Statistical Mechanics 2011–12 — Problem Set 7

Due: At review tutorial (check website)
NOTE: late submissions will not be accepted

7.1 RG for the Ising model on a 2D triangular lattice

The ising model on a 2D triangular lattice with no external field is defined by the Hamiltonian

$$\beta\mathcal{H} = -K \sum_{\langle ij \rangle} S_i S_j, \tag{1}$$

where $S_i = \pm 1$, and the sum runs over all nearest-neighbor pairs. The spins are located on the vertices of a triangular lattice.

(a) Perform a single RG transformation by decimating the spins indicated in figure 1a, and write down the resulting renormalized Hamiltonian. Identify the new interactions that are generated by this transformation.

(b) To obtain a closed set of recursion relations, use a different RG transformation according to the following two-step Migdal-Kadanoff scheme. First, erase all the spins indicated in figure 1a, and move the bonds associated with them to the remaining bonds. This should result in the lattice shown in figure 1b (the double lines in figure 1b indicate that the strength of the new bonds differs from the original bond strangth K. You should determine what this new strength is). Then, decimate the spins indicated in figure 1b, resulting in the lattice shown in figure 1c. Write down the recursion relations for $K'(K)$ resulting from this RG transformation.

(c) Identify all fixed points and note their stability. Draw the resulting flow diagram.

(d) Find the critical temperature and the scaling exponents y_t and ν within this Migdal-Kadanoff approximation.

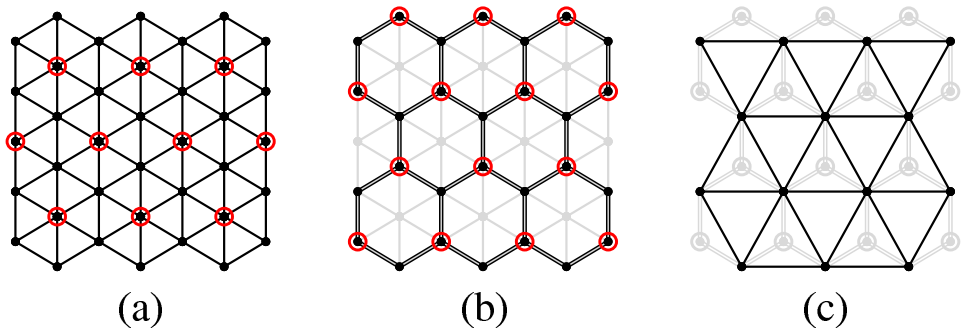

Figure 1: RG for the Ising model on a triangular lattice.