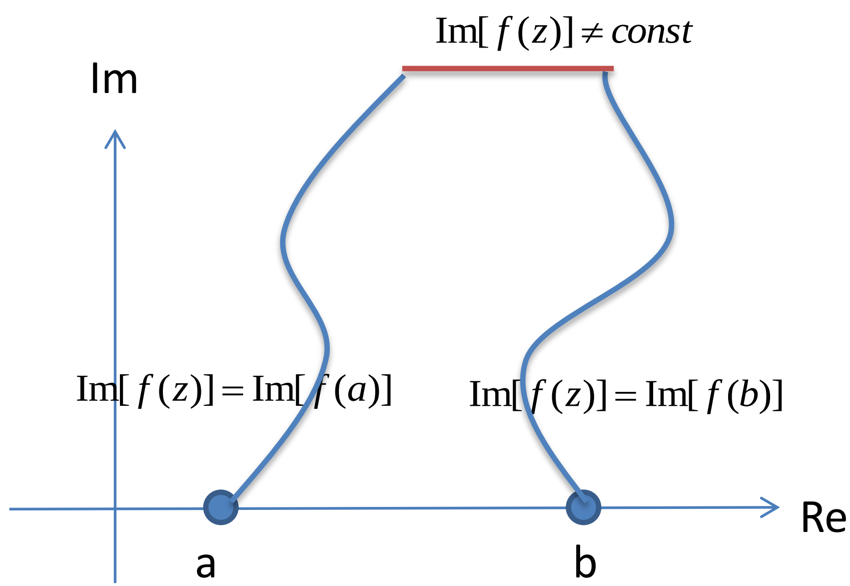
Clarification regarding the stationary wave method

In class we wanted to compute $\int_a^b g(x)e^{f(x)}dx = \int_a^b g(z)e^{f(z)}dz$

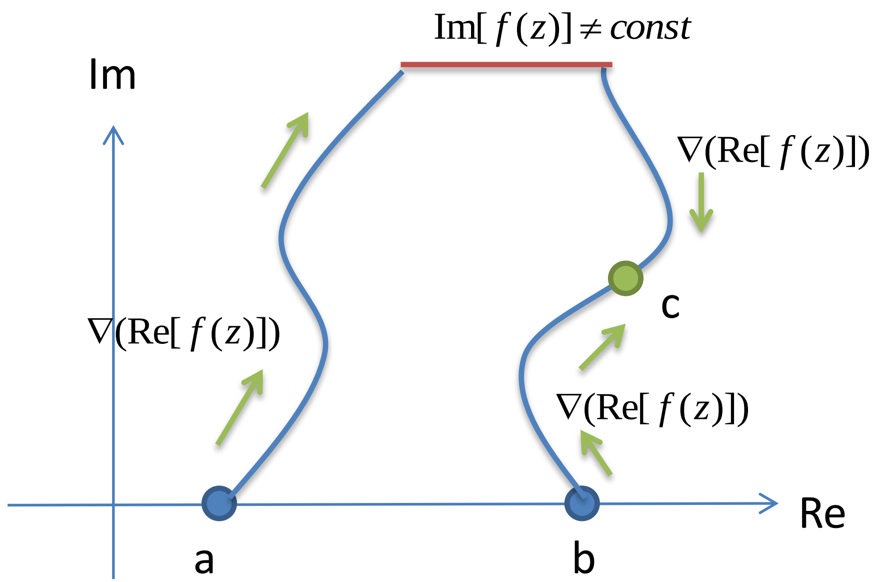
where x is real and f(x) is complex. We replaced it by an integration over complex z.

In class you we're told that there is a path in complex plane from a to b we the imaginary part of f(z) is constant. How could it be if Im[f(a)] ≠ Im[f(b)] ?

From a and b we follow in the complex plane a path where the phase is indeed constant



On the blue line the phase is contant and it changed only on the red line.
When the blue lines are extended to infinity the integral over the red line can be shown to be negligible.



Since f(z) is analytic, constant phase path (blue line) where the derivative of the imaginary part is zero are also gradient paths of the real part (steepest descent or steepest ascent). Therefore the saddle point has to be found somewhere on one of the blue lines.

On the blue lines the integral can be evaluated by it's maximum (at the saddle point) using the usual Laplace method in two dimensions :

$$\int_a^b g(x)e^{Lf(x)}dx \approx \sqrt{\frac{2\pi}{L\left|H[\mathrm{Re}(f(c))]\right|}}\,g(c)e^{f(c)}$$

were H denotes the Hessian matrix.