QUANTUM MECHANICAL EFFECTS IN MULTIPLE COULOMB EXCITATION

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The quantum-mechanical effects in multiple Coulomb excitation and their effects in quadrupole moment measurements are investigated analytically (for a simple one dimensional case) and numerically. It is found that their effect might be sometimes quite important although for the experimental conditions of the recent quadrupole moment measurements they are of the order of a few percents.

Multiple Coulomb Excitation (M. C. E.) experiments are now commonly used as a means to measure the static quadrupole moments of excited states [1-3]. The interpretation of such experiments is carried out using the Semi Classical (S. C.) theory of the M. C. E. process, while Quantum Mechanical (Q. M.) corrections are either ignored (e.g. polarization effects) or corrected for in an approximate way (the "symmetrization" procedure [4, 6]). The purpose of this note is twofold:

a) To solve analytically a simple M. C. E. problem, the results of which can give an idea of the order of magnitude of some Q. M. effects.

b) To present some results obtained from a coupled-channels code, in which all Q. M. effects are automatically included.

The simple problem to be solved is that of the one-dimensional motion of a particle exciting a nucleus, the energy levels of which are all degenerate. This same case was solved in the S. C. limit by Alder and Winther [5] ($\xi = 0$ approximation), and their notation will be used in this work.

The Schrödinger equation can be written as a set of coupled equations:

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{z_2\, z_2\, e^2}{x}\right)\psi_n(x) + \sum_m \langle n\,|V_{\text{int}}|\,m\rangle\,\psi_m(x) = E\,\psi_n(x) \tag{1}$$

where n denotes the channel (nuclear state) quantum numbers, and V_{int} is the quadrupole part of the electromagnetic interaction which causes the excitation. More specifically eq. (1) takes the form

$$\left(-\frac{\partial^2}{\partial\rho^2}+\frac{2\eta}{\rho}\right)\psi_n(\rho)+\frac{3\chi\eta^2}{\rho^3}\sum_m R^M_{nm}\,\psi_m(\rho)=\psi_n(\rho) \tag{2}$$

where

$$\rho=\sqrt{\frac{2mE}{\hbar}}\,x=K\,x$$

$$\chi=\sqrt{16\pi}\,\frac{1}{5!!}\,\frac{z_1 e}{\hbar v}\,\frac{\langle I_1||m(\text{E2})||I_2\rangle}{a^2\sqrt{2I_1+1}}$$

$$R^M_{nm}=(-)^{I_n-M}\sqrt{5}\,\sqrt{(2I_1+1)}\begin{pmatrix} I_n & 2 & I_m\\ -M & 0 & M\end{pmatrix}\times\frac{\langle I_n||m(\text{E2})||I_m\rangle}{\langle I_1||m(\text{E2})||I_2\rangle}$$

I_1 and I_2 are the spins of the ground and first excited states respectively.

The boundary conditions on the ψ_n are:

a) $\psi_n(\rho)\xrightarrow[\rho\to\infty]{}\frac{1}{\sqrt{K}}\left(\delta_{n,1}\,e^{-i\rho}+T_n\,e^{i\rho}\right)$

b) $\psi_n(\rho)$ vanishes exponentially for ρ less than the classical turning point.

The set (2) can be reduced to an equivalent set of non-coupled equations by choosing proper linear combinations of the $\psi_n(\rho)$. These equations can be solved separately, and the boundary conditions determine the coefficients T_n. It can be readily shown that the J. W. K. B. solution gives:

$$T_k=-i\sum_l U_{kl}\,U^{-1}_{l1}\,e^{2iI_l} \tag{3}$$

where U_{kl} diagonalize the matrix R^M_{mn}

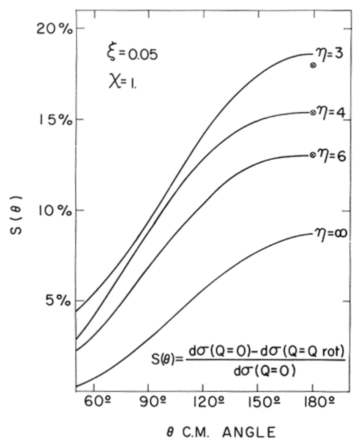

Fig. 1. The function $S(\theta)$ for $\chi = 1$ and $\xi = 0.05$, $\eta = 3$, 4, 6, ∞. The crosses on the right indicate the values of $S(\theta = 180^{\circ})$ as given by eq. (9).

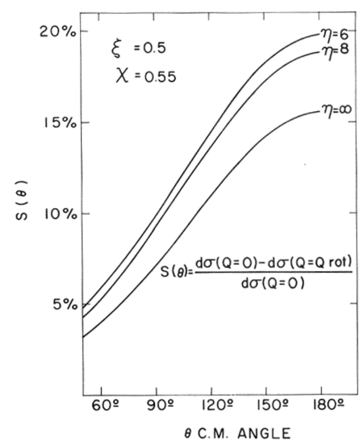

Fig. 2. The function $S(\theta)$ for $\chi = 0.55$ and $\xi = 0.5$, $\eta = 6$, 8, ∞. χ is defined here in a symmetrized way.

$$I_l = \lim_{R\to\infty} \left[\int_{\rho_{0l}}^{R} K_l(\rho)\,\mathrm{d}\rho - R \right],$$

$$K_l = \left[1 - \frac{2\eta}{\rho} - \frac{3\chi\eta^2}{\rho^3}\lambda_l \right]^{\frac{1}{2}},$$

λ_l is the l'th eigenvalue of R^M_{mn}, ρ_{0l} is the classical turning point in l'th channel. The excitation probabilities are thus

$$P_k = |T_k|^2 = |\sum_l U_{kl}U^*_{1l}\,\mathrm{e}^{2\mathrm{i}I_l}|^2 \qquad (4)$$

The integrals I_l can be expanded in power series of $1/\eta$. The leading terms are

$$I_l = I_0 - \tfrac{1}{2}\chi\lambda_l + \frac{9}{70}\frac{\lambda_l^2}{\eta}\chi^2 + \ldots \qquad (5)$$

where

$$I_0 = \lim_{R\to\infty} \left[\int_{2\eta}^{R} \sqrt{1 - \frac{2\eta}{\rho}}\,\mathrm{d}\rho - R \right].$$

For $\eta \to \infty$, the expression (4) reduces to that obtained in ref. 5 in the S. C. approximation.

For the sake of simplicity, the case of a target nucleus of only two rotational levels, will be considered. For $I_1 = 0$, $I_2 = 2$, the excitation probability is

$$P_2 = \chi^2 \left[1 - \frac{36}{70}\frac{\chi}{\eta}R_{22} - \tfrac{1}{3}\chi^2\,(1 + \tfrac{1}{4}R_{22}^2) \right] + \ldots \qquad (6)$$

In quadrupole moment measurements, the important parameter is the sensitivity of the excitation probabilities to the quadrupole moment, i.e.

$$S = \frac{P_2(Q=0) - P_2(Q=Q_{\mathrm{rot}})}{P_2(Q=0)}. \qquad (7)$$

Q_{rot} is the value of Q given by the rotational model. In the simple case considered here:

$$S = \left(\frac{36}{70}\frac{\chi}{\eta}R_{22} + \frac{1}{12}\chi^2 R_{22}^2 \right) / (1 - \tfrac{1}{3}\chi^2)$$
$$\cong (0.33\,\chi/\eta + 0.034\,\chi^2)/(1 - \tfrac{1}{3}\chi^2) \qquad (8)$$

According to this expression, there is a wide range of values of χ and η for which the added Q. M. term in S is of the same order of magnitude as the S. C. one.

In order to investigate the importance of the Q. M. corrections more closely, a coupled channels code was written. Two sets of results are being presented here. The first is considered as a check on the simple calculation described above. In fig. 1 the sensitivity S, as defined in (7), is plotted as a function of the scattering angle for cases with $\chi = 1$, $\xi = 0.05$ and $\eta = 3$, 4, 6, ∞. (The $\eta = \infty$ results were obtained from a S. C. code similar to the one discussed in ref. 4. It can be seen that the sensitivity S at $\theta = 180^{\circ}$ can be fitted by a form

$$S = \alpha + \beta/\eta; \qquad \alpha = 0.086; \qquad \beta = 0.27. \qquad (9)$$

α and β are different from those predicted by (8) because ξ is different from zero.

Fig. 2 shows S as a function of the scattering angle for more realistic values of χ and ξ, i.e. $\chi = 0.55$, $\xi = 0.5$ and $\eta = 6, 8, \infty$.

Assuming that the Q.M. correction to S is proportional to $1/\eta$, it is possible to extrapolate the values of fig. 2 to the region of interest where $\eta = 20$-40. The S.C. S should be corrected by 8% of its value for $\eta = 20$, and by 4% for $\eta = 40$. Another feature of the coupled channels calculation is that the cross-section for the case of $Q = 0$ is almost the same as the S.C. result, while the S.C. and the Q.M. cross-section for $Q = Q_{\rm rot}$ differ appreciably. This effect indicates that the "symmetrization procedure" is effective, while ignoring the polarization effects in the S.C. treatment leads to the discrepancy between the Q.M. and S.C. results.

In conclusion it may be said that Q.M. effects cannot be ignored in deducing quadrupole moments from M.C.E. experiments, especially when the target is a light nucleus. Application of the S.C. theory in the analysis of such experiments can be in error by as much as ten percent in realistic cases.

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References

1. R.G.Stokstad, I.Hall, G.D.Symons and J.de Boer, Nucl. Phys. A92 (1967) 319.
2. J.J.Simpson, D.Eccleshall, M.J.L.Yates and N.J.Freeman, Nucl. Phys. A94 (1967) 177.
3. G.Goldring and U.Smilansky, Phys. Letters 16 (1965) 151.
4. A.Winther and J.de Boer, CALTEC Technical Report (1965).
5. K.Alder and A.Winther, Mat. Fys. Medd. Dan. Vid. Selsk. 32 No. 8 (1960).
6. L.C.Biedenharn and P.J.Brussaard, Coulomb Excitation (Clarendon Press, Oxford, 1965) pp. 31-40.

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