Statistical Mechanics 2014/2015
Sample problem on Langevin equation and Fokker-Plank equation

1 Johnson thermal noise on a resistor

Consider an electric circuit composed of a resistor with resistance R connected in series to a capacitor with capacitance C, subject to thermal random noise $\xi(t)$.

- Write down the Langevin equation (in terms of the instantaneous charge Q on the capacitor) of this circuit and use it to calculate the non-equilibrium time correlation of the voltage across the capacitor. Assume the random driving voltage is uncorrelated white noise, $\langle \xi(t)\xi(t') \rangle = q\delta(t-t')$.

- Write down the energy stored in the capacitor, and use the equipartition theorem to determine q.

- Take the long time limit of the voltage fluctuations, and use it to calculate the power-spectrum of the voltage fluctuations. Take the limit $C \to 0$ and show that the fluctuations become white noise. This noise is the thermal Johnson noise on a resistor at temperature T.

- Write down the Fokker-Planck equation for the dynamics of this circuit and find the steady state distribution of the charge of the capacitor.