

QUANTUM NOISE, DETAILED BALANCE AND KUBO FORMULA IN NONEQUILIBRIUM QUANTUM SYSTEMS

U.GAVISH, Y. IMRY, Y. LEVINSON

Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel

Current quantum noise can be pictured as a sum over transitions through which the electronic system exchanges energy with its environment. We formulate this picture and use it to show which type of current correlators are measurable, and in what measurement the zero point fluctuations will play a role (the answer to the latter is as expected: only if the detector excites the system.) Using the above picture, we calculate and give physical interpretation of the finite-frequency finite-temperature current noise in a noninteracting Landauer-type system, where the chemical potentials of terminals 1 and 2 are $\mu + eV/2$ and $\mu - eV/2$ respectively, and derive a detailed-balance condition for this nonequilibrium system. Finally, we derive a generalized form of the Kubo formula for a wide class of interacting nonequilibrium systems, relating the *differential* conductivity to the current noise.

1 Introduction

A general expression for the current correlators is derived below: it is a sum over transitions between pairs of energy levels. This expression is very similar to the one obtained by Van Hove¹ in the framework of neutron scattering theory. Using this sum over transitions we will identify the positive and negative frequency part of the correlator as having distinct, well-defined, separately-measurable, physical interpretations, namely: the emission and absorption spectra. This picture will enable us to analyze in what type of measurement the zero-point fluctuations will have an effect on the measured spectrum. It will also enable us to derive a detailed balance relation out of equilibrium for a Landauer-type system which consists of a point scatterer connected through two one-dimensional single-channel conducting arms, to two terminals whose chemical potentials differ. This condition keeps the system in a stationary nonequilibrium state. Finally, a Kubo formula for the differential conductivity is derived for a wide class of systems out of equilibrium, and is verified analytically for the above Landauer-type case.

2 General Formulation for the Quantum noise.

Consider the current correlator for a stationary system (i.e. with no dependence on t'),

$$C(t) \equiv \langle \hat{J}(t') \hat{J}(t' + t) \rangle \equiv \sum_i P_i \langle i | \hat{J}(0) \hat{J}(t) | i \rangle, \quad (1)$$

of a quantum system ("antenna"), characterized by a density matrix which is diagonal in the eigenstate basis. $|i\rangle$ are the eigenstates of the antenna with energies E_i and populations P_i . \hat{J} is a space average² of the time-dependent current operator in the Heisenberg representation, $\hat{j}(x, t) = \exp(i\hat{H}t/\hbar) \hat{j} \exp(-i\hat{H}t/\hbar)$, where \hat{H} is the Hamiltonian of the antenna, taken to be time independent, but otherwise very general, *including interactions*.

If the (stationary) system were describable classically, then one would have $C(t) = C(-t)$ and it would be clear what the time-dependent current fluctuations were: the average-over-realizations of the product of the values of the current at different times. However, in the quantum case the operators $\hat{j}(t)$ for different times do not commute. Therefore the quantum correlator can not, in general, be given this simple interpretation. Actually, in the quantum case $C(t)$ is in general *complex* and not symmetric, but just satisfies: $C(t) = C(-t)^*$, which means that *it is not a directly measurable quantity* in the sense that the product of the (real) current values that appear, say, on some ampermeter screen at different times does not give $C(t)$. Its

Fourier transform (to which we shall also refer, for brevity, as the correlator):

$$S(\omega) \equiv \int_{-\infty}^{\infty} e^{i\omega t} \langle |\hat{J}(0)\hat{J}(t)| \rangle dt, \quad (2)$$

is also nonsymmetric, $S(\omega) \neq S(-\omega)$, although it is obviously real (see eq.(3)). This can be seen¹ by inserting the identity operator $\sum_f |f\rangle\langle f|$ in eq.(2) and performing the time-integration, which yields:

$$S(\omega) = 2\pi\hbar \sum_{if} P_i |\langle f|\hat{J}|i\rangle|^2 \delta(E_i - E_f - \hbar\omega). \quad (3)$$

This expression is not symmetric with respect to ω since P_i usually decreases with E_i .

$S(\omega)$ has the following important physical significance. If the system is coupled, through a small term which is linear in $\hat{J}(t)$, to a second system (see examples below), then, by the Fermi golden rule, $S(\omega)$ is proportional to the transition rate between the initial state $|i\rangle$ and the final state $|f\rangle$, for which $E_i - E_f = \hbar\omega$. Therefore, for $\omega > 0$ it is proportional to the emission rate and for $\omega < 0$ it is proportional to the absorption rate³.

For example, if the second system is the free EM field, then $S(\omega > 0)$ is proportional to the energy emission rate^{3,4} into the vacuum state of this field (i.e. the state of the EM field with occupation number $N_\omega = 0$), and for $\omega < 0$ it is proportional to the energy absorption rate from an EM field with a given photon, i.e., with $N_{|\omega|} = 1$. Another example is provided by a system which is coupled to a measuring device^{5,6,7,8} (e.g., a resonant circuit). Then $S(\omega)$ is proportional to the energy transfer rate between the system and the measuring-device: The terms with $E_i > E_f$ describe transitions in which an energy of $\hbar\omega = E_i - E_f > 0$ is transferred from the system to the measuring device, while terms with $E_f > E_i$ describe transitions in which an energy of $-\hbar\omega = E_f - E_i > 0$ is transferred from the measuring device to the system. When $\omega > 0$, only the first type of terms will remain and $S(\omega)$ will be the emission spectrum while $S(-\omega)$ will be the absorption spectrum. Thus, the two branches of $S(\omega)$ yield two physically interesting and separately-measurable quantities.

In the case when the antenna is in equilibrium at a temperature T and time-reversal symmetry holds, one finds the detailed balance relation¹

$$S(\omega) = S(-\omega)e^{-\hbar\omega/k_B T}. \quad (4)$$

This detailed balance relation ensures that the system remains in equilibrium, by taking care that the asymmetry $S(\omega) \neq S(-\omega)$, i.e., the difference between the upward transitions (absorption) and the downward ones (emission) is compensated by the difference between the higher and lower thermal occupations. In section 3 the above relation is generalized for a particular nonequilibrium system, where it serves to keep the latter in its nonequilibrium, though stationary, state. From eq.(4) one sees that only for low frequencies $\hbar|\omega| \ll k_B T$, will the classical symmetry, $S(\omega) = S(-\omega)$, hold. In the time domain this means that the classical symmetry, $C(t) = C(-t)$, becomes valid only for late times $|t| \gg \hbar/k_B T$.

The customary way to treat the quantum case is^{5,9} to consider the symmetrized correlator $C_S(t' - t) \equiv (1/2)\langle \hat{j}(t)\hat{j}(t') + \hat{j}(t')\hat{j}(t) \rangle$, which is real and symmetric like the classical one. However, for a wide class of noise detection schemes and in particular for a detector in its ground-state $C_S(t)$ is *not*, the measured correlator, since it contains the zero-point fluctuations. For example, if the antenna is in equilibrium at a temperature T , it follows from the fluctuation-dissipation theorem⁹ that for $\omega > 0$ one has $S_S(\omega) \sim [N_T(\omega) + (1/2)]\omega$, where $S_S(\omega)$ is the Fourier transform of $C_S(t)$ and $N_T(\omega) = [\exp(\omega/k_B T) - 1]^{-1}$ is the Planck function. This means that $S_S(\omega) \neq 0$ even when $T = 0$, and the antenna is in its ground state. Since being in the ground state the antenna can not radiate energy, $S_S(\omega)$ can not be considered as the correlator measured by detecting the radiation¹¹.

To conclude, the measured quantity will generally not contain the emission and absorption in a symmetric combination. This was shown for particular situations of quantum noise measurement in electronic transport^{5,8,6,11} as well as in quantum optics^{10,11,12}. The more physical correlator (and its power spectrum) is the one without symmetrization. Its transform is given by a sum over transitions - downward ones in the case of positive frequency, and upward ones in the case of negative frequency. Each of these two branches has its own distinct physical significance, the emission and absorption spectrum, that may in principle be detected separately.

3 Shot Noise

We now consider current fluctuations^{13,14} for the Landauer model. A point-like elastic scatterer is connected through two ideal single-channel conducting ballistic arms to two Fermion reservoirs, 1, and 2, with chemical potentials $\mu + eV/2$ and $\mu - eV/2$ respectively. V is the voltage, and it is assumed that $eV, \hbar\omega, k_B T \ll \mu$. We consider non-interacting electrons and ignore spin. The single-particle scattering-states with energy $\epsilon_n = \hbar^2 k^2 / 2m$, which corresponds to a wave that is incoming on arm $\alpha = 1, 2$, partially reflected back into it and partially transmitted into the other arms, is: $\varphi_n(x_\gamma) = L^{-1/2}[\delta_{\alpha\gamma} e^{-ikx_\gamma} + s_{\gamma\alpha}(k) e^{ikx_\gamma}]$. Here $n \equiv (\alpha, k)$ with $k > 0$; $\alpha, \gamma = 1, 2$, L is a normalization length, m the electron mass and x_γ the distance of a point on arm γ from the scatterer. $s_{\alpha\gamma}$ is the element of the unitary scattering matrix, and it is assumed to be energy independent unless otherwise stated. To specify that a state φ_n comes from terminal α , we shall write $n \in \alpha$. The current operator on arm α is

$$\hat{j}(x_\alpha) = -(ie\hbar/2m) \sum_{nm'} \hat{a}_n^\dagger \hat{a}_{n'} \varphi_n^* \nabla_\beta \varphi_{n'} + h.c.,$$

where \hat{a}_n and \hat{a}_n^\dagger are the annihilation and creation operators of the φ 's.

We assume that the *measured* current is the average

$$\hat{J}(t) \equiv \frac{1}{L_0} \int_{L_0} dx_2 \hat{j}(x_2) \quad (5)$$

over a segment L_0 far away from the scatterer which satisfies: $L_0 k_F \gg 1$ and $\omega L_0 m / (\hbar k_F) \ll 1$, where $k_F \equiv \sqrt{2m\mu}$, and ω is the frequency of the measured noise which is assumed to satisfy $\omega \ll \mu$. These conditions ensure that the correlators are independent of the length and position of the segment L_0 ; i.e, it has no spatial dependence, which is not addressed in experiments.

To describe the current noise consider the correlator, given by eq. (3) We emphasize again that at least for some types of noise detection, it is eq. (2) and *not* its symmetrized version, which gives the measured noise if the detector is cold enough, i.e., when excitation of the system by the detector is unlikely so that the absorption is negligible. The states $|i\rangle$ are given according to the Landauer picture by the eigenstates of the system (i.e., Slater determinants) labelled by specifying a set $\{n_i\}$ of the occupied single-particle states (the scattering states φ_n emanating from the two reservoirs):

$$|i\rangle \equiv |\{n_i\}\rangle = \prod_{n_i} \hat{a}_{n_i}^\dagger |vacuum\rangle, \quad (6)$$

and the corresponding probabilities (for a more general derivation see Ref. ¹⁶) are:

$$P_i \equiv \frac{1}{Z_1} \exp[-\beta \sum_{n \in 1} (\epsilon_{n_i} - (\mu + eV/2)) n_i] \times \frac{1}{Z_2} \exp[-\beta \sum_{n \in 2} (\epsilon_{n_i} - (\mu - eV/2)) n_i], \quad (7)$$

where $\beta \equiv (k_B T)^{-1}$ and

$$Z_\alpha \equiv \sum_{\{n_i\}} \exp[-\beta \sum_{n \in \alpha} n_i (\epsilon_{n_i} - (\mu - eV(-1)^\alpha/2))]$$

$\alpha = 1, 2$. The probabilities P_i correspond to a situation in which the occupations in the gas in the one-dimensional system are determined by grand-canonical probabilities that depend on the chemical potential and the temperature of the terminals that supply the electrons to the system. From eqs.(7) and (6) it follows that at zero temperature $P_i = 0$ for any i except for one state, which we name a *cold transport state* which is given by:

$$|cold\ transport\rangle \equiv \prod_{\substack{n \in 1; \epsilon_n \leq \mu + eV/2 \\ n \in 2; \epsilon_n \leq \mu - eV/2}} \hat{a}_n^\dagger |vacuum\rangle, \quad (8)$$

In this cold transport state all the φ_n 's are occupied up to an energy $\mu - eV/2$ if $n \in 2$ and up to $\mu + eV/2$ ($eV \ll \mu$) if $n \in 1$ (see fig. 1). Therefore, this state, although it is the stationary state with the lowest energy among those that are made possible by the two terminals, is not a ground state, and since it carries current, it is not even an equilibrium state.

In the sum in eq.(3), the non-diagonal matrix element $\langle i | \hat{J}(0) | f \rangle$ is nonzero only if $|f\rangle$ differs from $|i\rangle$ by moving one particle from an occupied state, φ_n , to a previously unoccupied state, $\varphi_{n'}$, i.e., $|f\rangle$ is of the form $\hat{a}_{n'}^\dagger \hat{a}_n |i\rangle$ (up to a fermionic factor of ± 1 , that will play no role below.) The diagonal elements $\langle i | \hat{J}(0) | i \rangle$ appear in a term $\sim \delta(\hbar\omega)$. In what follows we consider only $\omega \neq 0$ and therefore neglect this term. In experiments the integration in eq.(2) is limited by the sampling time of the experiment, T_s , and as a result $\delta(\hbar\omega)$ is smoothed into a peak with a width of $\simeq \hbar/T_s$ which means that the condition $\omega \neq 0$ actually means $\omega T_s \gg 1$. We therefore have:

$$S(\omega) = 2\pi \sum_i P_i \sum_{nn'} |J_{nn'}^i|^2 \delta(\epsilon_n - \epsilon_{n'} - \omega). \quad (9)$$

where $J_{nn'}^i \equiv \langle i | \hat{J}(0) \hat{a}_{n'}^\dagger \hat{a}_n | i \rangle$, and where now the summation over n and n' is over all *single-particle* states φ_n and $\varphi_{n'}$ (with *single-particle* energies ϵ_n and $\epsilon_{n'}$) which are occupied and unoccupied, respectively, in $|i\rangle$. Now we divide the summation in eq. (9) into four partial sums, according to the four possible types of transitions: two auto-terminal ones and two cross-terminal ones (shown in fig. 1 for $\omega < 0$ in the cold transport state):

$$S(\omega) = \sum_{\alpha, \gamma=1,2} S_{\alpha \rightarrow \gamma}(\omega), \quad (10)$$

where $S_{\alpha \rightarrow \gamma}(\omega)$ contains only the transitions from states with $n \in \alpha$ to states $n \in \gamma$. Explicitly:

$$S_{\alpha \rightarrow \gamma}(\omega) = 2\pi \sum_i P_i \sum_{n \in \alpha, n' \in \gamma} |J_{nn'}^i|^2 \delta(\epsilon_n - \epsilon_{n'} - \hbar\omega) \quad (11)$$

Each of these four sums can be evaluated separately by calculating the current matrix elements in eq.(9), transforming the sums over k and k' (which are implicit in the sums over n and n') into integrals, performing the summation over i according to Eqs. (6) and (7), and integrating using the condition $\hbar\omega, eV, k_B T \ll \mu$ and the unitarity of the scattering matrix, $\sum_\gamma s_{\alpha\gamma} s_{\beta\gamma}^* = \delta_{\alpha\beta}$. The final results are:

$$S_{1 \rightarrow 2}(\omega) = \frac{e^2 \tilde{T}(1 - \tilde{T})}{h} \frac{\hbar\omega - eV}{e^{\beta(\hbar\omega - eV)} - 1} \quad (12)$$



Figure 1: The Landauer model, states and transitions.

$$S_{2 \rightarrow 1}(\omega) = \frac{e^2 \tilde{T}(1 - \tilde{T})}{h} \frac{\hbar\omega + eV}{e^{\beta(\hbar\omega + eV)} - 1} \quad (13)$$

$$S_{1 \rightarrow 1}(\omega) = S_{2 \rightarrow 2}(\omega) = \frac{e^2 \tilde{T}^2}{h} \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}. \quad (14)$$

Where, $\tilde{T} \equiv |s_{21}|^2$ is the transmission from arm 1 to 2. Substitution in eq.(10) yields:

$$S(\omega) = \frac{e^2 \tilde{T}(1 - \tilde{T})}{h} [F(\hbar\omega - eV) + F(\hbar\omega + eV)] + \frac{2e^2 \tilde{T}^2}{h} F(\hbar\omega) \quad (15)$$

where $F(x) \equiv x(e^{\beta x} - 1)^{-1}$. Eq. (15) is the non-symmetrized power spectrum, at finite frequency (positive or negative), voltage and temperature. It has been previously obtained by Aguado and Kouwenhoven⁸ (they use an opposite convention for the sign of ω). Eq. (15) is consistent with the zero-temperature limit, i.e., the cold transport case, that was obtained by Lesovik and Loosen⁵. Unlike the symmetrized version that was derived by Yang¹⁴ and de Jong and Beenakker¹⁷, here $S(\omega) \neq S(-\omega)$.

Since the system is not in equilibrium, the detailed-balance condition eq.(4) is not satisfied, however, a modified version of it does exist. To obtain it, note that Eqs.(7) and (11), or (12),(13) and (14) imply:

$$S_{1 \rightarrow 1}(\omega) = S_{1 \rightarrow 1}(-\omega)e^{-\beta\hbar\omega}, \quad S_{2 \rightarrow 2}(\omega) = S_{2 \rightarrow 2}(-\omega)e^{-\beta\hbar\omega} \quad (16)$$

$$S_{1 \rightarrow 2}(\omega) = S_{2 \rightarrow 1}(-\omega)e^{-\beta\hbar\omega + \beta eV}, \quad S_{2 \rightarrow 1}(\omega) = S_{1 \rightarrow 2}(-\omega)e^{-\beta\hbar\omega - \beta eV}$$

In equilibrium, $eV = 0$ and the last two relations have the form of first two and then, by eq.(10), the ordinary detailed-balance relation, eq. (4) is recovered. The finite voltage creates a nonequilibrium but stationary state which is maintained by virtue of more complicated detailed-balance relations between upward and downward transitions, given by the above four equations. In particular these relations imply that $S(-\omega) \neq 0$ even at zero temperature which is not surprising since in the cold transport state, eq. (8), downward transitions are possible from occupied states with $n \in 1$ within the energy window $[\mu - eV/2, \mu + eV/2]$ into the empty states with $n \in 2$ and within the same energy window. That is, emission is possible.

Another conclusion arising from the above four equations is that in order to recover the classical symmetry $S(\omega) = S(-\omega)$, the condition $\hbar|\omega| \ll k_B T$ is not longer enough and should be replaced by: $eV + \hbar|\omega| \ll k_B T$.

4 Kubo Formula for Nonequilibrium Systems

Here we would like to make another application of the point of view of section 2. We again allow^{14,18} for the antenna a large class of nonequilibrium situations - all those in which the density matrix is diagonal in the eigenstate basis (and it therefore commutes with the Hamiltonian). This may include electron-electron interactions and the possibility that $\langle \hat{j}(t) \rangle \neq 0$.

A useful example to have in mind is a Landauer-type transport system of section 3, which has a two-terminal linear conductance, per spin channel, of

$$G = \frac{e^2}{2\pi\hbar} \tilde{T}. \quad (17)$$

We use the explicit exact expression of eq.3 for the current power-spectrum $S(\omega)$. As emphasized in section 2, its important physical significance is that it gives^{4,3} the emission rate into the vacuum (i.e. the state of the EM field where all $N_\omega = 0$) for $\omega > 0$ and the absorption rate for $\omega < 0$ and an EM field with one photon, $N_{|\omega|} = 1$.

We first take the (possibly D.C. driven) "antenna" to be in a given (usually the lowest) state consistent with the external driving (this will be valid, for example, when the reservoirs that feed the antenna are at $T = 0$ and there is either no coupling with any additional thermal bath, or when that coupling is with a bath at $T \rightarrow 0$, and no appreciable heating by the D.C. current has taken place. In the latter case, some tendency toward equilibrating the chemical potentials of the left- and right- moving electrons may occur, with a reduction of the D.C. current, which is of no particular concern to us). In the case of the Landauer-type system of section 3, this lowest state is the cold transport one, defined in eq. 8. The energy absorption rate, $R_a(\omega)$, by the antenna from a *classical field* (with $N_\omega \gg 1$ photons and a negligible spontaneous component of the emitted radiation) with a frequency $\omega > 0$ is given via the usual treatment⁴ by:

$$R_a(\omega) = \omega |A(\omega)|^2 S(-\omega) / (\hbar c^2). \quad (18)$$

The emission rate, $R_e(\omega)$ is given by

$$R_e(\omega) = \omega |A(\omega)|^2 S(\omega) / (\hbar c^2). \quad (19)$$

The *net* absorption rate, $R_{a,net}(\omega)$ is given by the difference between eqs. (18) and (19). It is also given, writing the infinitesimal "tickling" electric field as $E(\omega) = iA(\omega)\omega/c$, by

$$R_{a,net}(\omega) = -2G_d(\omega)(\omega/c)^2 |A(\omega)|^2, \quad (20)$$

where the volume of the system is defined as unity.

$G_d(\omega)$ is the differential ac conductance. It is defined as the in-phase (dissipative) linear response (A.C. current) to the tickling A.C. field at frequency ω . A finite D.C. current which in turn flows in response to the finite applied D.C. voltage²¹ V , is allowed.

Using eqs. 18, 19 and 20, we reach our principal conclusion that *the antisymmetric part of the current noise in our nonequilibrium system (i.e. typically including quantum shot-noise¹³) is related to the differential ac conductance $G_d(\omega)$ at the same frequency ω .*

$$S(-\omega) - S(\omega) = 2\hbar\omega G_d(\omega), \quad (\omega \geq 0), \quad (21)$$

Eq. 21 is the simple but nontrivial generalization of the Kubo formula¹⁹ for the current-carrying, nonequilibrium, case. The antisymmetric combination appearing on the LHS corresponds to the Fourier transform of the commutator of the Heisenberg current operators at different times. A similar expression was obtained by Lesovik and Loosen⁵ and by Lesovik²⁰. Here we have interpreted it physically and put it in a general context

It is straightforward to generalize the above treatment to the case where the antenna is not at its lowest possible state, but has a density matrix (assumed to be diagonal in the eigenstates' basis) which allows the population of a number of states. The general form of the result of the net absorption rate is still valid. *Therefore, our principal result, eq. 21, is unchanged.*

We now verify the above results with the example of the Landauer-type model, of section 3. From eq. (15) and (17) it follows that:

$$S(-\omega) - S(\omega) = e^2 \frac{\tilde{T}}{\pi} \omega = 2\hbar\omega G, \quad (22)$$

in agreement with the generalized Kubo formula, eq. 21. To consider a more general case we now relax the assumption that the scattering matrix is energy independent and assume instead that the scale on which it changes is of the order of eV , and that $eV \gg \hbar\omega$. We emphasize that the energy dependence of the scattering matrix must be evaluated including the self-consistent changes in the potential of the scatterer due to the voltage V . A similar derivation to the one that led to eq. (15) and (22) now gives:

$$S(-\omega) - S(\omega) = e^2 \frac{1}{2\pi} \omega (1 - R^2(\mu - eV/2) + \tilde{T}^2(\mu + eV/2)) \quad (23)$$

where $R = 1 - \tilde{T}$. Approximating $\tilde{T}(\epsilon) = \tilde{T}(\mu - eV/2) + \lambda(\epsilon - \mu + eV/2) + O(V^2)$, one has:

$$S(-\omega) - S(\omega) = e^2 \frac{\tilde{T}}{\pi} \omega (1 + \lambda eV), \quad (24)$$

where $\tilde{T} = \tilde{T}(\mu - eV/2)$. This, together with eq.21, is a new prediction for the low, but finite frequency²¹ dynamic conductivity, which may be different from the slope of the DC I-V characteristics.

The generalization of the above to the case of many transport channels, along the lines of the usual theories of quantum shot-noise^{13,14}, is straightforward. One uses the representation where the transmission part of the scattering matrix is diagonal. The results are expressed in terms of the transmission eigenvalues.

When the field probing the system has a finite number, N_ω , of photons, the net flow of energy, including the spontaneous process, from the system to the field is^{5,6}:

$$R_M(\omega) = S(\omega)(N_\omega + 1) - N_\omega S(-\omega). \quad (25)$$

Using eq.15 for the quantum shot-noise, and taking $G_d = G$, we see that the spontaneous term is unimportant for $N_\omega \gg eV(1 - \tilde{T})/\hbar\omega$. In the opposite limit the sample just emits noise into the "cold detector".

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References

1. L. van Hove, Phys. Rev **95**, 249 (1954).
2. The relevant quantity is often the space average, because of the excellent dipole approximation. An example⁴ is a system whose size is much smaller than the radiation wave-length, coupled to the EM field. In section 3, the space averaging is taken only on the portion of the antenna which is assumed to interact with the measuring device.
3. The transition rate contains the properties of the system with which energy is exchanged. E.g, a factor A^2 for the EM field (eqs. 18 and 19). Due to the factor⁴ of $1/\omega$ in A^2 , $S(\omega)$ and the energy emission rate times the photon number have the same ω -dependence.
4. G. Baym, *Lectures on Quantum Mechanics*, pp. 271-276, Addison-Wesley (1993).
5. G.B. Lesovik and R. Loosen, JETP Lett., **65**, 269 (1997).
6. U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. **B RC**, R10637 (2000).
7. Y. Levinson and Y. Imry, in Proc. of the Nato ASI "Mesoscope 99" in Ankara/Antalya, Kluwer 1999.
8. R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett., **84**, 1986 (2000); C.W.J. Beenakker and H. Schomerus, Phys. Rev. Lett., **86**, 700 (2001).
9. L.D. Landau and E.M. Lifshitz, Statistical Physics, § 118, 124, 125; Pergamon, Oxford, 1980.
10. M. Lax, Phys. Rev. **145**, 110-129 (1966)
11. C. W. Gardiner, *Quantum Noise*, Springer Verlag, Berlin (1991), chapter 1.
12. E.g., R.J. Glauber, in *Quantum Optics and Electronics*, C. DeWitt, A. Blandin, and C. Cohen-Tannoudji (Eds.), Les Houches Summer School (1964), p.63, Gordon and Breach.
13. V.A. Khlus, JETP **66**, 1243 (1987); G.B. Lesovik, JETP Lett., **49**, 592 (1989), M. Büttiker, Phys. Rev. B **46**, 12485 (1992).
14. S.-R. Eric Yang, solid state communications, **81**, 375, (1992).
15. Y. Imry, *Introduction to Mesoscopic Physics*, Oxford University Press (1997).
16. S. Hershfield, Phys. Rev. Lett. **70**, 2134 (1993).
17. M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön (Kluwer, The Netherlands, 1997).
18. R. M. More, J. A. Harte, G. B. Zimmerman, S. B. Libby, F. R. Graziani, K. B. Fournier and T. Kato, Non-LTE Linear Response Theory, Report UCRL-JC-129348.
19. R. Kubo, J. Phys. Soc. Japan, **12**, 570 (1957); **17**, 975, (1962). We refer to the result in Eq. 21 as a generalized Kubo formula. The term Fluctuation-Dissipation Theorem (FDT) is often reserved for the corresponding relationship between the linear response function and the *symmetrized* correlator. We are not using that correlator and believe that the truly fundamental relationship (generalized here to nonequilibrium systems) is the one between the differential linear response function and the *antisymmetrized* correlator.
20. G.B. Lesovik, Uspekhi **41**, 145 (1998).
21. The dissipated power is the time-averaged product of the driving field and the resulting current. If one of those is D.C. and the other has finite ω , the average power vanishes.