MEASUREMENT OF GROUND STATE QUADRUPOLE MOMENTS USING SIMULTANEOUS COULOMB EXCITATION OF TARGET AND PROJECTILE

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The angular distribution of γ rays following simultaneous Coulomb excitation of target and projectile is shown to provide a new method of measuring ground state quadrupole moments.

Recently several authors [1-3] have investigated the process of mutual target and projectile excitation which occurs in heavy-ion Coulomb excitation. These processes are a result of the multipole-multipole $[El_p \times El_t]$ interaction between the target and projectile; they have been shown to be much smaller than the usual monopole-multipole $(E0 \times El]$ terms. In this note, we report on a study of the angular distribution of gamma rays following such an excitation and show that it leads to a new method of measuring ground state quadrupole moments.

The interaction potential between two non-overlapping arbitrary charge distributions has recently been given by Alder and Winther [3]

$$H = 4\pi Z_p \sum_{l_t l_p l} G(l_t l_p l) \times \sum_m \frac{1}{(2l+1)} T^l_m (l_t l_p) Y^*_{lm}(r) \frac{1}{r^{l+1}} \tag{1}$$

Here T^l_m is the tensor product of the target and projectile multipoles:

$$T^l_m = [M_t(El_t) \times M_p(El_p)]^l_m \tag{2}$$

while $G(l_t l_p l)$ is given by

$$(-)^{l_t + l} \frac{\sqrt{4\pi}}{Z_p} \left[\frac{(2l+1)!}{(2l_p+1)!(2l_t+1)}\right]^{1/2} \delta(l_t + l_p - l) \tag{3}$$

The sum in (1) is extended over all possible multipolarities l_t and l_p of the target and projectile (designated by subscript t and p respectively).

The terms in eq. (1) in which both l_p and l_t do not vanish correspond to the multipole-multipole interaction, which leads to multual target and projectile excitations. Note that the value of l is limited to the algebraic sum of l_p and l_t; this is a consequence of the requirement that the two charge distributions do not overlap. In order to calculate the cross section for this process, we may use the usual semi-classical theory of Coulomb excitation [4]. The result for the first-order calculation can be cast in the notation of ref. 4:

$$d\sigma_{El} = \left(\frac{Z_p e}{\hbar v}\right)^2 a^{-2l+2} B_{\text{eff}}(El)\, df_{El}(\theta, \xi) \tag{4}$$

with $B(El)_{\text{eff}}$ equal to

$$\frac{1}{e^2} G^2 (l_t l_p l) \frac{2l+1}{(l_p+1)(2l_t+1)} B(El_p) B(El_t) \tag{5}$$

For the case of dual E2 excitation the cross section is typically of the order of 1% or less of the cross section for target excitation alone. For the case where the projectile ground state spin is larger then 1/2, one of the possible $[E2 \times E2]$ interactions leads to excitation of the target, but a reorientation of the projectile ground state. The cross section for this particular process will be proportional to the square of the projectile ground state quadrupole moment Q. (An interference term between $[E0 \times E2]$ and $[E2 \times E2]$ would give a term linear in Q, but it appears only when a polarized beam is used). The cross section for this process is small, but we show now that observation of the angular distribution of the target de-excitation γ ray can serve to magnify this effect enough to make a measurement of the quadrupole moment possible. The reason for this is as follows:

A projectile scattered at 180° from a target nucleus cannot transfer angular momentum along the direction of its motion. Therefore, if the target has a 0^+ ground state, an [E0 × E2] excitation will populate only an $m = 0$ excited substate. The de-excitation γ ray angular distribution will be a pure E2 pattern with zeros at integral multiples of $\theta = \frac{1}{2}\pi$. In the case of an [E2 × E2] interaction angular momentum can be transferred to the target if an equal amount but of opposite sign is transferred to the projectile. Consequently, the excited state of the target will not be so sharply aligned as in the [E0 × E2] case and this will be reflected in a less pronounced angular distribution. Specifically, the zeros at $\frac{1}{2}n\pi$ will be filled in. The observed γ yield at $\theta = \frac{1}{2}n\pi$ in coincidence with backscattered projectiles will only contain a contribution from the [E2 × E2] term, and this will be a direct measure of Q^2.

For particles scattered at a given angle θ, a calculation of the angular distribution of the target de-excitation γ ray after mutual excitation gives the following formula (the decay γ ray from the projectile is *not* observed):

$$\frac{dW}{d\Omega_\gamma} = \frac{1}{\sqrt{4\pi}} \sum_{K\kappa} F_K(J_t, J_{t'})\, Y_{K\kappa}(\Omega_\gamma)\, \alpha_{K\kappa}(\theta, \xi)$$

with $\alpha_{K\kappa}(\theta, \xi)$ equal to

$$(4\pi)^2 G^2(l_t l_p l)\,[(2l+1)(2l_p+1)]^{-1} \times$$

$$\times (-)^{l-l_p-J_t-J_{t'}} \begin{Bmatrix} l & l & K \\ l_t & l_t & l_p \end{Bmatrix} \begin{Bmatrix} J_{t'} & J_{t'} & K \\ l_t & l_t & J_t \end{Bmatrix} \times$$

$$\times \sqrt{2J_{T'}+1} \sum_{m\bar{m}} (-)^{l+m} \begin{pmatrix} l & l & K \\ -m & \bar{m} & \kappa \end{pmatrix} S_{El,m}(\theta, \xi)\, S^*_{El,\bar{m}}(\theta, \xi)\,.$$

Here J_t and $J_{t'}$ refer to the ground and excited state, respectively, of the target. Values of the integrals [4] $S_{El,m}(\theta, \xi)$ and the γ - γ correlation coefficients [5] $F_K(J_t, J_{t'})$ can be found in the literature. The resulting normalized distributions for the [E0 × E2] and [E2 × E2] excitations are given in fig. 1, and verify the intuitive discussion given above. The two ways of populating the same target level (namely via an [E0 × E2] process or an [E2 × E2] process) do not interfere, and each will give its own contribution to the distribution.

$$\frac{d\sigma_{(E2\times E2)}}{d\sigma_{(E2\times E0)}} = \frac{567}{50} \frac{1}{(Z_p e\, a^2)^2} \frac{(I+1)(2I+3)}{I(2I-1)} Q^2 \frac{df_{E4}(\theta, \xi)}{df_{E2}(\theta, \xi)}$$

For the case of an 11 MeV ^{7}Li scattered at 180° from a ^{24}Mg target, this ratio becomes 1.211×10^{-2}. If the experimental value of the $W(45^o)/W(0^o)$ ratio for the [E0 × E2] γ distribution is 20:1 then the quadrupole effect will enhance the counting rate in the valley by 14%. This sensitivity is of the same order as in most reorientation measurements.

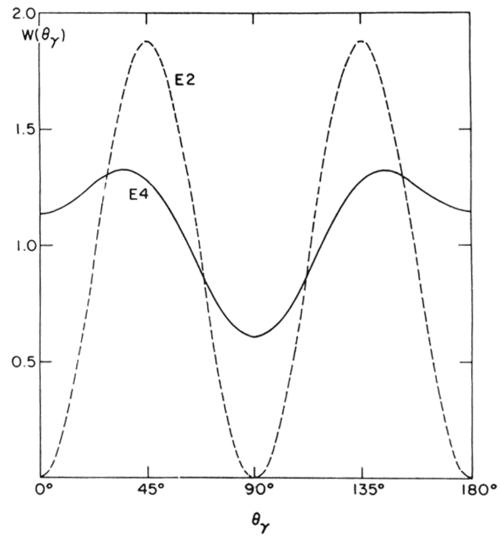

Fig. 1. The normalized γ angular distribution following [E2 × E0] (dashed line) and [E2 × E2] (solid line) Coulomb excitation by back-scattered ions.

A few remarks concerning this effect are in order:

1) In an actual experiment, the finite size of the γ and particle detectors and also the attenuation of the γ distribution due to hyperfine interactions will tend to obscure the quadrupole enhancement. However, these effects can be measured very accurately by repeating the experiment with a projectile of zero quadrupole moment. (e.g. ^{4}He, or even ^{6}Li which has a very small quadrupole moment, can be used for the case cited above).

2) Our considerations with regard to the vanishing intensity of γ rays at 0° are valid even in cases involving multiple-order processes in the [E0 × E2] interaction, since they follow from the symmetry of the problem.

3) A competing [E2 × E2] effect which can lead to a similar enhancement of intensity at 0° is one in which an excited level in the projectile is populated simultaneously with the excitation of the target. However, such effects can be

eliminated by performing the particle -γ coincidence only with backscattered particles having the correct energy. In cases where this is not feasible, their effect may be calculated from the formulas presented here and subtracted away.

References

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