

- One electron in the valence shell; “Alkali like”  $^2S_{1/2}$  ground state.

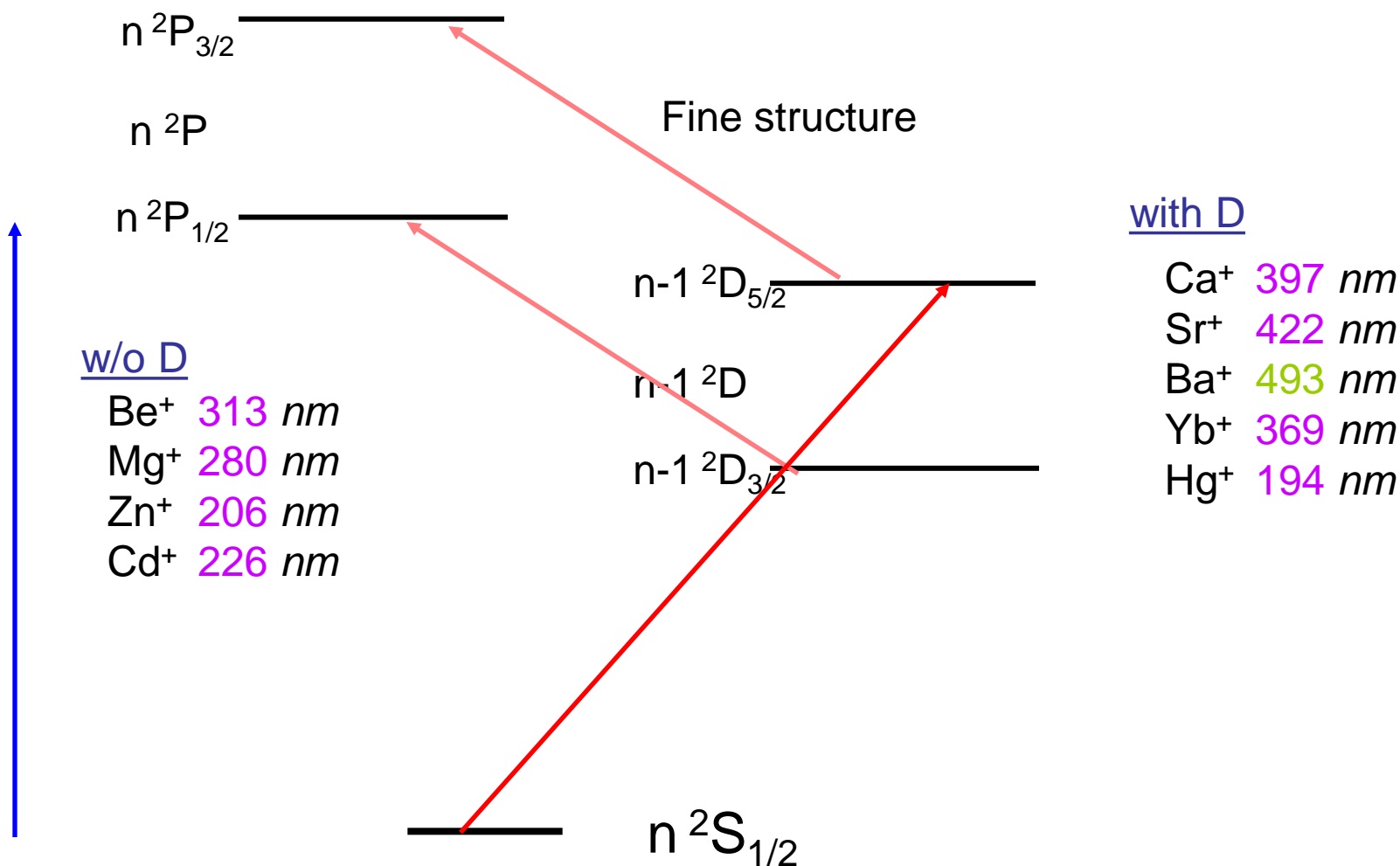
hydrogen 1 H 1.0079														helium 2 He 4.0026																							
lithium 3 Li 6.941		beryllium 4 Be 9.0122														boron 5 B 10.811		carbon 6 C 12.011		nitrogen 7 N 14.007		oxygen 8 O 15.999		fluorine 9 F 18.998		neon 10 Ne 20.180											
sodium 11 Na 22.990		magnesium 12 Mg 24.305														aluminium 13 Al 26.982		silicon 14 Si 28.086		phosphorus 15 P 30.974		sulfur 16 S 32.065		chlorine 17 Cl 35.453		argon 18 Ar 39.948											
potassium 19 K 39.098		calcium 20 Ca 40.078		scandium 21 Sc 44.956		titanium 22 Ti 47.867		vanadium 23 V 50.942		chromium 24 Cr 51.996		manganese 25 Mn 54.938		iron 26 Fe 55.845		cobalt 27 Co 58.933		nickel 28 Ni 58.693		copper 29 Cu 63.546		zinc 30 Zn 65.39		gallium 31 Ga 69.723		germanium 32 Ge 72.61		arsenic 33 As 74.922		selenium 34 Se 78.96		bromine 35 Br 79.904		krypton 36 Kr 83.80			
rubidium 37 Rb 85.468		strontium 38 Sr 87.62		yttrium 39 Y 88.906		zirconium 40 Zr 91.224		niobium 41 Nb 92.906		molybdenum 42 Mo 95.94		technetium 43 Tc [98]		ruthenium 44 Ru 101.07		rhodium 45 Rh 102.91		palladium 46 Pd 106.42		silver 47 Ag 107.87		cadmium 48 Cd 112.41		indium 49 In 114.82		tin 50 Sn 118.71		antimony 51 Sb 121.76		tellurium 52 Te 127.60		iodine 53 I 126.90		xenon 54 Xe 131.29			
caesium 55 Cs 132.91		barium 56 Ba 137.33		lanthanum 57-70 ✱		lutetium 71 Lu 174.97		hafnium 72 Hf 178.49		tantalum 73 Ta 180.95		tungsten 74 W 183.84		rhenium 75 Re 186.21		osmium 76 Os 190.23		iridium 77 Ir 202.22		platinum 78 Pt 195.08		gold 79 Au 196.97		mercury 80 Hg 200.59		thallium 81 Tl 204.38		lead 82 Pb 207.2		bismuth 83 Bi 208.98		polonium 84 Po [209]		astatine 85 At [210]		radon 86 Rn [222]	
francium 87 Fr [223]		radium 88 Ra [226]		actinium 89-102 ✱ ✱		lawrencium 103 Lr [262]		rutherfordium 104 Rf [261]		dubnium 105 Db [262]		seaborgium 106 Sg [266]		bohrium 107 Bh [264]		hassium 108 Hs [269]		meitnerium 109 Mt [268]		ununilium 110 Uun [271]		unununium 111 Uuu [272]		unundwium 112 Uub [277]		unungquadium 114 Uuq [289]											

\*Lanthanide series

\* \* Actinide series

lanthanum 57	cerium 58	praseodymium 59	neodymium 60	promethium 61	samarium 62	europium 63	gadolinium 64	terbium 65	dysprosium 66	holmium 67	erbium 68	thulium 69	ytterbium 70
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb
138.91	140.12	140.91	144.24	[145]	150.36	151.96	157.25	158.93	162.50	164.93	167.26	168.93	173.04
actinium 89	thorium 90	protactinium 91	uranium 92	neptunium 93	plutonium 94	americium 95	curium 96	berkelium 97	californium 98	einsteinium 99	fermium 100	mendelevium 101	nobelium 102
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No
[227]	232.04	231.04	238.03	[237]	[244]	[243]	[247]	[247]	[251]	[252]	[257]	[258]	[259]

# Electronic levels Structure



# Measurement: state selective fluorescence

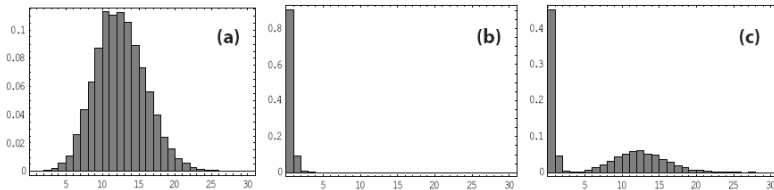
Optical qubit

$n^2P_{3/2}$

Fine structure

$n^2P_{1/2}$

After 200  $\mu\text{sec}$ :



$n-1^2D_{5/2}$

$n-1^2D_{3/2}$

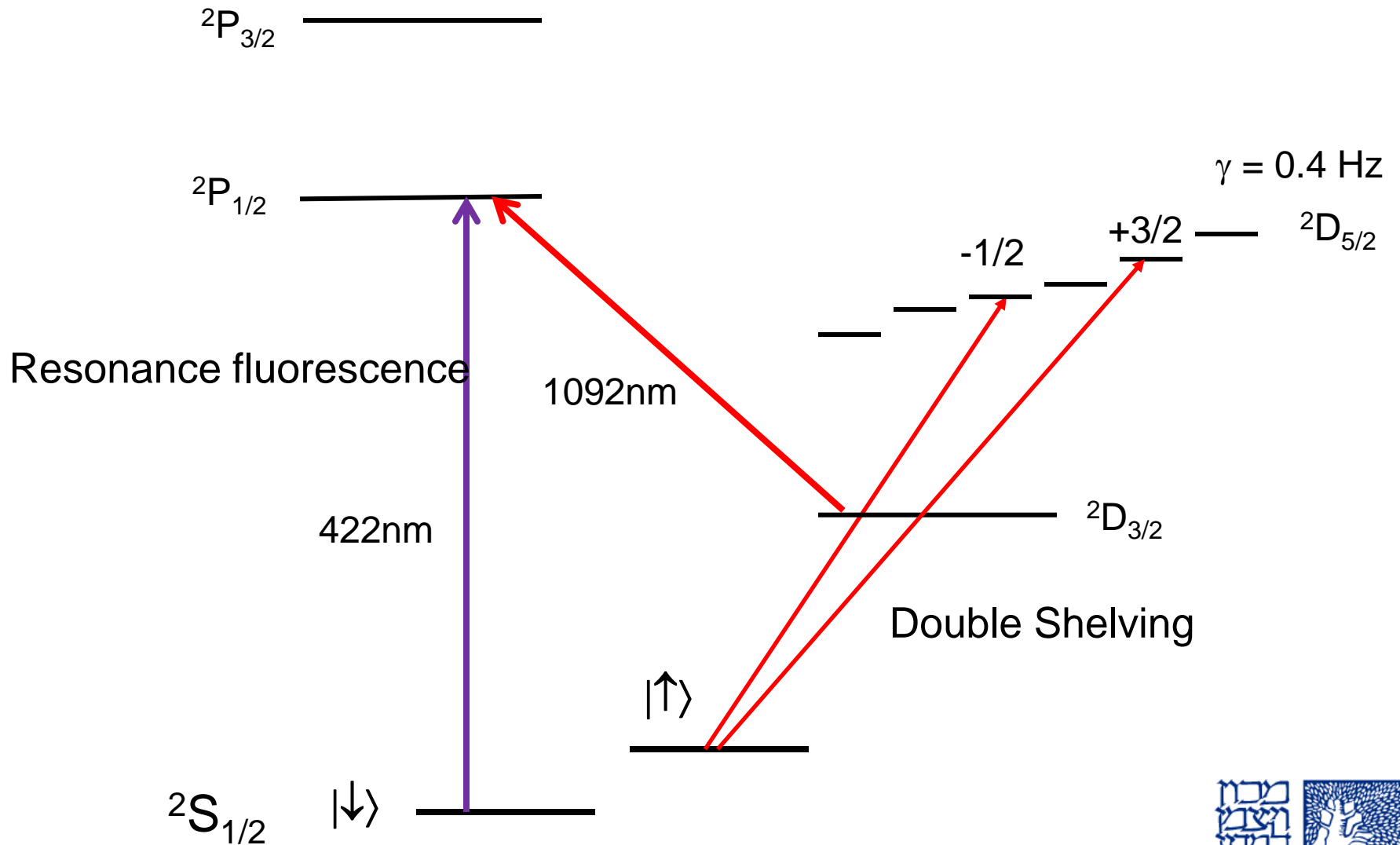
Threshold test

$n^2S_{1/2}$

Error sources:

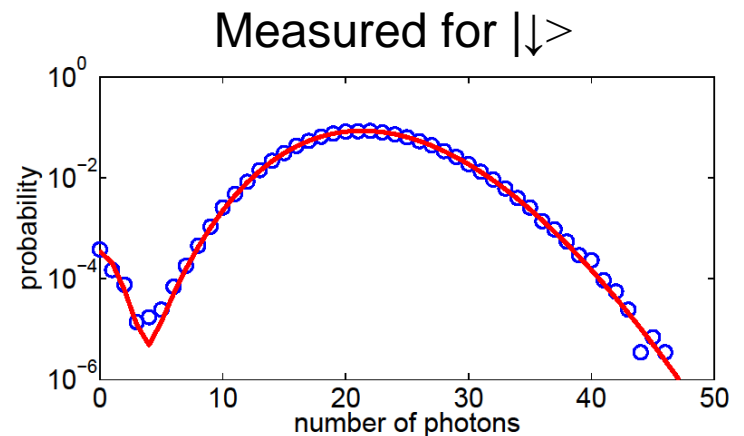
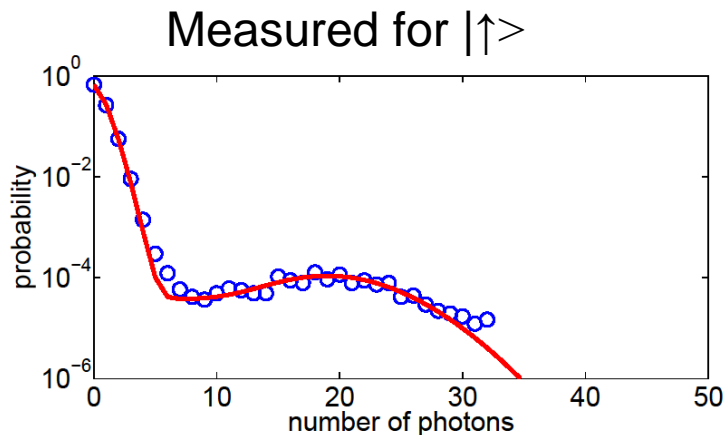
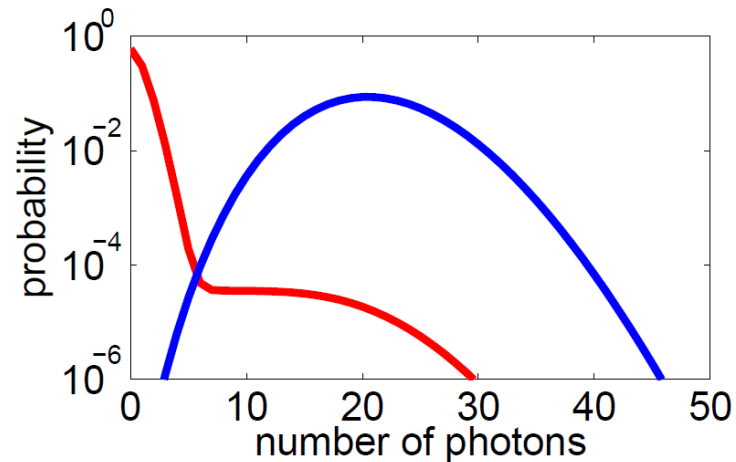
- Dark counts (10/S)
- Laser scatter (100/S)
- State decay.

# Zeeman Qubit Detection



# $^{88}\text{Sr}^+$ Zeeman Qubit Detection

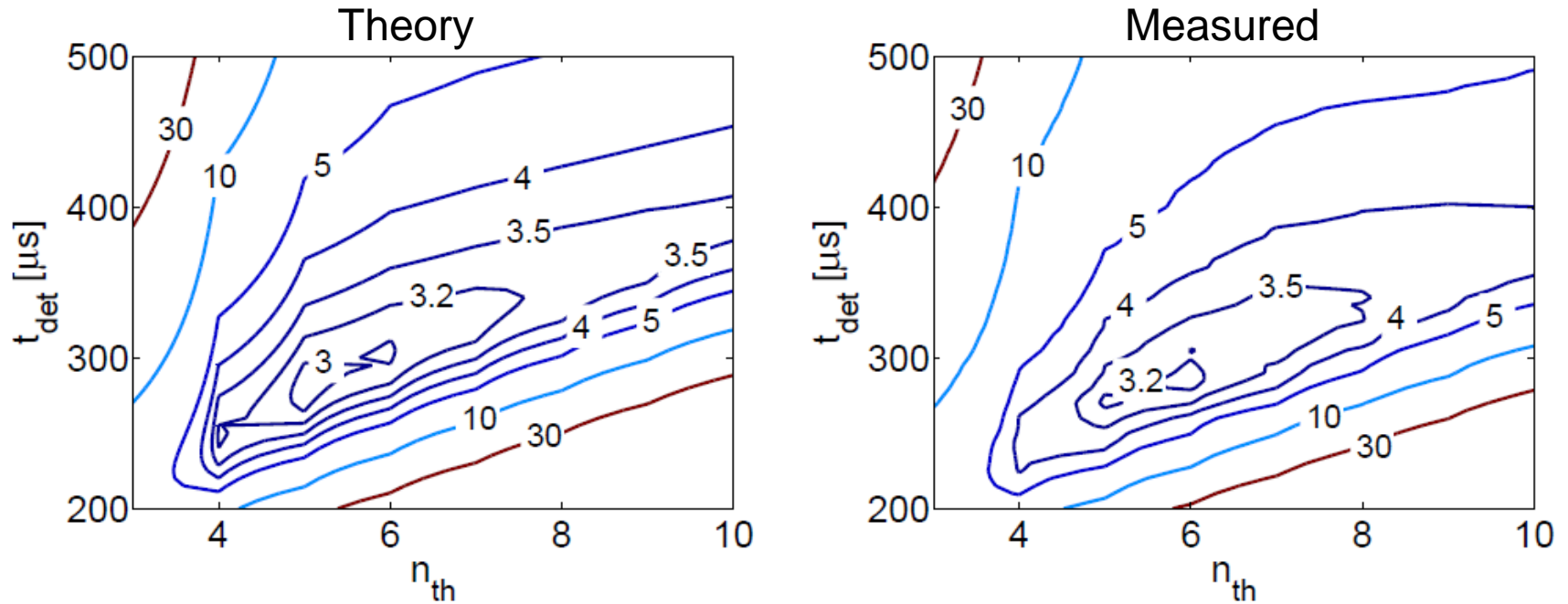
Expected distributions:  
Bright photon detection rate: 73.5 kHz  
Dark photon detection rate: 1.75 kHz  
D level lifetime: 390 ms  
Detection time: 285  $\mu\text{s}$



Initialization and shelving error:  $\varepsilon_{\uparrow} = 1 \cdot 10^{-3}$ ;  $\varepsilon_{\downarrow} = 0.6 \cdot 10^{-3}$

# $^{88}\text{Sr}^+$ Zeeman Qubit Detection

State discrimination error ( $10^{-4}$ )



Minimal State discrimination error =  $0.3 \cdot 10^{-3}$  @  $\tau_{\text{det}} = 285 \mu\text{s}$  and  $n_{\text{threshold}} = 6$

Average Detection fidelity: 0.9989

# fluorescence

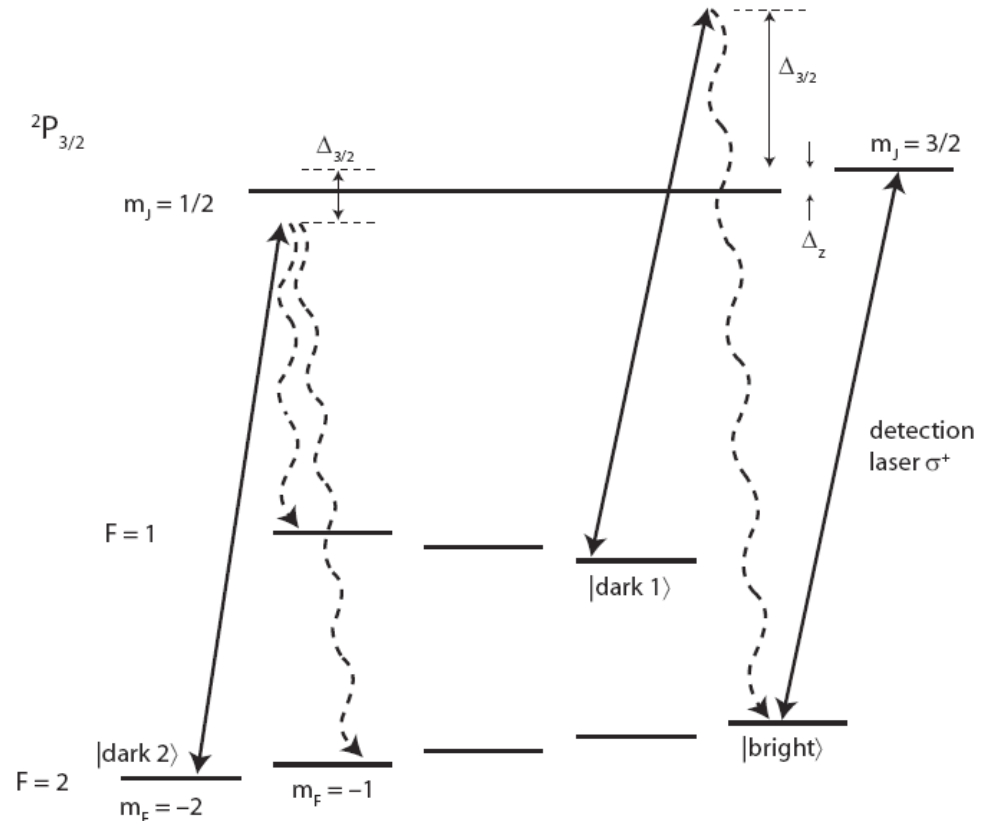
# Hyperfine qubit

## Error Sources:

- Polarization purity (Bright->dark optical pumping).
- Off resonance dark->bright optical pumping.
- Dark counts (10/S).
- Laser scatter (100/S).

## Benefit from:

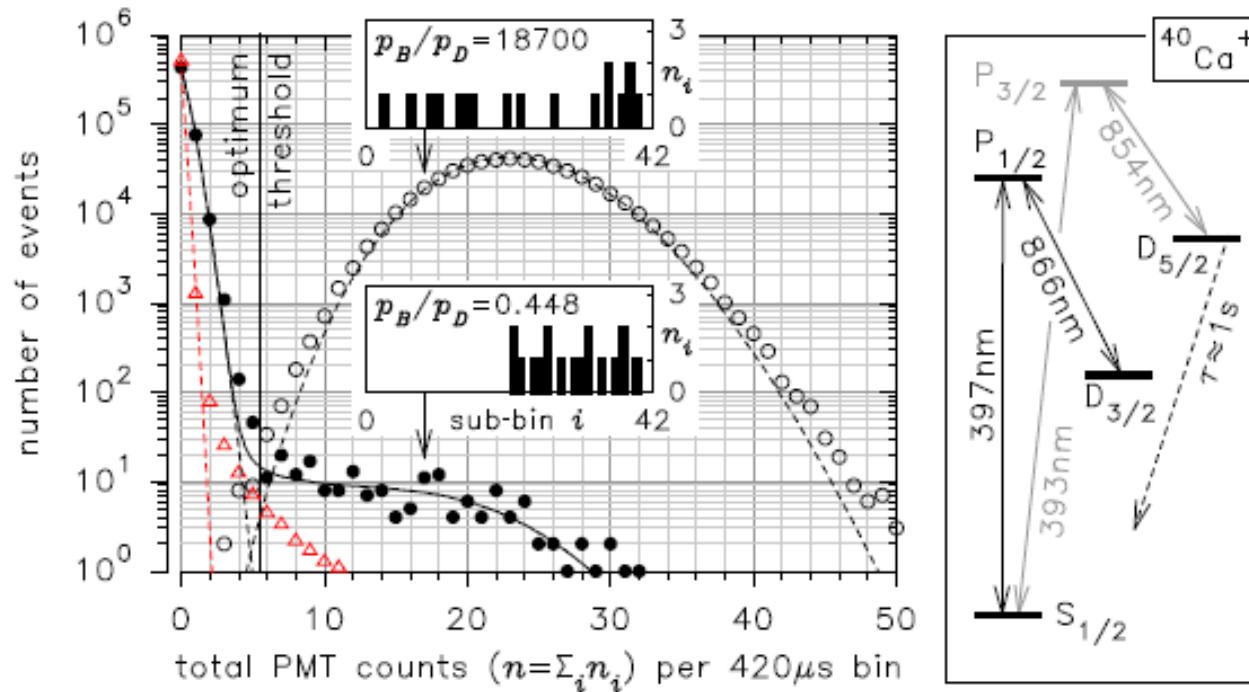
- Large hyperfine splitting.
- Large angular momentum splitting between bright and dark states.



Threshold test:  $\varepsilon \sim 8 \times 10^{-5}$

(C. Langer, Ph.D thesis, 2006) estimated, 200  $\mu$ sec, no background

# Measurement: Photon arrival time analysis



- Maximum likelihood test:  $\varepsilon \sim 8.7 \times 10^{-5}$

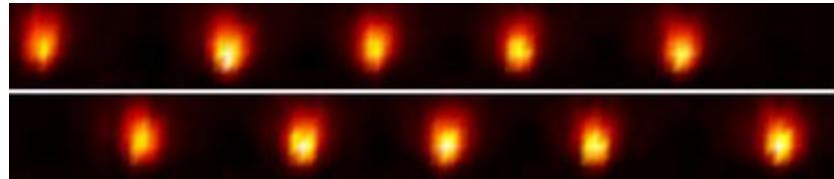
(Myerson et. al. Phys. Rev. Lett. 100, 200502 (2008); Oxford ions)  
225  $\mu\text{sec}$

(As compared with  $1.8 \times 10^{-4}$  threshold error in 420  $\mu\text{s}$ )



# Measurement with CCD

Antiferromagnetic ground-states in quantum magnetism (JQI, Maryland)



- Multiple ions: histograms overlap
- Which ion is bright?
- Slow readout
- Readout noise
- Cross-talk

Highest Camera fidelity (optical qubit; Oxford): 0.9991

Acton et. al. , Quant. Inf. Comp. 6, 465, (2006)

Burrell, Szwer, Webster and Lucas Phys. Rev. A 81, 040302, (2010)

# Measurement: Other

- Photon detection efficiency:

0.6 NA gives 0.99 in 10  $\mu\text{s}$  and 0.9915 in 100  $\mu\text{s}$  (hyperfine qubit, Duke)

Noek et. al. arXiv1304.3511

- Ancilla qubits:

- Entangled ancilla for twice the fluorescence (hyperfine qubit; NIST)

Schaetz et. al. PRL 94, 010501 (2005)

- State transfer to a different species ion (optical qubit; NIST)

Hume et. al. PRL 99, 137205 (2007)

# Single qubit gates

Reviewed in many places e.g. :

- D. Wineland, *Les Houches lecture notes*. (2003).
- D. Wineland *et.al.*, *J. Res. Natl. Inst. Stand. Technol.*, **103**, 259, (1998).
- D. Leibfried *et. al.*, *Rev. Mod. Phys.*, **75**, 281, (2003).
- Coupling between the two qubit levels using e.m. traveling plane waves (far-field).

For a single ion:

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t)$$

$$\hat{H}_0 = \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z + \hbar \omega_m (\hat{a}^\dagger \hat{a} + \frac{1}{2}) \text{ and } \hat{V}(t) = \hbar \Omega_0 (\hat{\sigma}^+ + \hat{\sigma}^-) \cos(k\hat{x} - \omega t + \phi)$$

$$\hat{\sigma}_- = \hat{\sigma}_x + i\hat{\sigma}_y = |\downarrow\rangle\langle\uparrow|$$

$$\hat{\sigma}_+ = \hat{\sigma}_x - i\hat{\sigma}_y = |\uparrow\rangle\langle\downarrow|$$

Coupling strength

$$x_0 = \sqrt{\frac{\hbar}{2M\omega_m}}$$

$$k\hat{x} = kx_{\text{eq}} + kx_0(\hat{a}^\dagger + \hat{a}) \equiv kx_{\text{eq}} + \eta(\hat{a}^\dagger + \hat{a})$$

Typ. 0.05-0.2

# Single qubit gates

In the interaction representation and within the Rotating Wave Appr. (RWA)

$$H_{int}(t) = \hbar\Omega_0/2\hat{\sigma}_+ \exp(i\eta(\hat{a}e^{-i\omega_m t} + \hat{a}^\dagger e^{i\omega_m t}))e^{i(kx_{eq} + \phi - \delta t)} + H.C.$$

When  $\delta = s\omega_m$ , only  $|\downarrow, n\rangle$  and  $|\uparrow, n+s\rangle$  will be resonantly coupled (another RWA).

$$\Omega_{n,n+s} = \Omega_{n+s,n} = \Omega_0 |\langle n+s | e^{i\eta(\hat{a}+\hat{a}^\dagger)} | n \rangle| \equiv \Omega_0 D_{n+s,n}$$

$$D_{n+s,n} = \exp(-\eta^2/2) \eta^{|s|} \left( \frac{n_{<}!}{n_{>}!} \right)^{1/2} L_{n_{<}}^{|s|}(\eta^2) \quad \text{Debye - Waller factor}$$

Carrier:  $s = 0$

$$\hat{H}_{\text{carrier}} = \frac{\hbar\Omega_{n,n}}{2} (\hat{\sigma}_+ \exp(i\phi) + \hat{\sigma}_- \exp(-i\phi))$$

Red sideband (RSB):  $s = -1$

$$\hat{H}_{\text{RSB}} = \frac{\hbar\Omega_{n-1,n}}{2} (\hat{a}\hat{\sigma}_+ \exp(i\phi) + \hat{a}^\dagger\hat{\sigma}_- \exp(-i\phi))$$

Blue sideband (BSB):  $s = +1$   $\hat{H}_{\text{BSB}} = \hat{H}_{\text{int}} = \frac{\hbar\Omega_{n+1,n}}{2} (\hat{a}^\dagger\hat{\sigma}_+ \exp(i\phi) + \hat{a}\hat{\sigma}_- \exp(-i\phi))$

# Lamb – Dicke regime

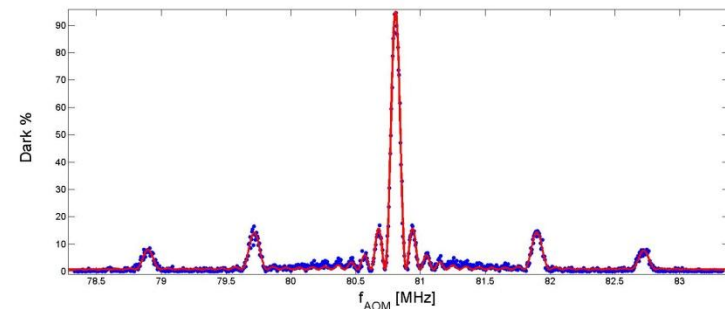
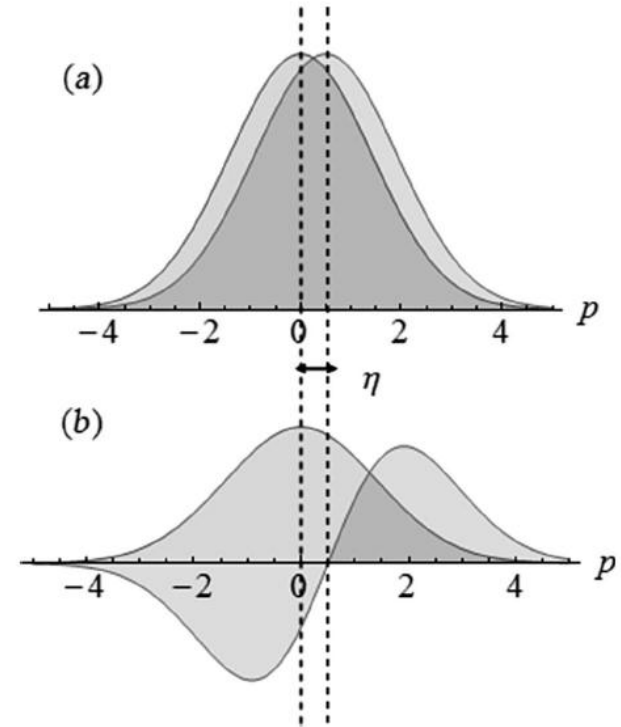
$$\eta \sqrt{\langle (\hat{a}^\dagger + \hat{a})^2 \rangle} \ll 1$$

$$\Omega_{n,n} \simeq \Omega_0 [1 - (n + 1/2)\eta^2],$$

$$\Omega_{n-1,n} \simeq \Omega_0 n^{1/2} \eta,$$

$$\Omega_{n+1,n} \simeq \Omega_0 (n + 1)^{1/2} \eta.$$

- Momentum conservation.
- Bosonic amplification.



# Single qubit gates

Carrier rotations:

Since  $H_{int}$  is  $t$  independent:

$$|\psi(t)\rangle_{int} = e^{-i\hat{H}_{int}t/\hbar}|\psi(0)\rangle_{int} = e^{-i\theta\vec{\sigma}\cdot\vec{n}}|\psi(0)\rangle_{int} \equiv \hat{R}(\theta, \phi)|\psi(0)\rangle_{int}$$

$$\theta = \Omega_0 t \qquad \vec{n} = \begin{pmatrix} \cos(\phi) \\ i \sin(\phi) \\ 0 \end{pmatrix}$$

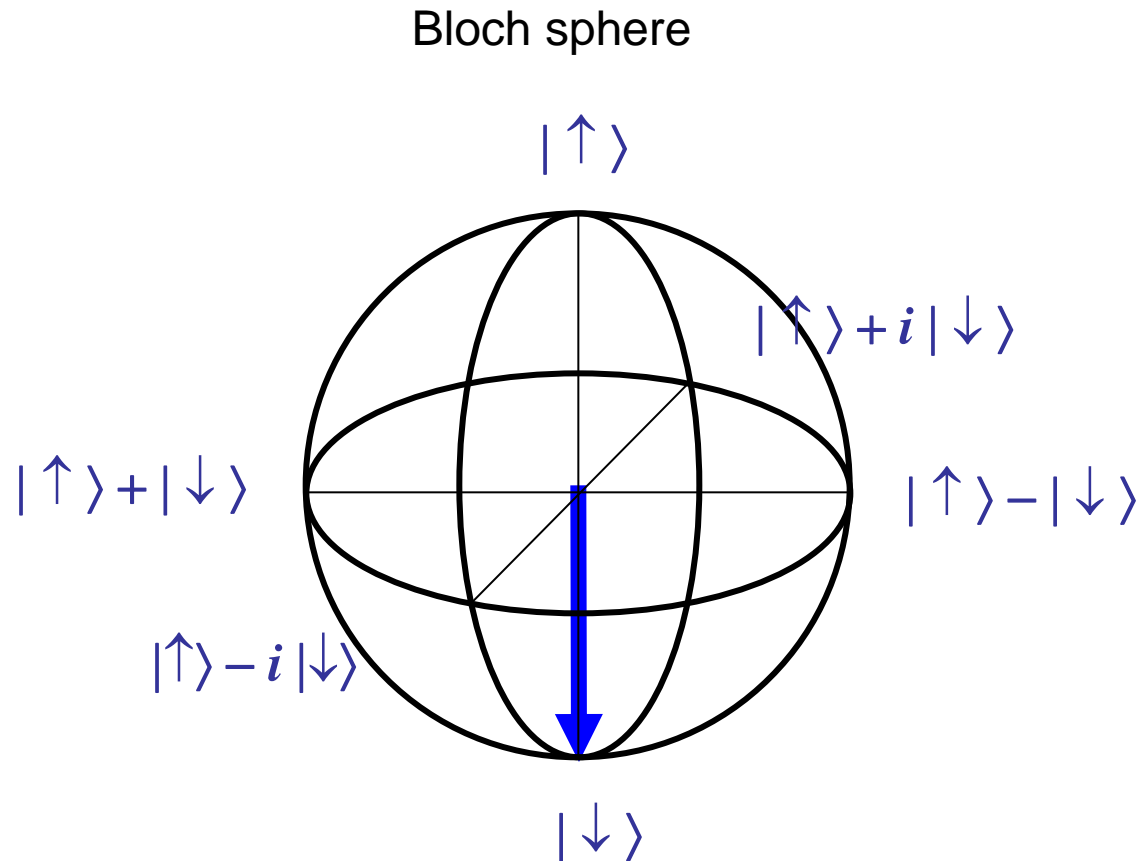
$$\hat{R}(\theta, \phi) = \begin{bmatrix} \cos(\theta/2) & -ie^{i\phi}\sin(\theta/2) \\ -ie^{-i\phi}\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

# Coherent qubit (carrier) rotations

$$R(\theta, \phi)$$

$$R(\pi, 0)$$

$$R(\pi/2, \pi/2)$$



Any single qubit rotation can be composed of 1-3 pulses

# Single qubit gates

RF qubit (Zeeman or Hyperfine)

Magnetic dipole coupling

$$V(t) = -\hat{\mu} \cdot B_0 \cos(k\hat{x} - \omega t + \phi)$$

$$\hat{\mu} = \mu_B(g_S\hat{\mathbf{S}} + g_L\hat{\mathbf{L}} + g_I\hat{\mathbf{I}}) \quad \longrightarrow \quad \Omega_0 = \langle \downarrow | \hat{\mu} \cdot \mathbf{B}_0 | \uparrow \rangle$$

e.g. for a Zeeman qubit:

$$\mathbf{B}_0 = B_0 \hat{x}$$

$$\hat{\mu} = g_S \mu_B \hat{\sigma}$$

$$\approx 2 \quad \approx 1.4 \text{ MHz/G}$$

$$\Omega_0 = 2\pi \times 2.8 B_0 \text{ MHz/G}$$

## Advantages

- Very classical and controlled.

## Disadvantages

- No momentum transfer ( $\eta = 0$ )
- No single qubit addressing.  
(... In the far field)

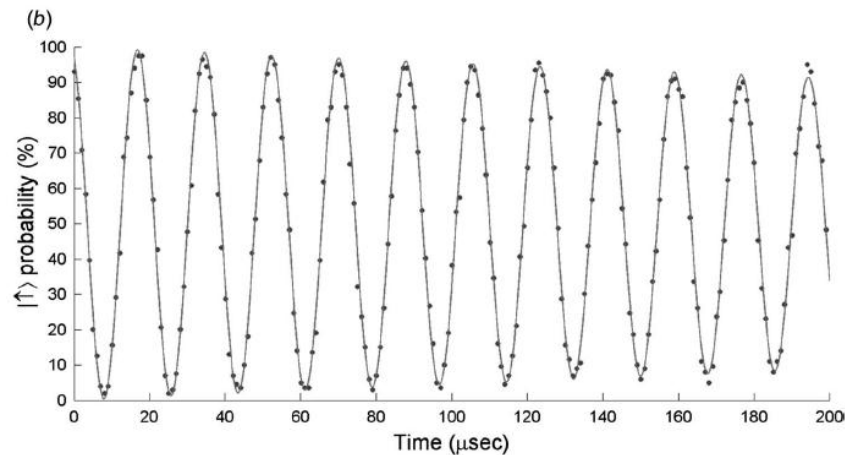


# Single qubit gates

RF qubit (Zeeman or Hyperfine)

Magnetic dipole coupling

e.g.  $^{88}\text{Sr}^+$  Zeeman qubit

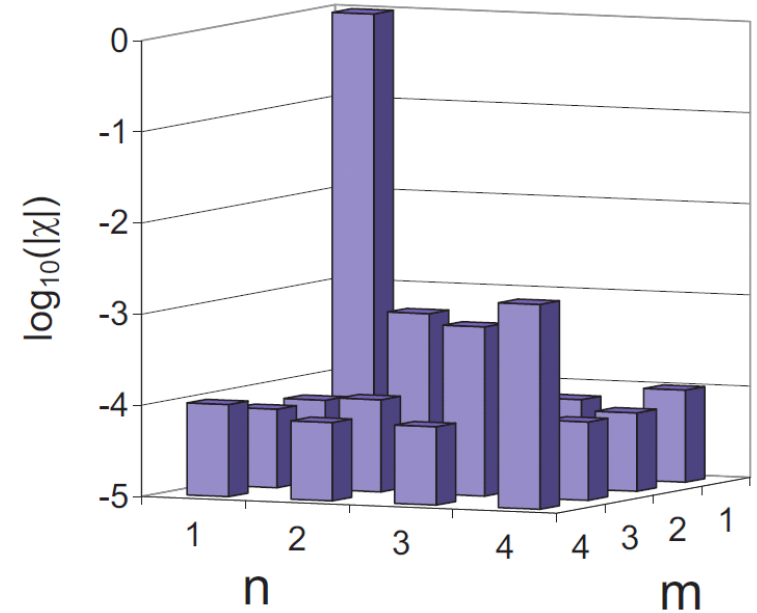
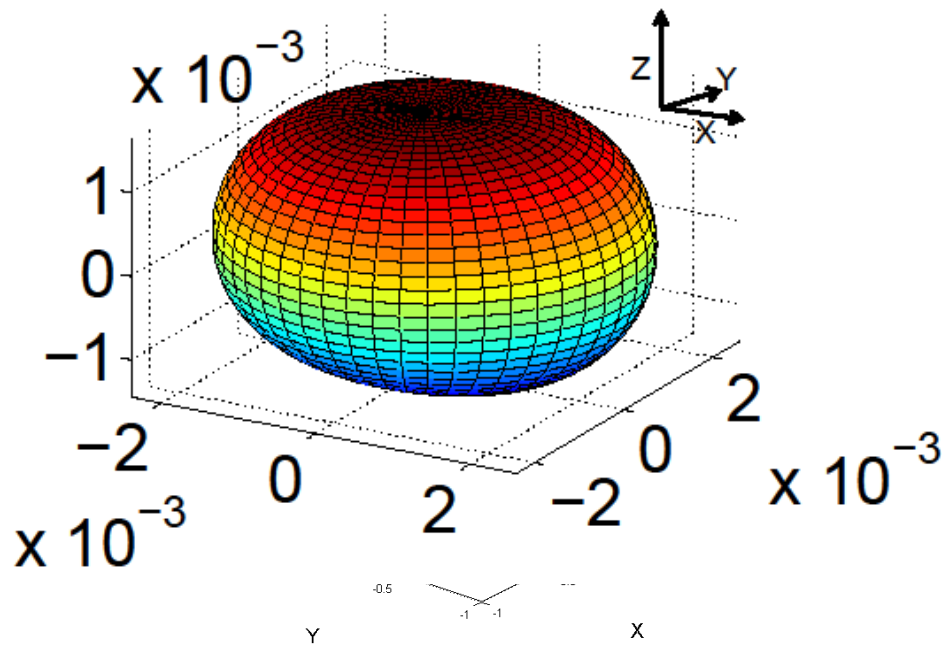


$$\varepsilon = 2 \times 10^{-3}$$

Error sources:

- Fluctuations in RF power.
- Relative phase/frequency noise (e.g. B field noise).

# Process Tomography of the Identity operation



$$E_1 = \hat{I}, E_2 = \hat{\sigma}_x, E_3 = -i\hat{\sigma}_y, E_4 = \hat{\sigma}_z$$

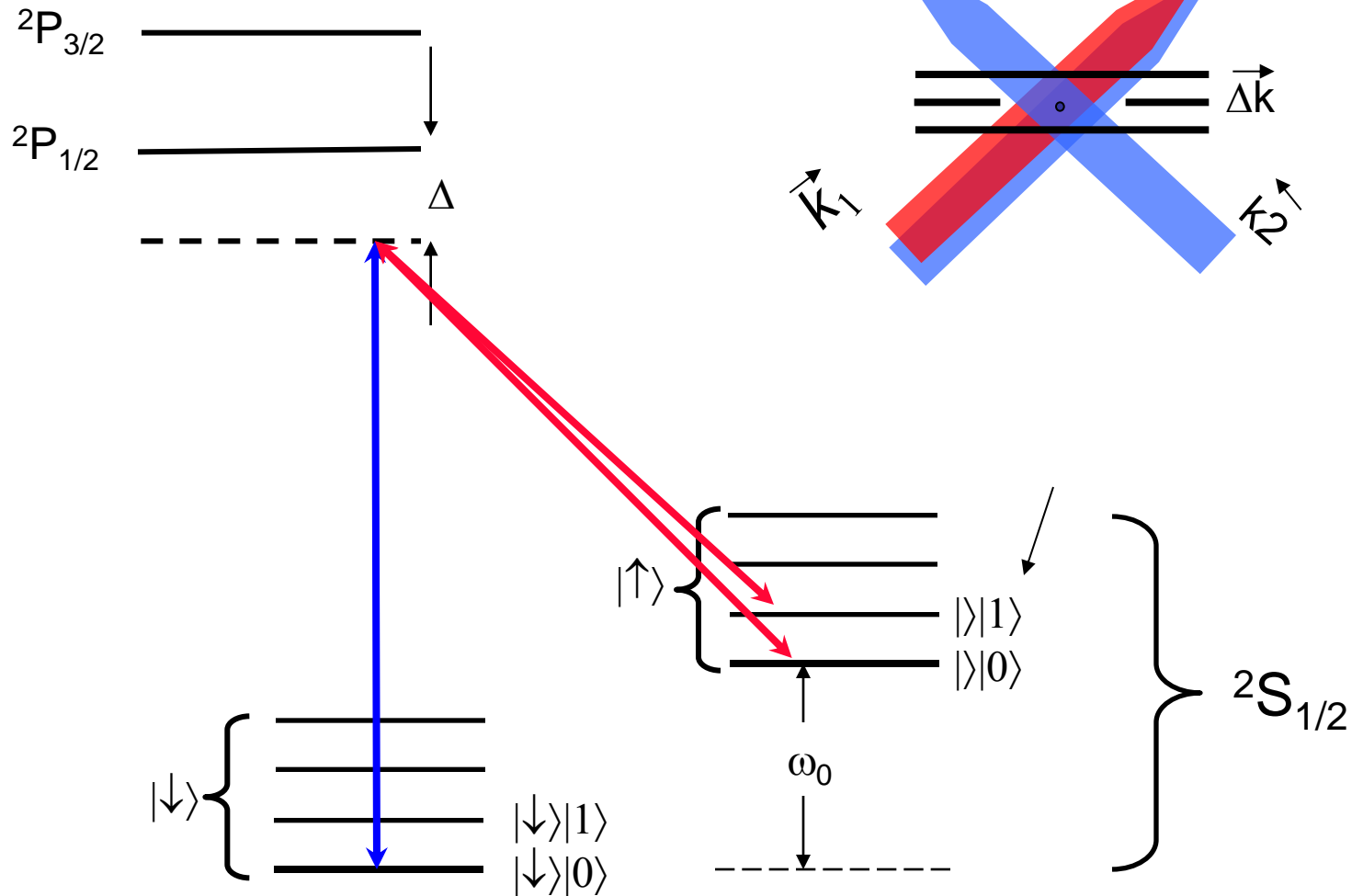
Process tomography Fidelity

$$F_{proc} = Tr(\chi_{ideal}\chi_{proc}) = 0.997(1)$$

# Single qubit gates

RF qubit (Zeeman or Hyperfine)

Two-photon Raman coupling



# Single qubit gates

RF qubit (Zeeman or Hyperfine)

## Two-photon Raman coupling

- Including the  $^2P$  levels we have three (or more) level coupling.
- For large enough  $\Delta$  excited states are “adiabatically eliminated”.
- Back to “effective” two level coupling.

$$\vec{E}_r = \hat{\epsilon}_r E_{r0} \cos(\vec{k}_r \cdot \hat{x} - \omega_r t + \phi_r) \quad \vec{E}_b = \hat{\epsilon}_b E_{b0} \cos(\vec{k}_b \cdot \hat{x} - \omega_b t + \phi_b)$$

For a single excited state:

$$\phi = \phi_b - \phi_r$$

$$\Omega_0 = \frac{E_{r0} E_{b0}}{4\hbar^2} \sum_i \frac{\langle \uparrow | \hat{\mathbf{d}} \cdot \vec{\epsilon}_r | e_i \rangle \langle e_i | \hat{\mathbf{d}} \cdot \vec{\epsilon}_b | \downarrow \rangle}{\Delta_i}$$

$$\vec{k} = \Delta \vec{k} = \vec{k}_b - \vec{k}_r$$

# Single qubit Raman gates

## Stark shifts

$$\hat{H}_{int} = \frac{\hbar\Omega_0}{2} D_{n,n} (\hat{\sigma}_+ e^{i\phi} + \hat{\sigma}_- e^{-i\phi}) + (\Delta_{\uparrow} - \Delta_{\downarrow}) \hat{\sigma}_z$$

$$\Delta_{\uparrow} = \frac{|E_r|^2}{4\hbar^2} \sum_i \frac{|\langle \uparrow | \hat{\mathbf{d}} \cdot \vec{\epsilon}_r | e_i \rangle|^2}{\Delta_{i,r}} + \frac{|E_b|^2}{4\hbar^2} \sum_i \frac{|\langle \uparrow | \hat{\mathbf{d}} \cdot \vec{\epsilon}_b | e_i \rangle|^2}{\Delta_{i,b}}$$

$$\Delta_{\downarrow} = \frac{|E_r|^2}{4\hbar^2} \sum_i \frac{|\langle \downarrow | \hat{\mathbf{d}} \cdot \vec{\epsilon}_r | e_i \rangle|^2}{\Delta_{i,r}} + \frac{|E_b|^2}{4\hbar^2} \sum_i \frac{|\langle \downarrow | \hat{\mathbf{d}} \cdot \vec{\epsilon}_b | e_i \rangle|^2}{\Delta_{i,b}}$$

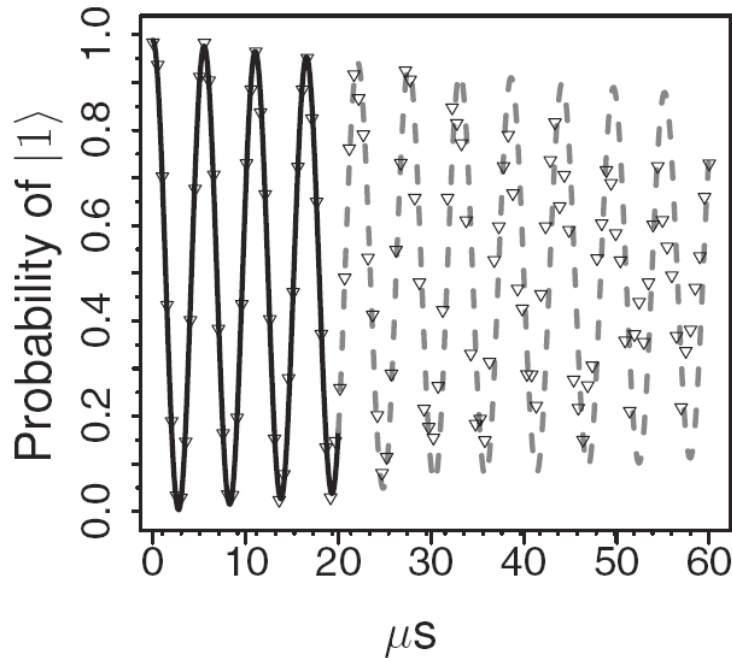
- Differential Stark shift can be tuned to zero with beam polarizations and detuning.

# Single qubit gates

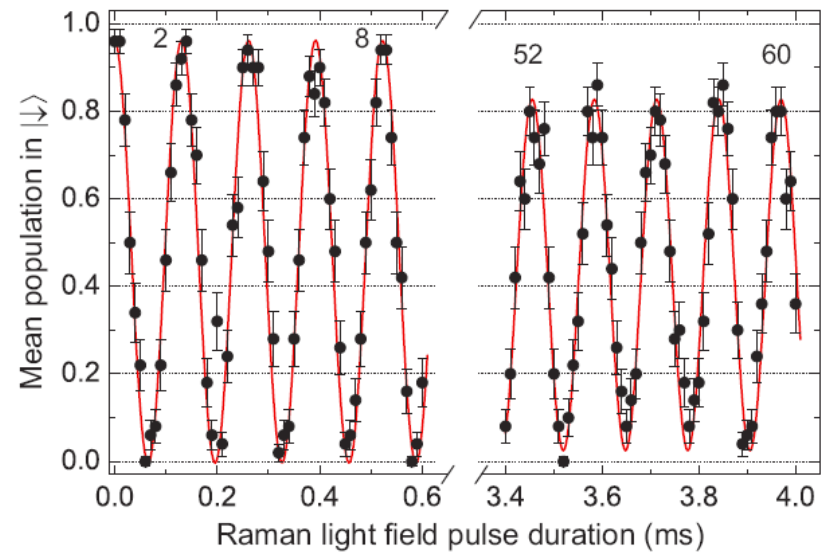
RF qubit (Zeeman or Hyperfine)

Raman carrier transitions: co-propagating beams.

${}^9\text{Be}^+$ ,  $|F=1, m_f = -1\rangle; |F=2, m_f = -2\rangle$



${}^{43}\text{Ca}^+$ , Clock transition



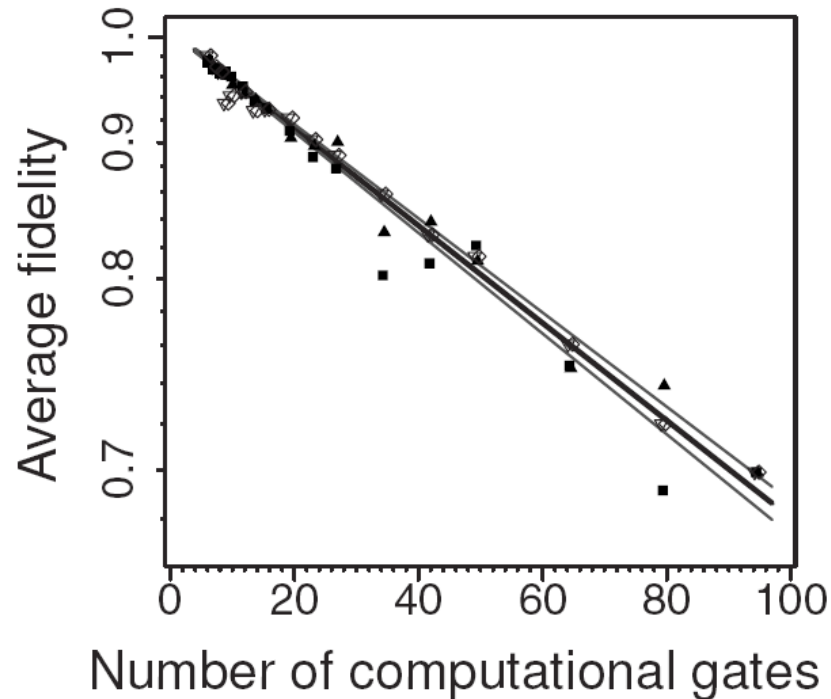
# Single qubit gates

RF qubit (Zeeman or Hyperfine)

Two-photon Raman coupling

Randomizing gates:

$\varepsilon$  in a  $\pi/2$  gate = 0.0048

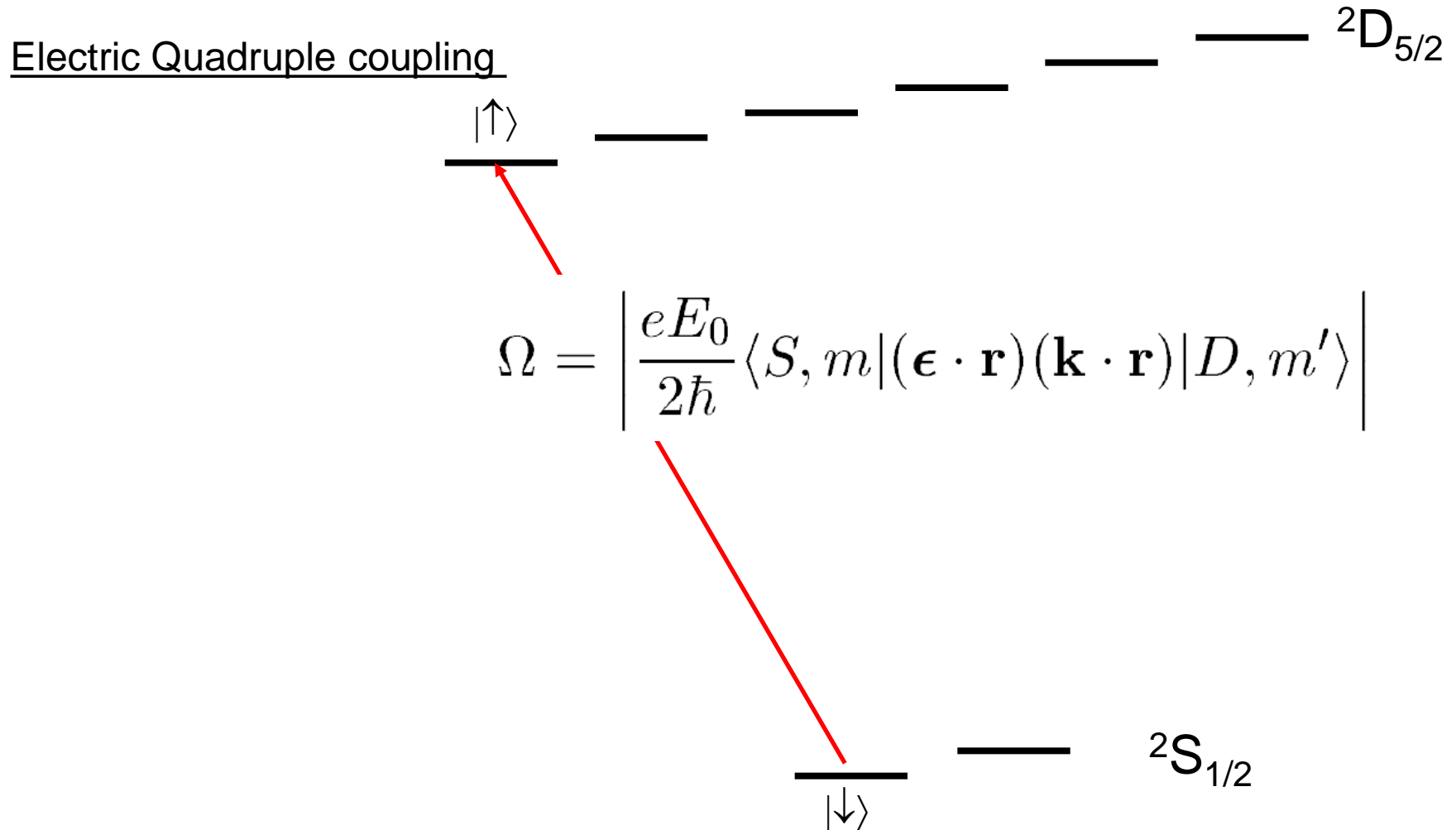


Error sources:

- Laser intensity and beam pointing noise.
- Inelastic spontaneous scattering of photons.
- B field noise.

# Single qubit gates

Optical qubit

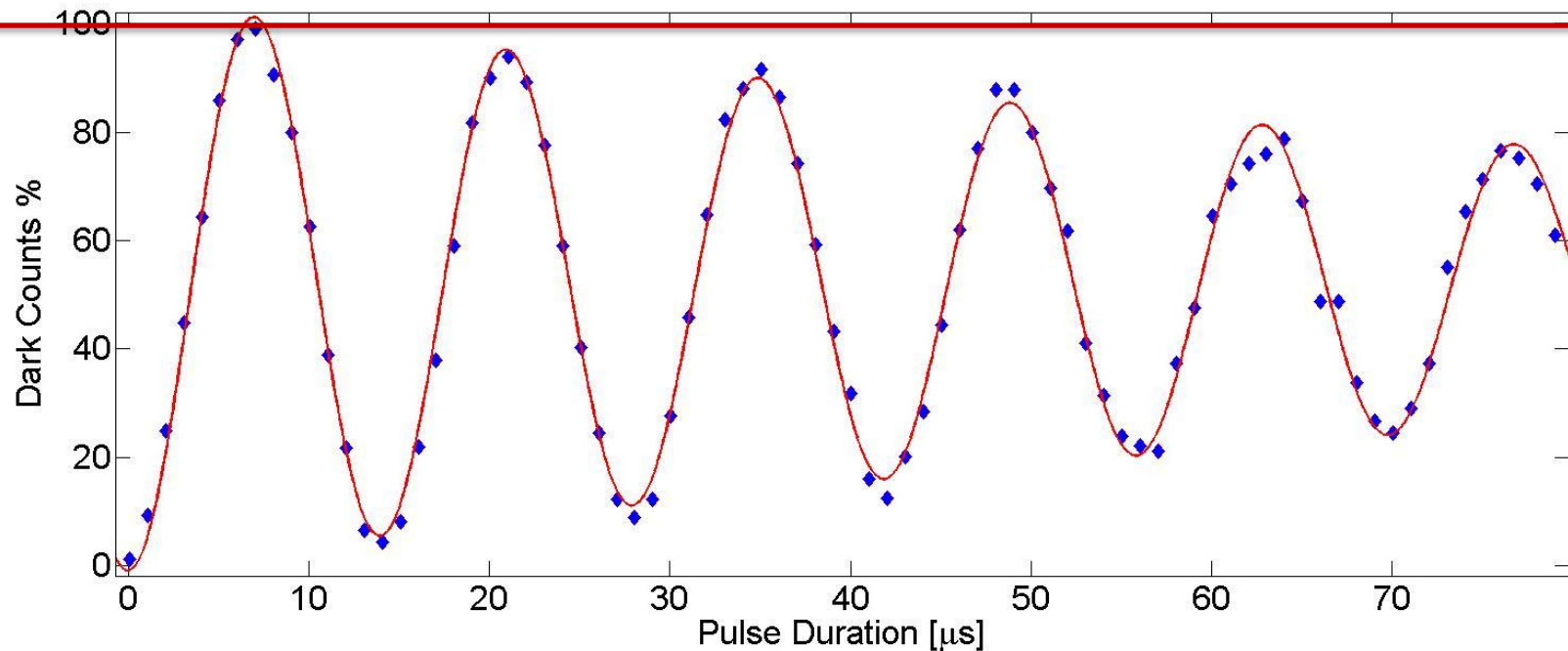




# Single qubit gates

Optical qubit: Electric Quadruple coupling

99 %



## Required:

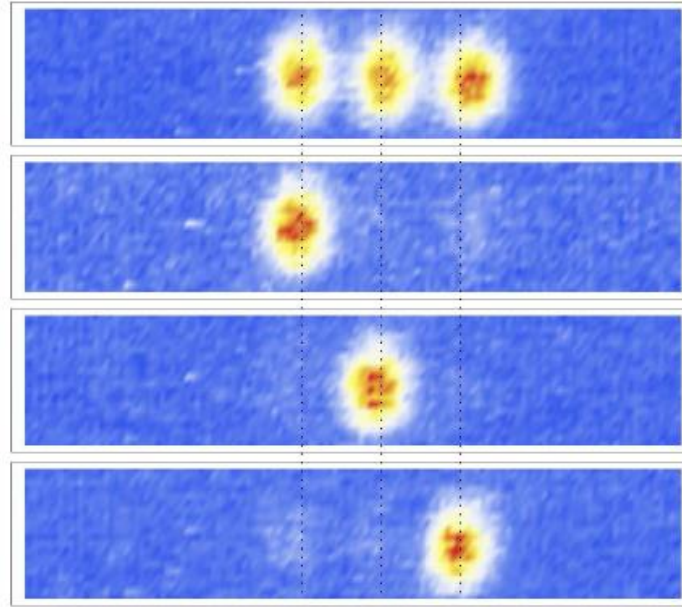
- Ground-state cooling
- Intensity noise-eater.
- Frequency auto-calibration every 100 s.

## Error sources:

- Beam pointing~ 0.3%
- Frequency drift 0.3%
- Laser linewidth 0.2%.
- Magnetic field noise 0.1%.



# Single qubit gates: Individual addressing



- Spatial: tightly focused laser beams (Innsbruck)
- Spatial: Large gradients in MW fields (NIST)
- Spectral: Large B field gradients (Siegen)
- Spectral: Inhomogeneous dressing field (Weizmann)