THE QUANTUM MECHANICAL EFFECTS ON GAMMA ANGULAR CORRELATIONS FOLLOWING MULTIPLE COULOMB EXCITATION

U. SMILANSKY
Department of Nuclear Physics, Weizmann Institute of Science, Rehovoth. Israel

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The quantum mechanical effects on the gamma angular distribution following multiple Coulomb excitation are investigated in relation to a new method for measuring quadrupole moments of excited states by the reorientation effect. It is shown that in this case the quantal effects might be important, and an estimate of their magnitude is given.

Several quantum mechanical calculations of the multiple Coulomb excitation process were carried out in the past [1-3]. A comparison of the results with those obtained from the semi-classical treatment of the Coulomb excitation [4], shows the following general features: (a) The "symmetrized" [4] version of the semi-classical theory gives a good approximation to most of the calculated quantities. (b) The largest discrepancies appear in those quantities which come about from the interference between the first and second order contributions such as the "reorientation" term. The quantum-mechanical effects in these cases are proportional to $1/\eta$ where η is the Sommerfeld parameter. It was found that for most practical situations where the quadrupole moment of excited states have been measured, the q.m. corrections were smaller than the experimental uncertainties.

Recently, a new method for measuring the reorientation effect was suggested by Eichler and de Boer [5-6]. The target is excited by heavy projectiles which are scattered through an angle of 90^o, and the de-excitation gamma rays are measured in coincidence with the exciting particles. The gamma intensity is measured in two counters which are positioned in the reaction plane, at 22.5^o to the left and to the right of the bisector of the classical trajectory (see fig. 2). According to first order semi-classical calculations, the counting rate in the two counters is the same. This symmetry is broken when higher order contributions are included, and it is shown by Eichler and de Boer that in the second order semi-classical treatment, the difference in counting rates is proportional to the quadrupole moment of the excited state.

Thus this is a "zero-effect" measurement of the quadrupole moment contrary to all present experimental techniques where the quadrupole effect appears as a small correction term. Eichler and de Boer point out another advantage, namely that the determination of the static quadrupole moment is less sensitive to contributions arising from the excitation of the level by

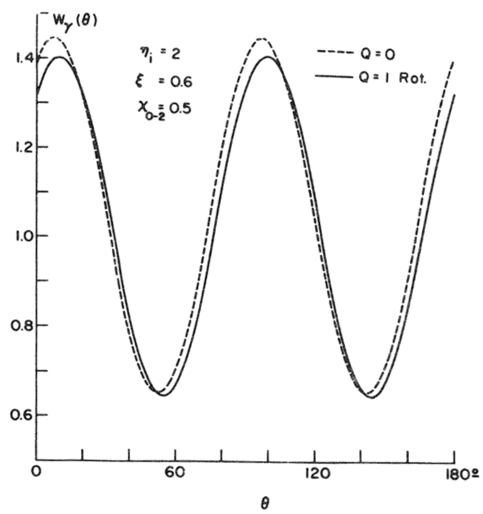

Fig. 1. The angular distribution of the de-excitation gamma radiation, following Coulomb excitation with particle scattered through 90^o. The Coulomb excitation parameters are: $\eta_i = 2$, $\xi = 0.6$, $\chi_{0\to2} = 0.5$ and $Q = 0$ or $Q = 1$ rot.

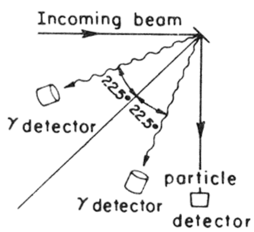

Fig. 2. This figure shows in a schematic way the experimental arrangement.

virtual transitions via higher lying states. Such transitions usually introduce an ambiguity in the results obtained for the quadrupole moment [2], and its weakening makes this experimental arrangement desirable.

It is well known that the gamma angular distribution is more sensitive to quantum mechanical effects than is the excitation cross-section. It is the purpose of this note to present the results of the exact quantum mechanical calculation and its influence on the analysis of the above mentioned experiment. As the measured quantity is the deviation of the angular correlation from being symmetric with respect to the bisector of the projectile's trajectory, most of the attention will be directed towards the deviations from this symmetry.

The gamma angular correlations were calculated using the Coupled Channels computer code which was described elsewhere [2]. Only two levels have been considered in these calculations: the ground state being a 0^+ state and an excited 2^+ state. The effects of higher states are not included.

In order to parametrize the results the four non-dimensional parameters which are listed below were introduced:

(a) η_i - the Sommerfeld parameter for the incoming projectile;

(b) ξ - $(=\eta_f - \eta_i)$ symmetrized adiabaticity parameter;

Table 1
The ratio $R = (W_L - W_R)/(W_L + W_R)$ calculated in the first order approximation as a function of η for $\chi = 0.5$, $\xi = 0.3$ and $\xi = 0.6$.

$\eta \rightarrow$	2	2.667	4	0.8	∞
$\xi = 0.3$	0.63%	0.37%	0.20%	0.084%	0
$\xi = 0.6$	2.34%	1.68%	1.09%	0.54%	0

Table 2
The ratio $R = (W_L - W_R)/(W_L + W_R)$ calculated by the exact quantum mechanical computer code. The quantity R is given as a function of η for $\chi = 0.5$ and $\xi = 0.3$ or 0.6 for $Q = 0$ and $Q = 1$ rot. unit. The column $\Delta\theta$ indicates the deviation of the $\boldsymbol{q}$ axis from the bisector of the classical trajectory.

η	$\xi = 0.3$			$\xi = 0.6$		
	$\Delta\theta^{\circ}$	R $(Q=0)$	R $(Q=1$ Rot)	$\Delta\theta^{\circ}$	R $(Q=0)$	R $(Q=1$ Rot)
2	3.99	1.63%	6.36%	7.43	0.21%	5.94%
2.667	3.05	1.54%	5.59%	5.77	0.27%	5.00%
4	2.07	1.43%	5.23%	3.99	0.33%	4.53%
8	1.05	1.22%	4.81%	2.07	0.43%	4.20%
∞	0.	0.91%	4.39%	0.	0.52%	3.91%

(c) $\chi_{0\rightarrow 2}$ - this parameter measures the strength of the interaction and its symmetrized form is given by

$$\chi_{0\rightarrow 2} = \frac{\sqrt{16}\,\pi}{15} \cdot \frac{K_i K_f}{(\eta_i \eta_f)^{\frac{1}{2}}} \cdot \frac{\langle 0||M(\mathrm{E2})||2\rangle}{Z_{\mathrm{target}}}$$

where K_i and K_f are the wave numbers of the incoming and outgoing particles respectively;

(d) Q - the quadrupole moment of the excited state. This parameter is measured in "rotational" units, i.e. the reduced matrix element $\langle 2||M(\mathrm{E2})||2\rangle$ is given in units of:

$$\langle 2||M(\mathrm{E2})||2_{\mathrm{rot}}\rangle = 1.195\,\langle 0||M(\mathrm{E2})||2\rangle \times (\eta_f/\eta_i)^{\frac{1}{2}}$$

These four parameters specify the E2 Coulomb excitation process in an unambiguous way. The choice of these parameters is very convenient in comparing semi-classical and quantal calculations and it was already shown [2] that for $\eta_i \rightarrow \infty$, the quantal results approach the semi-classical ones. The calculations were carried out both in first order and in the exact manner of solving the coupled channels equations.

The most important quantal effect occurs already in the first order calculations. It turns out that an approximate symmetry axis is the direction of the momentum transfer $\boldsymbol{q} = \boldsymbol{K}_f - \boldsymbol{K}_i$, rather than the bisector of the projectile's trajectory. This result is known to hold exactly in the case of a degenerate target nucleus (ξ = 0 limit) [7] and also in the first order Born approximation where the distortion in the incoming and outgoing waves are neglected. In the general case the symmetry axis and the direction of $\boldsymbol{q}$ coincide only in an approximate way but the deviation is rather small and is decreasing rapidly as $\eta_i \rightarrow \infty$. This behaviour is demonstrated in table 1. Having in mind the special experiment which was discussed above, the quantity which is presented in table 1 is:

$$R = (W_L - W_R)/(W_L + W_R)$$

where W_L and W_R are the gamma intensities measured at 22.5° to the left and to the right of the *momentum transfer direction* respectively. Table 1 gives R as a function of the η for $\chi = 0.5$, $\xi = 0.3$ and 0.6. The fact that R for $\xi = 0.3$ is larger than for $\xi = 0.6$ does not contradict the statement made above about the $\xi \to 0$ limit. It only reflects the way in which the slope of the angular correlation affects R. The slope in the $\xi = 0.3$ case is much higher than that of the $\xi = 0.6$ case.

The results of the exact quantal calculations are given in fig. 1 and table 1. Fig. 1 presents the complete angular correlation functions for the cases with $\chi = 0.5$, $\eta = 2$, $\xi = 0.6$ and with $Q = 0$ and 1 rotational unit. The value of χ is quite small and therefore the $Q = 0$ correlation is very similar to that obtained in the first order calculations. The correlation for $Q = 1$ rot. unit deviates from the one for $Q = 0$, and this deviation is the measured effect. In table 2 the ratio R, as defined above, is presented for $\chi = 0.5$ and for several values of ξ, Q and η. It is shown that for $Q = 0$, R is about four times smaller than R for a rotational Q. The R-values for $Q = 0$ are of the same order of magnitude as the first order results, but they do not vanish exactly and approach the semi-classical analogue when η increases. This non vanishing contribution comes from higher order corrections to the cross-section. It can also be observed that the ratio R (as defined with respect to the direction of $\boldsymbol{q}$) depends linearly on $1/\eta$; and that the deviation of the $\eta = 8$ results from the semi-classical ones amount to about 10% only.

The quantum mechanical effects are therefore important for the analysis of such experiments. The first, which appears also in the first order results is of a kinematical origin, and it can be corrected for by rotating the calculated semi-classical distribution so that its symmetry axis will coincide with the momentum transfer axis. The effect is very important when a quadrupole moment is to be deduced from such an experiment. As an example one can take the case of 34 MeV ^{16}O bombarding ^{70}Ge. Here $\xi = 0.538$ and $\eta = 27.7$ so that the shift in the angle must be 1.1°. It can be seen from fig. 1, that at the position of the γ detectors the slope of the angular distribution is the largest, and therefore a slight shift of 1.1° will induce a 2.3% effect. This quantity should be compared to the 4% effect which is expected to be caused by a quadrupole of 1 rotational unit. The second quantum mechanical effect comes from corrections to the second order contributions, and it is relatively small.

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References

1. U. Similansky, Phys. Letters 25B (1967) 385.
2. U. Similansky, Nucl. Phys. A112 (1968) 185.
3. K. Adler and H. Pauli, preliminary report (unpublished) (1968).
4. A. Winther and J. de Boer in "Coulomb excitation", Perspectives in Physics (Academic Press, New York 1966) pp. 303.
5. J. Eichler, Bull. Am. Phys. Soc. 9 (1964) 152.
6. J. de Boer and J. Eichler, Advances in Nucl. Phys. 1 (1968) 11.
7. A discussion of the first order quantum mechanical angular distributions can be found in L.C. Biedenharn and J.P. Brussard, Coulomb Excitation (Clarendon Press, Oxford 1965), Chapter 4-4.

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