THE QUADRUPOLE MOMENT OF THE 2^+ STATE AT 122 keV IN Sm^{152}

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High order Coulomb excitation has been suggested as a tool for measuring static quadrupole moments of excited nuclear states in a well known paper by Breit et al. [1]. Attempts at carrying out such measurements have so far been generally unsuccessful, due in a large measure to the inadequacy of existing calculations in Coulomb excitation theory. It was found [2] that if one wishes to treat the process as a second order excitation, the condition that higher orders than the second may be neglected, places quite stringent requirements on the parameters of the process and severely restricts the experimental maneuverability. Multiple Coulomb excitation theory is free of these restrictions but calculations carried out to date [3, 4] are not accurate enough for this purpose, mainly in the treatment of the energy of excitation. Also these calculations are all for specific nuclear models, implying fixed ratios between the relevant E2 moments and transition probabilities, whereas if a measurement of any one of these moments is intended, one has to study the *dependence* of the excitation process on these parameters and calculations of much greater scope are therefore required.

In one experiment of this nature [5, 6] an attempt was made to determine the quadrupole moment of the 2^+ state in Sm^{152} in a precision measurement of the excitation function for Coulomb excitation with back scattered oxygen ions. The excitation function was found to agree very well with the calculations of ref. 3 for a rotational band. This agreement could be taken as a confirmation that the rotational band description is in general correct in this case. As the quantity $Q(2^+)$, the quadrupole moment of the 2^+ state, is one of the significant parameters in this case - the general agreement also implies a value of $Q(2^+)$ which is not very far from the rotational value. However, because of the problems and difficulties discussed above, nothing more definite could be said about $Q(2^+)$.

In the present work a detailed and rigorous calculation was carried out for this excitation process and both $Q(2^+)$ and the transition probability $B(E2; 2^+ \rightarrow 4^+)$ were treated as variable parameters. In comparing these calculations with the measurements of ref. 5 some definite statements can be made about $Q(2^+)$.

Following the procedure of ref. 3 we describe the Coulomb excitation process by the Schrödinger equation

$$i\hbar\dot{a}_n = \sum_m H_E(t)_{mn} \exp\{\frac{i}{\hbar}(E_n - E_m)t\} a_m(t) \ .$$

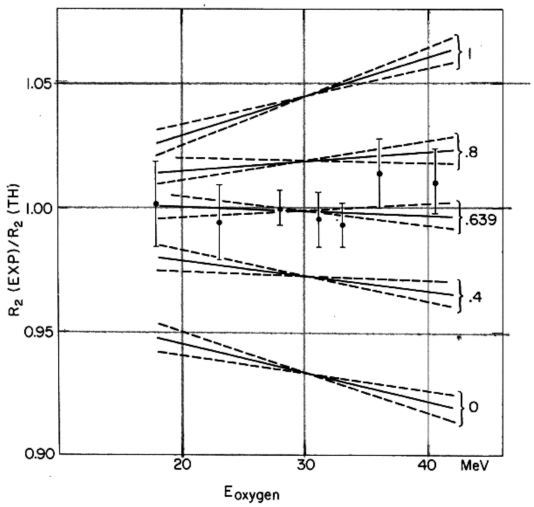

Fig. 1. The calculated ratio $R_2(E_2)/R_2(E_1)$ as function of $M(E2)_{2;2}/M(E2)_{0;2}$ for different bombarding energies and particles. The ratios are normalized to unity for the rotational value of $M(E2)_{2;2}/M(E2)_{0;2}$ (i.e. 0.639) by appropriate constants K_i

$$K_1 = 17.305 \; ; \quad K_2 = 7.463 \; ; \quad K_3 = 6.139 \; .$$

Exact solutions for these equations were found, mostly for the set of states: 0^+, 2^+, 4^+. In some typical instances larger sets including higher energy states were also considered but this addition was found to make no significant difference in the region of bombarding energies covered in the experiment of ref. 5. For the sake of simplicity the excitation process was in most cases calculated for exact back-scattering (at 180°). Finite angle calculations were also carried out for 160.7°, the mean angle of the counter, for most of the bombarding energies of ref. 5 and the rotational values for the E2 moments. All other calculations were then "normalized" to these standard solutions [7]. The quantities calculated were the R_i measured in the experiment of ref. 5:

$$R_2 = P_2 + P_4 + \ldots = 1 - P_0 \; ;$$

$$R_4 = P_4 + P_6 + \ldots = 1 - P_0 - P_2 \; ,$$

where P_i is the excitation probability for the state of spin i. The R_i were evaluated as functions of the nuclear reduced matrix elements $M(E2)_{2;2}$, $M(E2)_{2;4}$, $M(E2)_{4;4}$. For the parameter $M(E2)_{0;2}$ the value: $-1.85\, e \times 10^{-24}$ cm^2 was adopted. This corresponds to: $B(E2; 0 \rightarrow 2) = 3.43\, e^2 \times 10^{-48}$ cm^4 [8] and is the same as in ref. 5.

Fig. 1 presents the calculated ratios

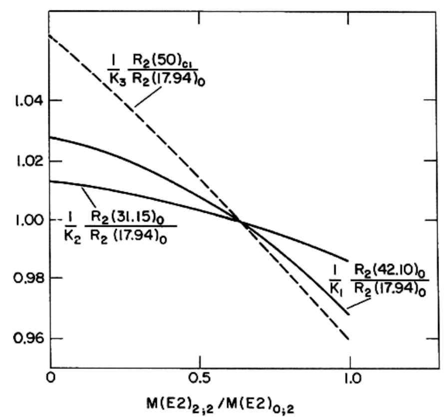

Fig. 2. The measured values of R_2 are here shown divided by the values calculated for $M(E2)_{2;2}/M(E2)_{0;2} = 0.639$. The full line through the points represents a straight line with a least square fit and the broken lines show the "probable error" limits. Straight lines representing least square fits are also shown (with the points left out) for $M(E2)_{2;2}/M(E2)_{0;2}$ equal to: 1; 0.8; 0.4; 0. The values of $M(E)_{2;2}/M(E)_{0;2}$ consistent with the measurement are those for which the probable error limits include a horizontal line. Ideally this line should be at the ordinate 1 but due to possible errors in the counter calibration, the permissible range of ordinates for such "horizontal" lines is: 1 ± 0.07.

$R_2(E_2)/R_2(E_1)$ for various bombarding energies of both oxygen and chlorine ions as a function of the parameter $M(E2)_{2;2}$. The parameter $M(E2)_{2;4}$ and $M(E2)_{4;4}$ are assumed in this case to have the rotational values. One gets in this way a clear indication of the sensitivity of Coulomb excitation as a measurement of static quadrupole moments. Quite obviously the sensitivity increases markedly with increasing bombarding energy. In choosing a suitable energy it is however important to consider also the dependence of these ratios on the other relevant parameters in particular $M(E2)_{2;4}$ and $M(E2)_{4;4}$. It was found that in the range of energies considered, R_2 depends on $M(E2)_{2;4}$ about as sensitively as on $M(E2)_{2;2} \cdot M(E2)_{2;4}$ can however be measured easily and well by the quantity R_4, as in ref. 5, and in this way it can essentially be eliminated. $M(E2)_{4;4}$ was found to have an almost negligible effect on R_2 in the range of energies considered and for these energies R_2 is therefore a direct measure of $M(E2)_{2;2}$. For higher bombarding energies the effect of $M(E2)_{4;4}$ and of higher excitations on R_2 increases rapidly and because of this limitation the optimum energy (for oxygen ions) for this type of measurement would in this case be about 45 MeV.

In analyzing the results of ref. 5 it was found that a correction had to be introduced due to a circumstance which was not realized at the time of measurement, namely - the fact that the recoiling Sm nuclei are not stopped in the backing but rather escape almost with the full velocity. During their lifetime these nuclei travel several millimeters and the efficiency of the gamma counter is thereby modified. In order to evaluate this correction a Co^{57} source was moved along the recoil path in a counter geometry similar to that which obtained at the original experiment and from the counter response and the known mean life of the 2^+ level, the efficiency was computed for the relevant recoil velocities. The relative correction for the various energies was found to be small but the absolute change in counter efficiency is substantial - about seven percent. The 4^+ level is short lived enough to make this correction completely negligible.

Interpreting the R_4 measurements we now conclude that $B(\mathrm{E2}, 2^+ \rightarrow 4^+)$ has the value:

$$B(\mathrm{E2}, 2^+ \rightarrow 4^+) =$$

$$= B(\mathrm{E2}, 2^+ \rightarrow 4^+)_{\mathrm{rotational}} \times (1.08 \pm 0.05) \; .$$

The excitation probability R_2 was calculated for this value of $B(\mathrm{E2}, 2^+ \rightarrow 4^+)$ and for the rotational value of $Q(4^+)$.

In fig. 2 the suitably corrected measured values are compared with calculations for various values of the parameter $Q(2^+)$. In this presentation the points drawn for the *correct* value of $Q(2^+)$ should be consistent with a horizontal line, but they may be somewhat removed from unity due to a possible inaccuracy in the counter calibration.

In an analysis of this type the value of the quadrupole moment was established as: $Q(2^+) = (-1.8 \pm 0.6) \times 10^{-24}\ \mathrm{cm}^2$ in good agreement with the rotational value of: $Q(2^+) = -1.68 \times 10^{-24}\ \mathrm{cm}^2$ and in good agreement also with the original estimate of ref. 5.

Because of the lack of adequate calculations the original measurements were in the nature of a shot in the dark with respect to the proper choice of bombarding energies, counting statistics, etc. With the calculations now available it seems to be possible to carry out such measurement to within about half the relative error of the present case. It also seems exceedingly difficult to improve the accuracy beyond this point in this type of measurement.

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References

1. G. Breit, R. L. Gluckstern and J. E. Russel, Phys. Rev. 103 (1956) 727.
2. D. L. Lin and J. F. Masso, Proc. Third Conf. on Reactions between complex nuclei, Asilomar, California, 1963 (University of California Press, Berkeley and Los Angeles, 1963) p. 267.
3. K. Alder and A. Winther, Kgl. Danske Videnskab Selskab, Mat. Fys. Medd. 32 No. 8 (1960).
4. K. Alder, Proc. Third Conf. on Reactions between complex nuclei, Asilomar, California, 1963 p. 253.
5. G. Goldring, J. De Boer and H. Winkler, Proc. Third Conf. on Reactions between complex Nuclei, Asilomar, California, 1963, p. 278.
6. G. Goldring, J. De Boer and H. Winkler, Phys. Rev. 134 (1964) B1032.
7. Similar calculations have recently been carried out for the first 2^+ state in Cd^{114} by J. De Boer, R. G. Stokstad, G. D. Symons and A. Winther (private communication).
8. E. M. Bernstein and E. Z. Skurnik, Phys. Rev. 121 (1961) 841.

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