Statistical Mechanics Fall 2014 — Problem Set 4

due: Wednesday, December 31, 2014

4.1 Renormalization of Ising model in one dimension (30 points)

In this problem we will repeat the decimation procedure for the one dimensional Ising model with zero magnetic field, but with $b = 3$, i.e. summing over two spins out of every three.

(a) Find the renormalization group equations in this case for the parameters of the free energy.

(b) Find the fixed points of the flow, and analyze their stability.

(c) Correlation length exponent (near unstable fixed point): using $t = e^{-4K}$, what is ν?

4.2 Renormalization of spin 1 Ising model in one dimension (25 points)

Consider the spin 1 Ising model in one dimension, i.e. the Ising model with $S_i \in \{-1, 0, 1\}$, in zero magnetic field:

$$\mathcal{H}_{\mathcal{B}} = -J \sum_i S_i S_{i+1}$$

(a) Show that decimation on every other spin ($b = 2$) generates new terms which are not included in $\mathcal{H}_{\mathcal{B}}$.

(b) Write down the simplest generalization of $\mathcal{H}_{\mathcal{B}}$ whose parameter space is closed under such RG.

4.3 One dimensional Potts model (45 points)

Consider a one dimensional array of N Potts spins $S_i = 1, 2, ..., q$, subject to the Hamiltonian $\beta\mathcal{H} = -J \sum_i \delta_{S_i, S_{i+1}}$ (here $\delta_{a,b}$ is the Kronecker Delta function, $\delta_{a,b} = 1$ if $a = b$ and 0 otherwise).

(a) Calculate (using the transfer matrix or any other method) the partition function Z, and the correlation length ξ.

(b) Is the system critical at zero temperature for antiferromagnetic couplings $J < 0$?

(c) By eliminating every other spin ($b = 2$ decimation), write down the recursion relations for the coupling J and the additive constant of the free energy.

(d) Discuss the fixed points (for $J \geq 0$), and their stability.

(e) Repeat the RG calculation of part (c), when a small symmetry breaking term $h \sum_i \delta_{S_i,1}$ is added to $\beta\mathcal{H}$. You will find that an additional coupling term $K \sum_i \delta_{S_i,1}\delta_{S_{i+1},1}$ is genererated under RG. Calculate the recursion relations in the three parameter space (J, K, h).