The behaviour of two-dimensional electron gases in strong magnetic fields at low temperatures has been a rich source of new physics for two decades, and continues to surprise theorists and experimentalists alike

Fractional quantum Hall effects

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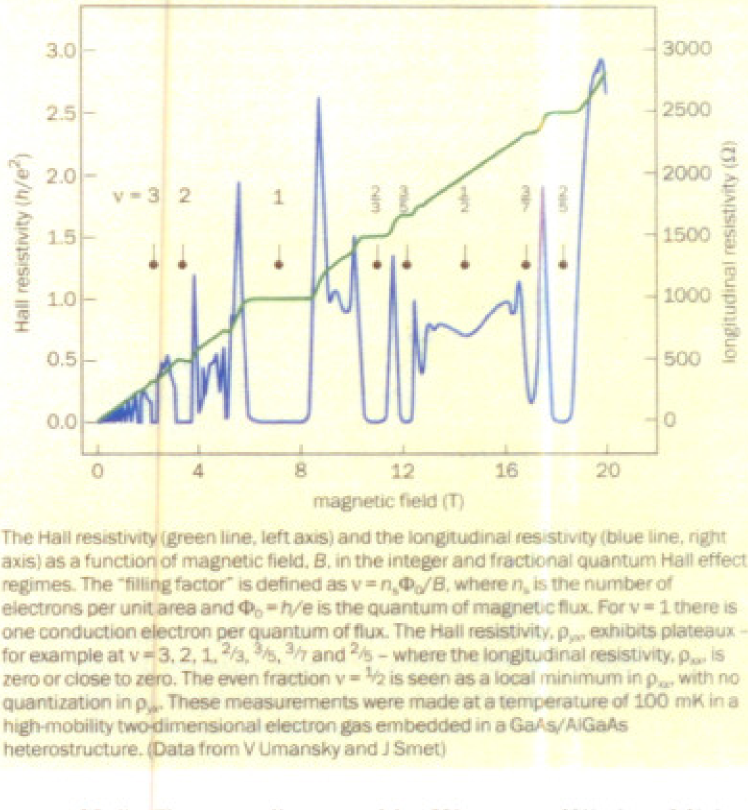
SINCE its discovery almost two decades ago, the quantum Hall effect has been investigated by an exceptionally broad community of physicists, ranging all the way from materials scientists to topologists. Although the integer and fractional quantum Hall effects were discovered in the early 1980s, they are still the focus of a large research effort worldwide, and many new phenomena have been discovered in recent years. These include objects as diverse as composite fermions, skyrmions and fractional charge carriers.

The classical Hall effect, which dates back to the 19th century, is widely used to characterize electronic materials. In the classical effect, a thin strip of a conducting material is placed in a magnetic field and a current is driven through it. The electrons experience a Lorentz force that is perpendicular to both the magnetic field and their initial direction. If the electrons are initially moving in the x direction through a strip in the x–y plane and the magnetic field is in the z direction, then the Lorentz force, and hence the Hall voltage, will be in the y direction.

The Hall resistance is defined as $R_{yx} = V_y/I_x$, where V_y is the Hall voltage and I_x is the current. Classical theory predicts $R_{yx} = B/en_s$, where B is the magnetic field, e is the electron charge and n_s is the number of electrons per unit area in the strip. Whereas the Hall resistance increases linearly with the magnetic field, the longitudinal resistance (i.e. the resistance in the direction of the current, $R_{xx} = V_x/I_x$) is predicted to be roughly independent of the magnetic field.

1 The quantum Hall effects



The Hall resistivity (green line, left axis) and the longitudinal resistivity (blue line, right axis) as a function of magnetic field, B, in the integer and fractional quantum Hall effect regimes. The "filling factor" is defined as $\nu = n_s\Phi_0/B$, where n_s is the number of electrons per unit area and $\Phi_0 = h/e$ is the quantum of magnetic flux. For $\nu = 1$ there is one conduction electron per quantum of flux. The Hall resistivity, ρ_{yx}, exhibits plateaux – for example at $\nu = 3, 2, 1, 2/3, 3/5, 3/7$ and $2/5$ – where the longitudinal resistivity, ρ_{xx}, is zero or close to zero. The even fraction $\nu = 1/2$ is seen as a local minimum in ρ_{xx}, with no quantization in ρ_{yx}. These measurements were made at a temperature of 100 mK in a high-mobility two-dimensional electron gas embedded in a GaAs/AlGaAs heterostructure. (Data from V Umansky and J Smet)

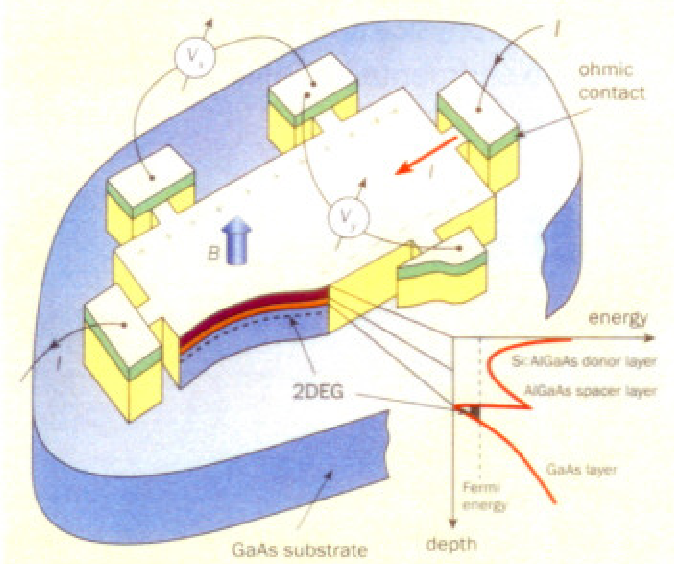
The quantum Hall effect

The quantum Hall effect violates the classical theory in a dramatic way. When the Hall effect is measured at low temperatures in a sample that is so thin that the electrons are confined to move only in the x–y plane, the Hall resistance is found to deviate from the classical $R_{yx} = B/en_s$ behaviour. At sufficiently high fields, a series of flat "steps" appear in the graph of the Hall resistance versus magnetic field (figure 1). The value of the resistance at these steps is $R_{QHE} = h/\nu e^2$, where h is the Planck constant and ν is an integer. This integer quantum Hall effect was discovered by Klaus von Klitzing, Michael Pepper and Gerhard Dorda in 1980.

Two years later Daniel Tsui, Horst Störmer and Arthur Gossard discovered that ν could take on fractional values of the form p/q, with the most prominent examples being $\nu = 1/3$, $2/5$ and $3/7$, and other fractions of the form $p/(2p+1)$.

Amazingly, the resistance is quantized with a precision that is close to one part in a billion, which makes the quantum Hall effect one of the most precise phenomena known in physics. Moreover, the quantized values of the resistance are independent of the material used and the shape of the sample, and depend only on universal physical constants. For

2 Measuring the quantum Hall effect



Modern quantum Hall experiments are carried out in semiconductor structures that contain layers of gallium arsenide (GaAs, blue) and aluminium gallium arsenide (AlGaAs, orange) that have been grown epitaxially on top of an insulating GaAs substrate. The electrons for the two-dimensional electron gas (2DEG, black dashed line) are supplied by silicon atoms in the Si:AlGaAs donor layer (purple). The resulting potential-well structure (red line on graph) confines the electrons to a quasi-two-dimensional region that is typically 10 nm thick and about 100 nm below the surface. The 2DEG is separated from the donor layer by an AlGaAs spacer layer (typically 20–60 nm thick). Current flows in and out of the device through ohmic contacts that penetrate into the structure and make contact with the 2DEG. The magnetic field is applied at right angles to the layers and various electrodes allow both the Hall and the longitudinal voltage to be measured.

these reasons the international standard for resistance is now defined in terms of the quantum Hall effect.

The behaviour of the longitudinal resistance, R_{xx}, is also interesting. In the regions where the Hall resistance is constant, the longitudinal resistivity vanishes. In condensed-matter physics such an absence of energy dissipation suggests the existence of an energy gap between the ground state of the system and its first excited state. This is a valuable clue in trying to understand the physics of these systems.

The essential technology behind the discovery of the integer quantum Hall effect was the silicon-based metal-oxide semiconductor field effect transistor (MOSFET), which is now a backbone of the microelectronics industry. In this device electrons are confined to a thin (~10 nm) layer, and this confinement leads to strong energy quantization in the direction perpendicular to the interface. At low enough temperatures the electrons occupy only the lowest quantized energy state and their motion is confined to the two dimensions parallel to the interface. In addition to their scientific value, these two-dimensional electron gases also form the basis of high-performance electronic components in many mobile-communications and satellite-television systems.

Electrons moving in a two-dimensional electron gas (2DEG) can have extremely high mobilities due to the low levels of impurity scattering. The integer quantum Hall effect was discovered in a silicon MOSFET with an electron mobility of 30–50 × 10^3 $cm^2 V^{-1} s^{-1}$, while the fractional effect was discovered in a gallium arsenide/aluminium gallium arsenide (GaAs/AlGaAs) structure with a higher mobility. The electron mobility in today's structures can be as high as 10^7 $cm^2 V^{-1} s^{-1}$ (figure 2).

Quantum Hall theory

The theory of the integer quantum Hall effect was worked out shortly after its experimental discovery. Classically, when an electron is confined to a plane and subjected to a perpendicular magnetic field, it performs cyclotron motion with a frequency $\omega_c = eB/m$ and a radius $R_c = P/eB$, where m is the mass of the electron and P is its momentum. Quantum mechanically, the energy of the electron is quantized into so-called Landau levels with energies $E_N = (N + ½)\hbar\omega_c$, where N is an integer and $\hbar$ is the Planck constant divided by 2π. The different Landau levels are therefore separated by the cyclotron energy, $\hbar\omega_c$.

Each Landau level is vastly degenerate and can therefore hold a large number of electrons. For a large sample, the number of states in each Landau level is macroscopically large and is approximately equal to the number of magnetic flux quanta, $\Phi_0 = h/e$, threading the entire sample. A given Landau level is full when there is one electron circulating around each flux quantum. The number of filled Landau levels is called the "filling factor", ν_c, and it can be an integer or a fraction. The filling factor can be thought of as the number of conduction electrons per quantum of flux, which is given by $\nu_c = n_s\Phi_0/B = n_s h/eB$.

At zero temperature the electrons populate all the states below the chemical potential or Fermi energy, and no states above this energy. Therefore, if the chemical potential lies between two Landau levels (i.e. if $\nu_c = N$) there will be an energy gap between the ground state, where N Landau levels are completely filled, and the excited states, where some electrons occupy states in the $N + 1$ Landau level. Indeed, for samples with very few impurities the plateaux found in the integer quantum Hall effect are centred around $B = n_s\Phi_0/N$ (figure 1), while the corresponding energy gaps are of the order of the cyclotron energy, $\hbar\omega_c$. The chemical potential can be varied by changing the applied magnetic field, B, or by using electrostatic or optical fields to vary the number of electrons per unit area, n_s.

Scattering by impurities, which is bound to happen in real samples, broadens the range of energies in each Landau level, and localizes most of the electronic states. While localized states do not carry current, states with energies close to E_N remain extended and therefore capable of carrying current. Hence, when the chemical potential is varied within the energy range in which all the states are localized, there is no dissipation (i.e. the longitudinal resistance is zero) and the number of electrons that can carry current remains constant (i.e. the Hall resistance is constant) – just as is seen in the experiments. The integer quantum Hall effect is therefore a consequence of a strong magnetic field and weak impurity scattering.

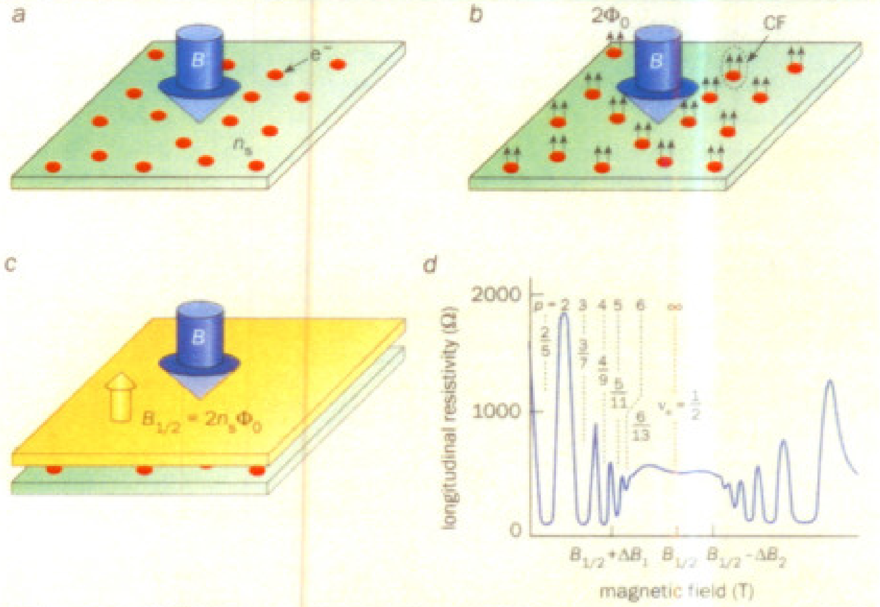
In contrast, the fractional quantum Hall effect cannot be explained without taking into account the Coulomb repulsion between electrons. Most of the fractional quantum Hall effect states are observed for $\nu_c < 1$: that is when there are more flux quanta than conduction electrons in the system. The strong magnetic field needed to observe the fractional effect makes the cyclotron energy significantly larger than the typical interaction energy between the electrons, which means that most of the action takes place in the lowest Landau level.

This observation motivated Robert Laughlin to develop, in 1983, many-body trial wavefunctions for the $\nu = 1/(2m+1)$ fractional quantum Hall states. These wavefunctions are composed solely of single-particle wavefunctions from the lowest Landau level and are built in a way that minimizes the probability of the electrons getting close to each other. Using these trial wavefunctions, Laughlin was able to explain the $\nu = 1/3$ and $\nu = 1/5$ fractional quantum Hall effects.

Moreover, by drawing insights from his trial wavefunctions and from the experimental observations of energy gaps and quantized Hall resistivity, Laughlin predicted that excitations above the $\nu = 1/(2m+1)$ ground state would be composed of localized lumps of charge – referred to as "quasiparticles" or "quasiholes" – and that the charge on each lump would be *a fraction* of the electron charge.

Following this, Bertrand Halperin of Harvard University predicted that these fractionally charged excitations should not obey either of the sets of statistics – that is neither Fermi–Dirac statistics or Bose–Einstein statistics – that are obeyed by all other particles. Instead they obey so-called fractional statistics.

3 How to make a composite fermion



(*a*) The quantum Hall effects occur when a two-dimensional electron gas is placed in a magnetic field, B (blue arrow). (*b*) The electrons (red dots) can be mathematically transformed into composite fermions, which consist of an electron plus two magnetic flux quanta that point in the opposite direction to B. (*c*) In the "mean field approximation" these flux quanta are replaced by a uniform magnetic field of magnitude $B_{1/2}$. The net magnetic field that each composite fermion experiences is now $\Delta B = B - B_{1/2}$. The behaviour of composite fermions in a field ΔB can be related to the behaviour of electrons in the stronger field, B. (*d*) The behaviour of the longitudinal resistivity, ρ_{xx}, near $B_{1/2}$, where the composite fermions behave as charged fermions in a relatively weak magnetic field.

Composite fermions

In many ways the integer and fractional quantum Hall effects seem similar. For instance, they both involve the precise quantization of the Hall resistivity and the flow of current without dissipation. However, it appeared difficult, at first, to put the theories of the two effects on an equal footing. In particular, electron–electron interactions were an essential ingredient for the fractional effect but not for the integer effect.

A common theory of the two effects was first developed by Jainendra Jain of the State University of New York (SUNY) at Stony Brook in 1989. Jain exploited the similarity between Laughlin's wavefunction for the $\nu = 1/(2m+1)$ fractional states and the wavefunction of the filled $\nu = 1$ Landau level.

A different approach was the development in the late 1980s of so-called Chern–Simons field theories – first used by theoretical particle physicists – for the fractional effect. This approach transforms the Schrödinger equation describing electrons in a strong magnetic field into a different Schrödinger equation that is, in some senses, easier to handle (see box on page 40 for the mathematical details).

The transformation can be visualized as the attachment of an integer number ϕ of magnetic flux quanta to each electron. These flux quanta point in the opposite direction to the applied magnetic field (see figure 3). The combination of the electron and the flux quanta is called a composite particle.

More precisely, in quantum theory a many-body problem is defined by its Hamiltonian and by the symmetry of its wavefunction. In the fractional quantum Hall effect the Hamiltonian (before the transformation) describes electrons that experience a uniform magnetic field, B, and repel each other as a result of Coulomb forces. The wavefunction has a fermionic symmetry.

After the Chern–Simons transformation the Hamiltonian contains a new ingredient – each particle is now subject to a magnetic field that is composed of B, which is static, and a new field, $b(r) = \phi\Phi_0\rho(r)$, where $\rho(r)$ is the electron density. This new field $b(r)$, which originates from the fictitious flux tubes carried by the composite fermions, is dynamic and points in the opposite direction to B. The symmetry of the transformed wavefunction is fermionic when ϕ is even and bosonic when ϕ is odd. Here, we will focus on the case of $\phi = 2$: this is known as composite fermion (CF) theory.

At first sight, the transformation makes the problem harder, since the magnitude of the magnetic field, $B - b(r)$, becomes a quantum operator, the value of which changes with the fluctuations in the density of the electrons. However, it suggests a natural approximation in which $b(r)$ is replaced by its average value, $B_{1/2} = \phi\Phi_0 n_s = \phi h n_s / e$.

In this "mean-field approximation", the transformed wavefunction is found to be the ground-state wavefunction for composite fermions in a magnetic field $\Delta B = B - B_{1/2}$. Furthermore, the composite-fermion filling factor, ν_{CF}, is related to the electron filling factor, ν_e, by the equation: $\nu_{CF} = 1/(\nu_e^{-1} - \phi)$.

Given that $\phi = 2$ for composite fermions, we find that $\nu_e = p/(2p+1)$ transforms into $\nu_{CF} = p$. Therefore, the fractional quantum Hall effect for electrons is the integer quantum Hall effect for composite fermions!

Experiments on composite fermions

The effective magnetic field, ΔB, also introduces a new length scale – the cyclotron radius for a composite fermion, which is larger than its electronic counterpart by a factor of $B/\Delta B$.

Composite fermions

The many-electron wavefunction $\psi(\{r_i\})$ satisfies the Schrödinger equation $H\psi(\{r_i\}) = E\psi(\{r_i\})$ where r_i is the position of the i-th electron in the x–y plane, E is the electron energy and H is the following Hamiltonian:

$$H = \sum[P_i - eB\times r_i/2]^2/2m + \sum e^2/\varepsilon|r_i - r_j|.$$

The first term is the sum of the kinetic energy of all the electrons in a magnetic field, B, while the second term is the potential energy due to the sum of all the electron–electron interactions. In the absence of these interactions, the ground state of a partially filled Landau level is vastly degenerate, making a perturbative analysis of electron–electron interaction all but impossible.

In the Chern–Simons approach one is able to define the following wavefunction:

$$\chi(r_i) = \psi(r_i)\exp\left[i\phi\sum_{i<j}\arg(r_i - r_j)\right].$$

Here ϕ is an integer and $\arg(r_i - r_j)$ is the angle of the vector $(r_i - r_j)$ relative to the x-axis. As explained in the text, the transformation is equivalent to attaching ϕ magnetic flux quanta to each electron.

The new wavefunction χ satisfies a Schrödinger equation with a modified Hamiltonian, and describes composite fermions interacting in a reduced magnetic field, $\Delta B = B - b$, where b is zero everywhere except at $r = r_1$, r_2 and so on. At these points the magnitude of b is equivalent to the magnetic field produced by an infinitely thin solenoid with a flux of $\phi h/e$. The transformed wavefunction, χ, is symmetric (or bosonic) for odd ϕ and anti-symmetric (fermionic) for even ϕ.

The mapping of electrons in a strong magnetic field onto composite fermions in a much weaker field may sound puzzling at first because, for example, the $\nu_e = ½$ state (in which the electrons perform fast cyclotron cycles) transforms into a state in which the magnetic field is zero, on average, and the composite fermions therefore move in straight lines. In fact, the transformation maps the longitudinal resistivity of electrons in a field B to be that of composite fermions in a field ΔB. However, the transverse or Hall resistivity for electrons turns out to be the sum of the Hall resistivity for the composite fermions plus $2h/e^2$. This additional Hall resistivity is induced by the magnetic flux quanta.

The mean-field approximation described in the main text neglects density fluctuations, which can be justified as a starting point because the Coulomb repulsion between electrons suppresses such fluctuations. However, when density fluctuations are taken into account, electron–electron interactions are found to affect the properties of composite fermions in a crucial way. In particular, the Coulomb interaction modifies the mass of the composite fermions, and this in turn modifies the scattering times and the energy gap.

Many physicists have contributed to the development of Chern–Simons composite fermion theory, including Steven Girvin and Allan MacDonald of Indiana University, Steven Kivelson of the University of California at Los Angeles and co-workers, Ana Lopez and Eduardo Fradkin of the University of Illinois at Urbana-Champaign, Bertrand Halperin of Harvard, Patrick Lee of the Massachusetts Institute of Technology and Nicholas Read of Yale University, and Vadim Kalmayer and Shou-Cheng Zhang, then at IBM's Almaden Research Center in California.

The effects of this new length scale have been seen in several experiments. For example, Robert Willett, Ken West and Loren Pfeiffer of Bell Laboratories in New Jersey have measured changes in the absorption and velocity of acoustic waves due to the resonant response of composite fermions when their cyclotron radius matches the wavelength of the acoustic wave.

Due to the inevitable disorder in the samples, the fractional quantum Hall effect is usually observed only when ΔB is large enough (since a large energy gap is needed), which corresponds to $\nu_{CF} < 10$. However, the resonant response to acoustic waves has been observed for much smaller ΔB, which proves that composite fermions can describe these systems even beyond the normally observable fractional quantum Hall effect states.

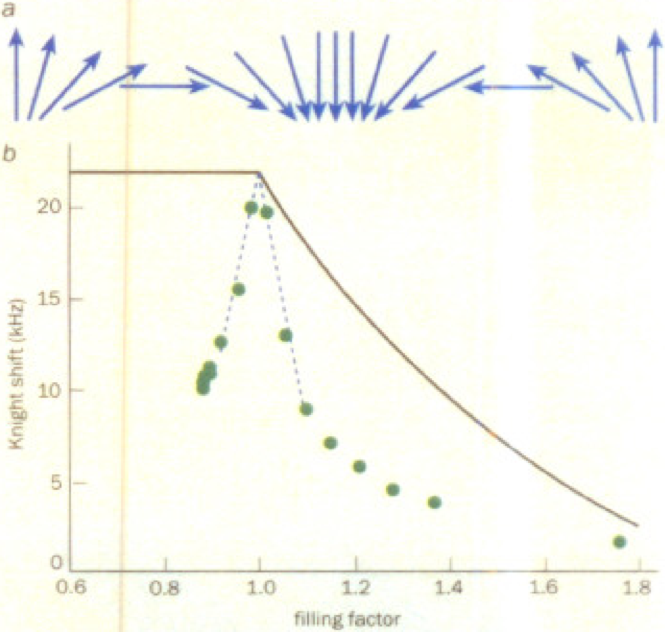
Interestingly, when $\nu_e = ½$, we find that ν_{CF} becomes infinite and ΔB becomes zero. In other words, the flux carried by the composite fermions fully cancels out the external magnetic field. Therefore the case in which electrons are in a $\nu_e = ½$ state is transformed into a state in which composite fermions experience, on average, no magnetic field. An immediate consequence of this is that the cyclotron radius for the composite fermions becomes infinite.

Despite this, electrons at $\nu_e = ½$ behave quite differently from electrons at zero magnetic field, as one might expect. One difference is in their longitudinal response to a spatially periodic electric field, such as that set up by an acoustic wave. This response is described by the longitudinal conductivity. For electrons at zero field, the longitudinal conductivity is proportional to the spatial period of the electric field. However, for electrons at $\nu_e = ½$, composite-fermion theory predicts that the conductivity is *inversely* proportional to the period. This prediction has been confirmed in measurements with acoustic waves.

Another unusual property of the $\nu_e = ½$ state is that the composite fermions are predicted to be strongly scattered by density fluctuations, since these are viewed as fluctuations of the magnetic field. This has been confirmed in measurements of the Coulomb "drag" between a pair of parallel two-dimensional electron gases (2DEGs). Jim Eisenstein and co-workers at the California Institute of Technology prepared two adjacent 2DEGs in the $\nu_e = ½$ state. Current was driven through one while the voltage induced in the other was measured. The ratio of the voltage to the current, called the trans-resistance, reflects the rate of momentum transfer (i.e. the scattering) between the two systems. The scattering at $\nu_e = ½$ was indeed found to be several orders of magnitude larger than its value at zero magnetic field.

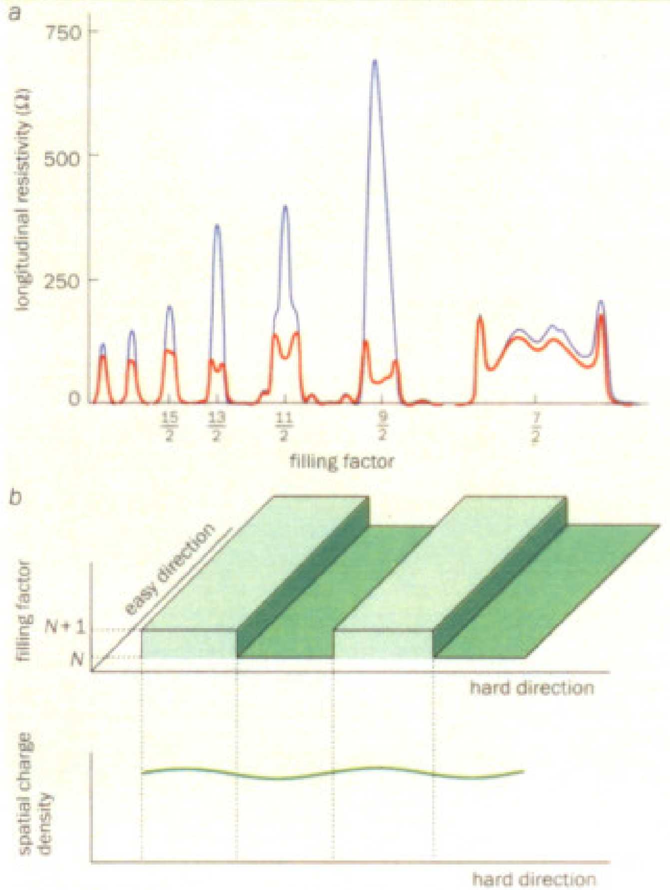
Now that we have identified the composite fermion as the basic ingredient of the fractional quantum Hall effect, what is the charge on a composite fermion? To answer this question, imagine the following *gedanken* experiment. An electron is placed at the origin and two flux quanta are slowly added to it to create a composite fermion. In addition to the charge e at the origin, the introduction of the magnetic-flux tubes generates an azimuthal electric field around the origin, and this field generates a radial Hall current. This current sends part of the initial charge to the edges of the system. The remaining charge turns out to be $e^* = e/(2p+1)$: this is the charge of the Laughlin quasiparticle.

5 Spin effects in the quantum Hall regime



(*a*) In the spin configuration known as a skyrmion, the direction of the electron spin gradually rotates from up at one side to down in the centre, and back to up at the other side. (*b*) Measurement of the electronic spin polarization (Knight shift) in a two-dimensional electron gas near $\nu = 1$ (green circles). The predictions of a model in which skyrmions are formed (blue dashed line) are in good agreement with the data, unlike predictions in which the spins are reversed one by one (black line). (S Barrett *et al.* 199 *Phys. Rev. Lett.* **74** 5112)

6 Anisotropy and stripes



(*a*) Experimental measurements of the longitudinal resistivity along the easy (red) and hard (blue) directions in a high-mobility two-dimensional electron gas at 25 mK. Note the large difference between the resistivity in these directions at $\nu = 7/2, 9/2, 11/2, 13/2$ and $15/2$. (Data from M Lilly and J Eisenstein) (*b*) A possible explanation of this anisotropy is the formation of stripes with $\nu = N$ and $\nu = N + 1$ along the easy direction (top). Although the stripes only lead to a small modulation in the spatial charge density, n_s (bottom), calculations show that current flowing perpendicular to the stripes encounters high resistance.

shot noise in both the transmitted and reflected currents would be $S_I = 2e^*I_R$, where I_R is the (small) reflected current (see figure 4*a*).

Such experiments were performed in 1997 by Christian Glattli and co-workers at the CEA laboratory in Saclay, near Paris (for $\nu_c = 1/3$), and by Rafi de-Picciotto and colleagues at our institute (for $\nu_c = 1/3$ and $2/5$). A small constriction, in the form of a quantum point contact, was embedded into the 2DEG to reflect the current. These experiments were very demanding because the predicted shot noise was extremely small, and various techniques were needed to improve the signal-to-noise ratio.

When the edge current was not partitioned, no noise was measured (since $I_R = 0$). However, when a weak reflection of the $\nu_c = 1/3$ or the $\nu_c = 2/5$ channel was introduced, there was good agreement between the measured noise and the noise predicted for $e^* = e/3$ and $e^* = e/5$, verifying the effect of fractionally charged quasiparticles (see figure 4*b*).

Spins in the lowest Landau level

Naively, one would expect all the electron spins in ground states with $\nu_c < 1$ to point in the direction of the magnetic field (i.e. to be "spin polarized"), and hence for the spin degree of freedom to be frozen. However, it so happens that the particular structure of the gallium arsenide (GaAs) lattice can lead to far more complicated arrangements of electron spins. The Lande g-factor, which determines the splitting of energy levels with different spin angular momentum in a magnetic field, is very small in GaAs and this allows a wide range of magnetic behaviour.

One example of such interesting behaviour was the destruction of the state with zero longitudinal resistance at $\nu_c = 5/2$. (See "A new spin on the quantum Hall effect" by Giles Davies *Physics World* September 1999 pages 20–21.)

A second example of surprising spin behaviour can be observed in the vicinity of the $\nu_c = 1$ state. In this state the electrons form a ferromagnet where the exchange interaction aligns all their spins in the same direction. However, as we move away from $\nu_c = 1$ by changing the magnetic field, the spin polarization falls dramatically. Moving away from $\nu_c = 1$ is equivalent to keeping the number of magnetic flux lines constant and adding (or taking away) electrons so that there is just more (or just less) than one electron per flux quantum.

Two scenarios are possible when an electron is removed from a $\nu_c = 1$ state. In the first, the spins of the remaining electrons are left intact, reducing the magnetization by the contribution of that particular electron. In the second, all the spins reorient themselves to form a "spin texture" known as a skyrmion (figure 5*a*). In a skyrmion the spin direction changes from being parallel to the magnetic field to being antiparallel. This happens over a length scale that is much larger than the distance between electrons, and has a dramatic effect on the overall magnetization.

Surprisingly, in 1993 Shivaji Sondhi, then at UCLA, and co-workers showed that the skyrmiom configuration has a lower energy than that in which the spins do not change direction. This was later confirmed in experiments by Sean Barrett and co-workers at Bell Labs (figure 5*b*). Skyrmions were also found to play a role in various other thermodynamic and transport phenomena near $\nu_c = 1$.

High Landau levels

Most work on the fractional quantum Hall effect has concentrated on the lowest Landau level. However, when physicists recently looked at samples with extremely high mobility at very low temperatures (<100 mK), they found surprising behaviour in the high Landau levels.

Measurements by Jim Eisenstein's group at Caltech, by Störmer, Tsui and co-workers at Princeton University, and by Mansour Shayegan and co-workers, also at Princeton, found that the longitudinal resistivity showed a strong peak around $\nu = 9/2, 11/2, 13/2$ and $15/2$ when the current was flowing in one direction (the so-called hard direction), and a minimum when the current was flowing in the orthogonal direction (the easy direction). The resistivity in the hard direction, ρ_{xx}, was typically 5–10 times greater than that in the easy direction, ρ_{yy} (figure 6*a*). Moreover, the Hall resistivity, ρ_{xy}, was not quantized.

This anisotropic state also shows several other interesting charge-transport properties. For instance, when a magnetic field of about 1 tesla is applied in the plane of the 2DEG along the easy direction, the anisotropy can disappear and, in some cases, reverse the hard and easy axes.

The new phase, which seems to be highly sensitive to disorder and to temperature, may be related to earlier theoretical predictions by Michael Fogler and co-workers of the University of Minnesota in the US, and Roderic Moessner and co-workers of Oxford University in the UK. They predicted a new ground state that is composed of narrow spatial stripes, each only a few cyclotron radii wide, with alternate filling factors of $\nu_c = N$ and $\nu_c = N + 1$, where N is larger than 4 (figure 6*b*). In high Landau levels the energy of this state is lower than that of a uniform density state. Although the modulation in the charge density is only a few per cent, theory suggests that this can account for the strong anisotropy observed in experiments.

Formation of this striped phase requires symmetry breaking in the plane of the electron gas. As Herbert Kroemer of the University of California at Santa Barbara has pointed out, while there is full cubic symmetry in gallium arsenide in momentum space, the electric field needed to confine the 2DEG breaks this symmetry and distinguishes between different directions in the crystal. Such asymmetry, although small, might well select the preferred direction for the stripes.

Conclusion

Even though the Hall effect has been studied for the past 120 years, research in the last two decades has resulted in new and unexpected discoveries by hundreds of researchers, and Nobel prizes for von Klitzing (1985), and Laughlin, Störmer and Tsui (1998). In addition to the subjects that have been covered in this article – such as skyrmioms, composite fermions, anisotropic states and fractional charge carriers – many other interesting phenomena have also been studied. Examples include: edge states; phase transitions between different quantum Hall states, and between quantum Hall and non-quantum Hall states; electron pairing in the quantum Hall effect; local probing of quantum Hall states; and interactions between quantum Hall states in two adjacent two-dimensional electron gases.

For an area of research that was believed to be well understood 20 years ago, the Hall effect has proved to be another example of how quantum mechanics revolutionizes classical and deeply held ideas in physics.

Further reading

T Chakraborty and P Pietilainen 1995 *The Quantum Hall Effects: Integral and Fractional* (Springer, New York)

S Das Sarma and A Pinczuk (ed) 1996 *Perspectives in Quantum Hall Effects: Novel Quantum Liquids in Low-Dimensional Semiconductor Structures* (Wiley, New York)

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R B Laughlin 1983 Anomalous quantum Hall effect: an incompressible quantum fluid with fractionally charged excitations *Phys. Rev. Lett.* **50** 1395

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The 1998 Nobel lectures by Robert Laughlin, Horst Störmer and Daniel Tsui were published in the July 1999 issue of *Reviews of Modern Physics*

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