(0,\lambda_{\min})\sqrt{1-(g_{\lambda_{\min}} d)^2}] = 0, \tag{8}$$

$$\lambda_{\max} - \Lambda_x[\sin(\theta_{\text{inc,max}}) + n(0,\lambda_{\max})] = 0. \tag{9}$$

We considered a specific example where the GRIN substrate of thickness $d=3$ mm is modeled as a stratified medium made of infinitesimally thin glass plates, each with a uniform refractive index. We assumed that the refractive index of the top plate is $n_{\text{sub}}(0,\lambda)=1.8083$, while that of the bottom plate $n_{\text{sub}}(d,\lambda)=1.4635$, both for $\lambda=480$ nm, corresponding to SF 11 glass and fused silica, as calculated from Sellmeir's dispersion formulas.[8] The corresponding gradient constant found by solving Eq. (2) with all orders of g_λ is $g_\lambda=0.2219\ \text{mm}^{-1}$, and consequently, from Eq. (7) with normally incident light, the grating period is $\Lambda_x=355.7$ nm.

We also calculated the refractive indices $n_{\text{sub}}(0,\lambda)$ and $n_{\text{sub}}(d,\lambda)$ and gradient constants $g(\lambda)$ for the entire spectrum of visible light. Then, using Eqs. (8) and (9), with $\Lambda_x=355.7$ nm and with $n_{\text{sub}}(0,\lambda)$, $n_{\text{sub}}(d,\lambda)$, and g_λ as variables, we determined the bounds $\lambda_{\min}$ and $\lambda_{\max}$ of the allowable range of wavelengths $\Delta\lambda$ for a selected three incidence angles $\theta_{\text{inc,max}}=-4°,0°,4°$. The results are summarized in Table 1. They reveal that the range of wavelengths decreases and shifts toward the longer wavelengths as the incidence angles vary from −4° to +4°.

The beam trajectories for three selected wavelengths were calculated by using Eq. (3). The results are presented in Fig. 3 for two successive bounces, al-

Table 1. Bounds and allowable range of wavelengths for $\theta_{\text{inc,max}}=-4°,0°,4°$

$\theta_{\text{inc,max}}$	$\lambda_{\min}$ (nm)	$\lambda_{\max}$ (nm)	$\Delta\lambda$ (nm)
−4°	454.2	608.9	154.7
0°	480	632.6	152.6
4°	505.6	656.4	150.8

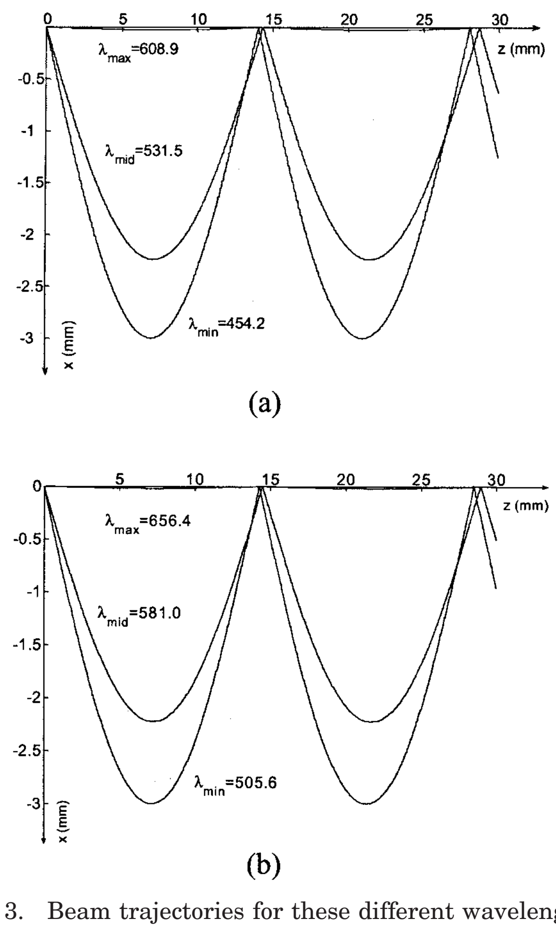

Fig. 3. Beam trajectories for these different wavelengths. (a) Incidence angle $\theta_{\text{inc,max}}=-4°$; (b) incidence angle $\theta_{\text{inc,max}}=4°$.

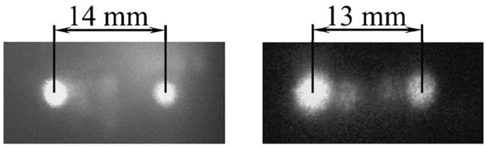

Fig. 4. Experimental detection of light beams decoupled from the stratified substrate, showing the distance between two successive bounces, z_p, for two different wavelengths: (a) $\lambda=476.5$ nm, (b) $\lambda=568.2$ nm. The relative color displacement is 1 mm.

though typically we assume only one bounce. Figure 3(a) shows the trajectories at the incidence angle $\theta_{\text{inc,max}}=-4°$, for wavelengths $\lambda_{\min}=454.2$ nm, $\lambda_{\text{mid}}=531.5$ nm, and $\lambda_{\max}=608.9$ nm. Figure 3(b) shows the trajectories at the incidence angle $\theta_{\text{inc,max}}=4°$, for wavelengths $\lambda_{\min}=505.6$ nm, $\lambda_{\text{mid}}=581.0$ nm and $\lambda_{\max}=656.4$ nm. As is evident, the distances z_p slightly vary with wavelength, indicating that we cannot completely eliminate color displacement.[13]

To experimentally verify our approach, we found it convenient to resort to a planar optics configuration with two laterally displaced gratings arranged on the stratified substrate prepared as a stack of five glass plates, K-10 (bottom plate), BAK-5, LAK-9, SF-10, and SF-6 (top plate), with the corresponding refractive indices at the sodium d line of $n_d=1.5014, 1.5570, 1.6910, 1.7283, 1.8052$. The glass plates, each of 1 mm thickness, were attached to each other with the proper index-matching liquids. The gratings were obtained by recording the interference pattern of two plane waves that were derived from the argon laser of wavelength $\lambda=363.8$ nm, in a Shipley Ultra-i 123 photoresist layer that was coated onto the uppermost SF-6 glass plate.

Both input and output gratings had a period of 360 nm. The input grating diffracts the normally incident light beam of wavelength $\lambda=476.5$ nm into the substrate at the angle $\theta_{\text{diff}}=47.2°$ and of wavelength $\lambda=568.2$ nm at the angle $\theta_{\text{diff}}=61°$. Thus, the corresponding distances between two successive bounces inside the stratified substrate, calculated by using Snell's law,[8] applied to each interface between two adjacent glass plates, will be about 14.02 mm and 13.29 mm, so color displacement should be about 0.73 mm. For a comparable planar optics configuration with a uniform SF-6 glass substrate of 5 mm thickness, these distances would be about 10.78 mm and 18.01 mm, so color displacement should be about 7.23 mm.

The input grating was first illuminated with a normally incident beam of wavelength $\lambda=476.5$ nm derived from an argon laser, and then with one of wavelength $\lambda=568.2$ nm derived from the krypton laser. Two successive spots indicating the distance between adjacent bounces, z_p, were detected at the output grating. The results are shown in Fig. 4. As predicted, distances between two successive bounces inside the stratified substrate were about 14 mm for the light beam of wavelength $\lambda=476.5$ nm, as depicted in Fig. 4(a), and about 13 mm for the light beam of wavelength $\lambda=568.2$ nm, as depicted in Fig. 4(b). Accordingly, experimental color displacement is about 1 mm.

In summary, we showed that color displacement can be significantly reduced in planar optics configurations by resorting to gradient- rather than uniform-refractive-index substrates. In our experimental demonstration the color displacement was reduced by a factor of 7 when a substrate with a uniform refractive index was compared with that of a gradient-refractive index formed by stacking five glass plates. This was achieved for a wavelength range of about 100 nm with gradient-refractive-index variation $\Delta n=0.3$. The wavelength range could be further increased by resorting to larger gradient-refractive-index variations.

L. Eisen's e-mail address is leon.eisen@weizmann.ac.il.

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