Maxwell's Eqns:

1. $\vec{\nabla} \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t}$ $\qquad \vec{\nabla} = \left(\dfrac{\partial}{\partial x}\hat{x}, \dfrac{\partial}{\partial y}\hat{y}, \dfrac{\partial}{\partial z}\hat{z}\right)$

2. $\vec{\nabla} \times \vec{H} = \varepsilon \dfrac{\partial \vec{E}}{\partial t}$ $\qquad$ (MKS)

In a charge free region.

3. $\vec{\nabla} \cdot \varepsilon \vec{E} = 0$

4. $\vec{\nabla} \cdot \mu \vec{H} = 0$ $\qquad$ Dialectric medium

μ - permeability

ε - permitivity

take $\vec{\nabla}\times$ of 1.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla} \times \mu \frac{\partial \vec{H}}{\partial t} = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

use vector field identity $\quad \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \vec{\nabla}^2 \vec{E}$

0 in a linear homogeneous medium

$$\Rightarrow \nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \text{wave Eq.}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad n = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$$

speed of light in vacuum.

set $\mu = \mu_0$ (non-magnetic material)

OA

what happens if $\varepsilon = \varepsilon(\vec{r}) \Rightarrow n = n(\vec{r})$

$$\vec{\nabla}\cdot\varepsilon\vec{E} = \frac{\partial}{\partial x}\varepsilon(\vec{r})E_x(\vec{r}) + \frac{\partial}{\partial y}\varepsilon(\vec{r})E_y(\vec{r}) + \frac{\partial}{\partial z}\varepsilon(\vec{r})E_z(\vec{r})$$

$$= \frac{\partial\varepsilon}{\partial x}E_x + \frac{\partial\varepsilon}{\partial y}E_y + \frac{\partial\varepsilon}{\partial z}E_z + \varepsilon(\vec{r})\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\varepsilon\cdot\vec{E} + \varepsilon\vec{\nabla}\cdot\vec{E} = 0$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = -\frac{\vec{\nabla}\varepsilon}{\varepsilon}\cdot\vec{E} = -2\vec{\nabla}\ln(n(\vec{r}))\cdot\vec{E}$$

because of $\sqrt{\ }$

$$\vec{\nabla}\left(-\frac{\vec{\nabla}\varepsilon}{\varepsilon}\cdot\vec{E}\right) - \vec{\nabla}^2\vec{E} = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$\Rightarrow$ wave equation:

$$\nabla^2\vec{E} + 2\vec{\nabla}\ln(n)\cdot\vec{E} - \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

lecture notes: Lesson # 1

optical propagation

the electric field wave equation for a $\left\{\begin{array}{l}\text{linear}\\ \text{homogeneous}\\ \text{isotropic}\end{array}\right.$ medium

$$\nabla^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad ; \quad \vec{E} = \vec{E}(\vec{r}, t) \quad ; \quad n = \sqrt{\mu\varepsilon} \simeq \sqrt{\varepsilon}$$

where ε is the dialectric constant $\varepsilon = 1 + 4\pi\chi$ $\quad P = \chi E$ ↳ susceptibility.

Assume we can decompose $\vec{E}$ into monochromatic contributions.

$$\vec{E}(\vec{r}, t) = \sum_{\omega} f_{\omega} e^{i\omega t} \vec{E}_{\omega}(\vec{r}) \to \int f(\omega) \vec{E}_{\omega}(\vec{r}) e^{i\omega t} d\omega$$

Get the Helmholtz equation:

$$(\nabla^2 + k^2)\vec{E}_k(\vec{r}) = 0 \quad ; \quad \text{where} \quad k = \frac{\omega n}{c}$$

A simple solution: plane wave

$$\vec{E}_k(\vec{r}) = \hat{\varepsilon} E_0 e^{i\vec{k}\cdot\vec{r}}$$

(polarization ind. of $\vec{r}$)

For a linear, homogeneous and isotropic then all components of the electric and magnetic field are described by the same scalar function $\varepsilon(\vec{r})$; reduce to a scalar Helmholtz eq.

$$(\nabla^2 + k^2) E(\vec{r}) = 0$$

nice, since the polarization is uniform and factors out: $\vec{E}(\vec{r}) = \hat{\varepsilon} E(\vec{r})$