MICROFLUIDICS

# For a few drops more

Experiments in microfluidics reveal long-range orientational correlations in the velocities of flowing droplets that can be rationalized in terms of an analytically solvable model.

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A recurring theme in physics is the dynamics of many-body systems. For example, we are familiar with two-body problems, such as the motion of a planet around the Sun. Moreover, we learn that for a three-body problem, such as the Sun–Earth–Moon system, there is, in general, no analytical solution and the dynamics is not predictable over long periods (although the equations are deterministic). Thus, unravelling the behaviour of many-body problems, which occur in all areas of physics on various time and length scales — vortices in superconductors, electrical propagation in neural networks, the formation of self-organized structures during the sedimentation of particles in a fluid, the flocking of birds or schooling of fish and so on — is a long-standing challenge. Typically, these (out-of-equilibrium) systems involve interactions between the many discrete elements that constitute them, and no general governing principles are available for how to think quantitatively about such problems, short of elaborate simulations.

But fresh insight is now provided by Itamar Shani and colleagues who, as they report in *Nature Physics* (I. Shani, T. Beatus, R. H. Bar-Ziv and T. Tlusty, http://dx.doi.org/10.1038/nphys2843; 2014), have found long-range orientational order in the velocities of water droplets in a two-dimensional oil flow, despite the positions of the droplets being disordered. The authors present a theoretical approach to rationalizing the experimentally observed correlations in the droplet velocities that might be applicable to other systems — although it may eventually turn out that their ideas are restricted to systems with some significant degree of confinement and linearity.

At equilibrium (minimal free energy), the principles of statistical mechanics allow the characterization of the average properties of many-particle systems — for example, the average pressure or density of the system. For out-of-equilibrium systems, however, no general variational principles are known yet and
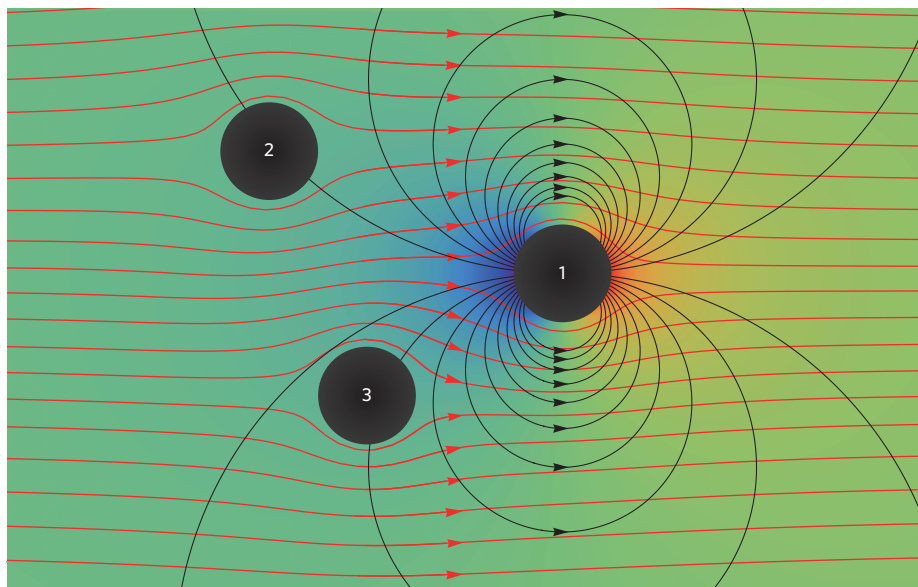


**Figure 1 |** Disc-shaped water droplets (black) translating along a confined oil layer deform the flow field as shown by the red streamlines (initially the oil flows uniformly to the right). Each disc acts as a so-called hydrodynamic dipole. The scalar field of the droplet labelled 1 is represented by the colour gradient, and the associated velocity field is indicated by black arrowed lines. Shani *et al.* show that in the case of many such drops, the effect of the interaction of a pair of droplets (for example, the ones labelled 2 and 3) with the rest of the system reduces to each of the two droplets responding to the velocity field of a third droplet (1).

system-specific descriptions are often required to obtain quantitative results. In hydrodynamics, for instance, the average features of single-phase materials are typically described by mean-field equations such as the continuity and Navier–Stokes equations for low-molecular-weight fluid flows. For multiphase materials, such as particles dispersed in a fluid, there are no systematic, widely applicable equations for the description of their dynamics. Whenever such equations are proposed they are generally appropriate for dilute systems. Hence, numerical simulations and experiments are the basic means of interrogation for many-body problems.

The experiments performed by Shani *et al.* involve pressure-driven flow in the rectangular space between two parallel plates, a so-called Hele–Shaw cell.

The suspending liquid (oil) wets the surrounding plates and small droplets of a second immiscible phase (water) are formed with diameters slightly larger than the narrow gap between the plates so that the droplets are shaped like pancakes. The droplets have a mean speed a little slower than the continuous phase, as they experience friction from the nearby planar boundaries. The flow is set up in such a way that it has a low Reynolds number (Stokes flow); the governing equations for the fluid are then linear but the dynamics of the individual droplets can be complex owing to the hydrodynamic interactions between them. In this dynamical regime, the individual droplets have a characteristic velocity signature that is referred to as a hydrodynamic dipole: the streamlines in the surrounding fluid are similar to those of

a two-dimensional dipole in electrostatics (Fig. 1). For this 'dipolar flow', the fluid velocity decays with distance $r$ from a droplet as $1/r^2$. This kind of dependence, where the inverse power law exponent of the velocity decay is equal to (or less than) the spatial dimensionality of the system, is referred to as a long-range interaction (A. Campa, T. Dauxois and S. Ruffo, *Phys. Rep.* **480,** 57–159; 2009).

As a result of these interactions, the droplets' velocities are constantly fluctuating around the mean velocity of the liquid. Shani *et al.* identified a distinctive feature of the fluctuations: although the droplet positions in this two-dimensional suspension flow are disordered, the fluctuations in the droplet velocity are correlated, even at modest concentrations. A visual qualitative analysis of the $x$ and $y$ components of the droplet velocity fluctuations reveals a kind of 'stripy-ness': for the $x$ components, there are positive correlations in directions parallel and perpendicular to the imposed flow (parallel to the $x$ direction) and anti-correlations in directions making an angle $\theta$ of ±45° with the flow direction, and vice versa for the $y$ components. The authors examined

these correlations in a quantitative way by evaluating the correlation functions of the velocity fluctuations, and found that they have several distinctive features including a decay again proportional to $1/r^2$ and a four-fold angular symmetry — the correlation functions are approximately proportional to $\cos(4\theta)$.

Although the situation of long-range orientational order in combination with spatial disorder of the individual components is not new — it is the hallmark of nematic liquid crystals, for example — Shani *et al.* succeed in rationalizing the experimentally observed fluctuations in a macroscopic system of hydrodynamic dipoles from first principles. Because of the linearity of the equations for Stokes flow, the principle of superposition can be invoked to add the effects of the individual two-dimensional point dipolar fields and analytically calculate the correlation function that describes the fluctuations of the droplet velocities — reproducing the experimentally obtained $\cos(4\theta)/r^2$ dependencies. What's more, under the simplifying assumption that the droplet positions are random, the authors' formulation of the correlation function

implies an elegant physical picture: the effect of the entire ensemble of droplets on a pair of droplets is simply that of a single third droplet (Fig. 1).

Armed with these new findings for a driven low-dimensional system, it is tempting to ask whether similar features might show up in other non-equilibrium systems, particularly in those with an internal driving as opposed to externally driven systems, as considered by Shani and colleagues. One promising example might be a population of micro-scale self-propelled particles, the dynamics of which is also governed by the physics of a flow with a low Reynolds number. Finally, it might be possible to examine the long-standing question regarding the existence of variational principles governing non-equilibrium statistical mechanics, by measuring energy dissipation in similar experiments or simulations. ❏

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