

## Dynamic creation and annihilation of metastable vortex phase as a source of excess noise

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**Abstract.** – The large increase in voltage noise, commonly observed in the vicinity of the peak-effect in superconductors, is ascribed to a novel noise mechanism. A strongly pinned metastable disordered vortex phase, which is randomly generated at the edges and annealed into ordered phase in the bulk, causes large fluctuations in the integrated critical current of the sample. The excess noise due to this dynamic admixture of two distinct phases is found to display pronounced re-entrant behavior. In the Corbino geometry the injection of the metastable phase is prevented and, accordingly, the excess noise disappears.

The appearance of large noise with the onset of motion of a condensate in the presence of random pinning potential has been studied extensively in incommensurate charge density waves [1], Wigner crystals in two-dimensional electron gas [2], and most notably, in vortex matter in type-II superconductors [3–13]. In all cases, the noise is thought to reflect spatio-temporal irregularities of the moving condensate due to its interaction with the underlying pinning potential, though its precise origin remains obscure and controversial. The voltage noise due to vortex motion in a current-biased superconductor is generally referred to as flux-flow noise for which various mechanisms have been considered (for an early review see [3]). They include vortex shot noise and the associated density fluctuations [3, 4], velocity fluctuations resulting from vortex-pin interactions [3] or turbulent flow of surface currents [5], critical slowing down of vortex dynamics [6], and several suggestions [7–12] and numerical simulations [13] of various plastic vortex flow mechanisms. Each of these mechanisms may make a substantial contribution to the total measured noise. Yet the puzzling observation, which has no satisfactory explanation, is that in a specific and narrow region of the  $H$ - $T$  phase diagram the noise is enhanced drastically. This *excess noise* exceeds the usual flux-flow noise level by orders of magnitude [3, 7–11]. In low- $T_c$  superconductors the excess noise occurs in the vicinity of the peak effect (PE) below  $H_{c2}$ , where the critical current  $I_c$  anomalously increases with field [3, 7–9]. In high- $T_c$  superconductors (HTS) similar noise enhancement was found in the

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vicinity of the melting or order-disorder transitions [10, 11, 14, 15]. This low-frequency excess noise is apparently inconsistent with the common flux-flow noise mechanisms due to its unusually high amplitude, strong field and current dependence, and non-Gaussian character [7–10].

In this paper we demonstrate that the excess noise does not result from any previously known flux-flow noise mechanism, but rather is due to a conceptually different phenomenon of random creation and annihilation of a metastable phase. The conventional models consider various types of irregularities and defects in the vortex lattice which are perturbations within an otherwise single thermodynamic phase of the vortex matter. In contrast, it has recently been suggested that the PE in low- $T_c$  superconductors, as well as the second magnetization peak in HTS, reflect a first-order phase transition between two distinct thermodynamic phases of the vortex matter [16–19]: an ordered phase (OP, or Bragg glass [20]), and an amorphous disordered phase (DP). The DP is the equilibrium thermodynamic phase above the peak field of the PE, whereas the OP is the equilibrium phase below the peak field. However, the DP can be also present as a “wrong” *metastable* supercooled phase below the PE instead of the equilibrium OP [17, 21, 22]. As shown below the excess noise results from the *dynamic co-existence of these two distinct phases of the vortex matter*. In the lower part of the PE the random penetration of vortices through the sample edges does not just create some defects in the OP, but rather generates a distinct, albeit metastable, phase. The random generation and annihilation of this strongly pinned metastable DP is shown to be the cause of large fluctuations in the instantaneous  $I_c$  of the sample, leading to greatly enhanced voltage noise. We demonstrate accordingly that the excess noise can be *eliminated* by preventing the formation of the “wrong” phase in the Corbino disk geometry. In addition, the excess noise, which was previously observed only in the vicinity of the PE where  $I_c$  increases with field  $H$ , is found also at low fields, where  $I_c$  *decreases* with  $H$ .

To investigate the origin of the excess noise we have studied 2H-NbSe<sub>2</sub> crystals using a special contact configuration (fig. 1b, inset) that enables measurements in both the Corbino and strip-like geometry in the same crystal [16]. By applying the current to the +S, −S contacts, the vortices penetrate through the edge and flow across the sample, similarly to the standard configuration. In contrast, by applying the current to the +C, −C contacts, the vortices circulate in the bulk without crossing the edges, as in a Corbino disk. In both configurations the voltage and the corresponding noise are measured across the same contacts +V, −V. The distance between the voltage contacts is 0.15 mm and the diameter of the Corbino is 1.1 mm. The data presented here were obtained on a Fe-doped (200 ppm) NbSe<sub>2</sub> single crystal  $2.2 \times 1.5 \times 0.04 \text{ mm}^3$  with  $T_c = 5.7 \text{ K}$  [16]. Similar results were obtained on a number of additional crystals.

The inset of fig. 1a shows the voltage response  $V$  *vs.* the field  $H \parallel c$ -axis at 4.4 K in the Corbino and strip configurations. The applied current  $I = 26 \text{ mA}$  in the strip configuration and  $34 \text{ mA}$  in the Corbino. Since the current density varies across the sample, this difference in  $I$  is chosen to give the same average current density between the +V, −V contacts in the two geometries. As a result, the measured  $V$  and the corresponding vortex velocity are identical at high fields. Upon decreasing the field from above  $H_{c2}(T)$ , the voltage decreases rapidly and vanishes in the PE region (4 to 8 kOe, fig. 1a inset) where  $I_c$  of the sample is large due to the presence of the strongly pinned DP. The voltage becomes finite at intermediate fields before vanishing again at  $H \lesssim 1.2 \text{ kOe}$ . We shall concentrate on the intermediate fields where the excess noise appears and where significantly a different response in the two geometries is observed.

In the Corbino,  $V$  increases linearly with  $H$  (fig. 1a) in accord with the well-known flux-flow behavior in undoped NbSe<sub>2</sub> [23], indicating that the lattice is in the OP. The voltage vanishes abruptly at both high and low fields. The high-field point  $H_{DT}^H$  is the sharp disorder-driven

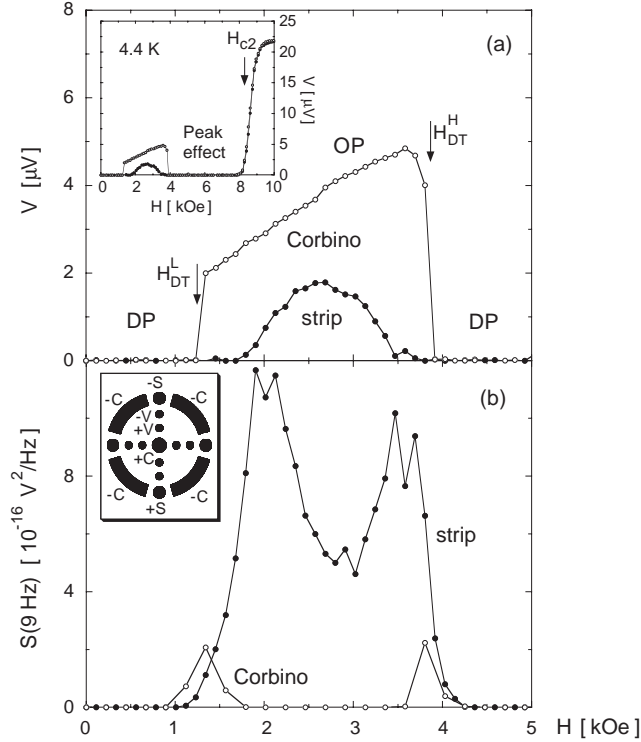


Fig. 1 – (a) The voltage response  $V$  vs. magnetic field at 4.4 K measured in the Corbino ( $\circ$ ) and strip ( $\bullet$ ) geometries. Inset: same data over a wider field range extending above  $H_{c2}$ . (b) The corresponding Corbino ( $\circ$ ) and strip ( $\bullet$ ) noise power at 9 Hz showing the absence of the excess noise in the Corbino at intermediate fields. Inset: the electrode configuration allowing measurements in both the Corbino and strip configurations.

transition at the PE between the equilibrium OP and the equilibrium DP, whereas the low-field point  $H_{DT}^L$  is the re-entrant disorder-driven transition [16,24]. The strip configuration displays qualitatively different behavior (fig. 1a). In contrast to Corbino, no sharp transitions are observed and the voltage remains vanishingly small below  $H_{DT}^H$  as well as above  $H_{DT}^L$ . At intermediate fields  $V$  increases, but it is still significantly suppressed relative to the Corbino response.

Figure 1b shows the low-frequency noise in the two geometries. The large noise in the strip was previously observed only in the lower part of the PE [7–9]. Here we find, for the first time, two peaks in the noise. The noise of the strip is maximal where voltage response is strongly suppressed, while the noise is reduced in the central region, where  $V$  of the strip recovers. The most important observation in fig. 1b, however, is that the large excess noise that is found in the usual strip configurations is absent in the Corbino geometry, and only two small peaks remain at  $H_{DT}^H$  and  $H_{DT}^L$ . This is a first demonstration that the excess noise depends dramatically on sample geometry, and therefore does not reflect an *intrinsic* bulk property of the vortex system.

As shown previously, the difference in the dc voltage response of the two geometries is a result of the injection of a metastable DP [16,21]. The DP, which is the *equilibrium* phase above  $H_{DT}^H$ , can exist as a *metastable* phase below  $H_{DT}^H$  [16,17,21,22]. In the presence of a driving current, vortices that penetrate through the rough edges locally destroy the equilibrium OP, and form instead the *metastable* DP near the edges. As the entire lattice flows deeper into the sample, this inadvertently injected “wrong” phase anneals into the OP

over a characteristic relaxation length  $L_r(H, I)$ . The value of  $L_r$  is the key parameter that determines the voltage response and the noise of the system. Since the DP has a significantly larger critical current density  $J_c^{\text{dis}}$  than  $J_c^{\text{ord}}$  of the OP, the contamination by the metastable DP enhances the integrated  $I_c$  of the strip:  $I_c = d \int_0^W J_c(x) dx = WdJ_c^{\text{ord}} + L_r d(J_c^{\text{dis}} - J_c^{\text{ord}})(1 - \exp[-W/L_r])$ . Here  $d$  and  $W$  are the thickness and width of the sample and we have assumed, for simplicity, an exponential relaxation of  $J_c(x) = J_c^{\text{ord}} + (J_c^{\text{dis}} - J_c^{\text{ord}}) \exp[-x/L_r]$ . Since usually experimentally  $L_r < W$  [21], we can approximate  $I_c \simeq L_r d(J_c^{\text{dis}} - J_c^{\text{ord}}) + WdJ_c^{\text{ord}}$ . Importantly,  $L_r(H, I)$  depends strongly on field as well as on current. Close to the order-disorder transition fields  $H_{\text{DT}}$  the free energies of the DP and OP are comparable; hence the metastable DP has a long lifetime and  $L_r$  is large [21]. This results in a large  $I_c$  and consequently in almost vanishing  $V$  in the strip at fields slightly below  $H_{\text{DT}}^{\text{H}}$  (fig. 1a). A mirror-image-like behavior is observed in the vicinity of  $H_{\text{DT}}^{\text{L}}$ , where  $L_r$  increases again upon approaching the re-entrant transition from above. At intermediate fields, away from the transitions, the metastable phase becomes less favorable energetically and therefore  $L_r$  shortens and  $V$  is enhanced.

We now illustrate that the above edge contamination mechanism, which accounts for both the dc and the ac response of the system [21], also provides a compelling explanation of the excess noise. The generation of the metastable DP results from the non-uniform vortex penetration through the surface barriers at the sample edge. The generation mechanism and the subsequent annealing of the DP in the bulk constitute random processes both in time and in space, resulting in complex spatiotemporal fluctuations [25] of the vortex-lattice disorder and of the corresponding value of  $J_c(\mathbf{r}, t)$ . Two mechanisms actually contribute to the spatiotemporal variations: fluctuations of  $J_c^{\text{dis}}$  at the sample edge,  $\delta J_c^{\text{dis}}$ , and local fluctuations in the relaxation length  $\delta L_r$  within the bulk of the sample. Here we discuss for simplicity only  $\delta J_c^{\text{dis}}$  fluctuations, although analogous conclusions are obtained on considering  $\delta L_r$  fluctuations. In order to simplify the analysis, we shall consider a one-dimensional  $J_c(x, t)$  problem, which effectively averages out the sample properties along the current direction. Since  $J_c^{\text{dis}}$  is typically an order of magnitude larger than  $J_c^{\text{ord}}$  [16, 22],  $\delta J_c^{\text{dis}}$  fluctuations will result in large variations in  $I_c$ , causing large voltage noise. The detailed microscopic properties of  $\delta J_c^{\text{dis}}$  fluctuations are presently unknown. However, from the above derivation of  $I_c$ , the fluctuation in the total critical current caused by a given  $\delta J_c^{\text{dis}}$  is  $\delta I_c \simeq L_r(H, I) \delta J_c^{\text{dis}} d$ , which shows that  $I_c$  fluctuations strongly depend on the value of  $L_r(H, I)$ .

The low-frequency voltage noise  $\delta V$  can be evaluated as follows. The dc voltage  $V$ , which appears at  $I > I_c$ , can be expressed as  $V = f(I - I_c)$ , where  $f$  is a general function describing the dc  $V$ - $I$  characteristics. Therefore,  $\delta V = (df/dI_c) \delta I_c = -(df/dI) \delta I_c = -(dV/dI) \delta I_c = -L_r(dV/dI) \delta J_c^{\text{dis}} d$ . Consequently, as described below, the commonly observed large excess noise in the lower part of the PE, seen at about 3.5 kOe in fig. 1b in the strip, is a result of the large  $L_r$  in the vicinity of  $H_{\text{DT}}^{\text{H}}$ . The novel observation in fig. 1b, however, is the existence of a second noise peak at about 2 kOe where  $L_r(H, I)$  becomes large again on approaching  $H_{\text{DT}}^{\text{L}}$ . This low-field peak was not previously observed since the noise studies were carried out on undoped NbSe<sub>2</sub> [7–9] which does not show a re-entrant disorder-driven transition. Our finding of the two peaks is an important manifestation of the proposed mechanism: It demonstrates that the excess noise is not a mere result of the fact that  $I_c$  increases with  $H$  at the PE, since at low fields the same excess noise is found in the region where  $I_c$  decreases with  $H$ . Furthermore, the same value of  $I_c$  is attained at three values of  $H$ : above the re-entrant  $H_{\text{DT}}^{\text{L}}$  where  $I_c$  decreases with  $H$ , below  $H_{\text{DT}}^{\text{H}}$  where  $I_c$  increases with  $H$ , and above  $H_{\text{DT}}^{\text{H}}$  where  $I_c$  decreases again in the upper part of the PE. The excess noise occurs only in the first two cases, where the metastable DP contaminates the equilibrium OP. In the third case, above

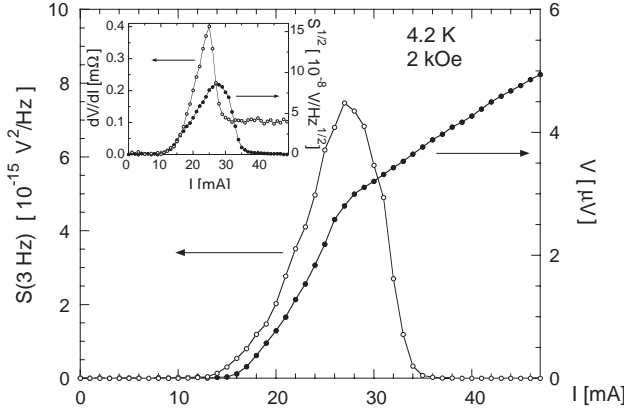


Fig. 2 – The  $V$ - $I$  characteristic ( $\bullet$ ) and the power spectral density of the noise  $S$  at 3 Hz ( $\circ$ ) in the strip configuration. Inset:  $dV/dI$  along with the noise intensity  $\sqrt{S}$ . The noise data were obtained by averaging 1000 spectra, whereas the  $V$ - $I$  data are not time averaged.

$H_{\text{DT}}^{\text{H}}$ , the DP is the thermodynamically stable phase and therefore no metastable phase is generated at the edges.

In order to test the validity of the described concept, we have performed noise measurements in the Corbino geometry. Strikingly, we find that the excess noise is entirely absent in Corbino in the central-field region, as shown in fig. 1b. This means that *the motion of vortex lattice within the bulk of the sample does not, by itself, create excess noise*. Any conventional bulk noise mechanism should have resulted in a similar noise level in the Corbino and strip geometries. One may even argue that some of the noise mechanisms, such as a bulk plastic vortex flow, could cause a larger noise level in the Corbino due to the enhanced vortex shear by the  $1/r$  radial current distribution, contrary to the observations. The absence of the noise in the Corbino therefore clearly indicates the dominant role of the edge contamination in the noise process. The residual small and narrow peaks in the Corbino noise in fig. 1b can be ascribed to small deviations from a perfect Corbino disk configuration. Since  $L_r$  diverges at  $H_{\text{DT}}^{\text{H}}$  and  $H_{\text{DT}}^{\text{L}}$ , any small non-radial part of the current or inhomogeneities may result in some injection of the metastable DP, giving rise to noise. In the vicinity of the mean-field  $H_{\text{DT}}$  non-uniform disorder distribution may result in some parts of the sample being in the equilibrium DP, whereas others in the OP [17], similar to the solid-liquid coexistence at melting [26]. Consequently, when the lattice is set in motion the DP drifts into regions of the OP, where it becomes metastable, and may cause noise in this narrow field region.

We now analyze the current dependence of the excess noise in the strip. Figure 2 shows the low-frequency spectral density of the noise  $S(3\text{ Hz}) \propto \delta V^2$  as a function of  $I$ , along with the  $V$ - $I$  characteristic. The  $V$ - $I$  characteristic displays a rapid upturn and approaches the linear behavior with a constant  $dV/dI$  at elevated currents.  $S$ , in contrast, displays a large peak and vanishes rapidly at higher currents. According to our simplified analysis, the voltage noise  $\sqrt{S} \propto \delta V = \delta J_c^{\text{dis}} L_r(H, I) \frac{dV}{dI} d$  is a product of three separate terms.  $\delta J_c^{\text{dis}}$  describes the statistical process of the generation of the DP at the rough sample edges. There is currently no theoretical description of this random process which should generally be field and current dependent [27]. There is also no theoretical description of the relaxation process; however, it is known experimentally that  $L_r(H, I)$  decreases rapidly with current [21, 22]. When the lattice is displaced very slowly, the lifetime of the metastable DP is long and  $L_r$  is large. However, as the pinning potential is tilted stronger by a larger driving force, the annealing process becomes progressively faster [22] resulting in a rapid decrease of  $L_r$  with vortex velocity. Finally, even

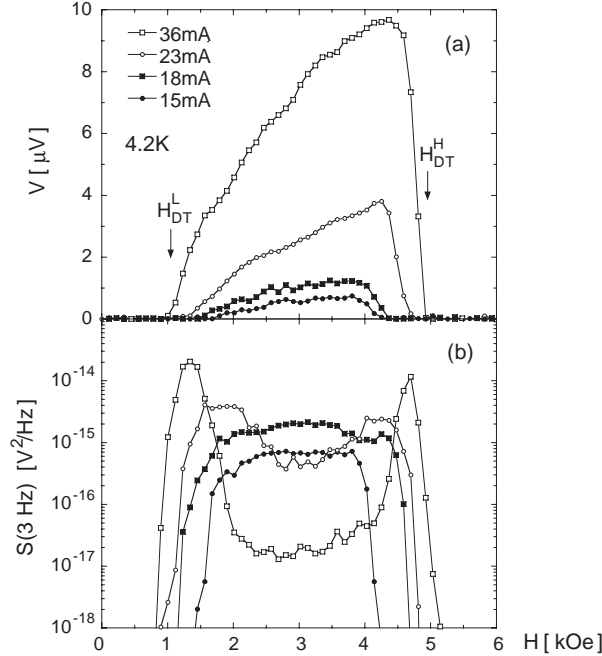


Fig. 3 – (a) Voltage response *vs.* field at 4.2K in the strip configuration for  $I = 36$  mA ( $\square$ ), 23 mA ( $\circ$ ), 18 mA ( $\blacksquare$ ) and 15 mA ( $\bullet$ ). (b) The corresponding noise power density at 3 Hz.

the third term  $dV/dI$  is not well-defined experimentally. Due to the metastable nature of the system  $dV/dI$  has large fluctuations and strong dependence on measurement frequency as well as on current ramp direction [23]. These uncertainties in all three terms prevent the quantitative analysis of the noise amplitude at this stage. Nevertheless, we can understand the general form of the noise knowing the qualitative behavior of  $L_r(H, I)$  and  $dV/dI$ . The inset of fig. 2 shows a typical  $dV/dI$  along with the noise intensity  $\sqrt{S}$ .  $dV/dI$  shows a pronounced peak and reaches a constant value above about 30 mA. At low currents  $L_r(H, I)$  is large and hence the noise initially follows  $dV/dI$ . Above about 15 mA, however, the increase of the noise is moderated due to the gradual decrease of  $L_r$ . At still higher current the noise starts to drop rapidly because of the rapid decrease of  $L_r(H, I)$  with current. Above 28 mA,  $dV/dI$  approaches a constant value of the flux-flow resistance of the OP, indicating that  $L_r$  is small and that most of the sample volume is in the OP. Accordingly, noise decreases rapidly in this region with decreasing  $L_r(H, I)$ . Since the ordered part of the sample does not contribute to the noise, the noise level vanishes as the width of the DP near the edge diminishes.

From the above considerations we can analyze the general noise behavior of the strip, presented in fig. 3. At the lowest current,  $I = 15$  mA, vortex motion occurs only in the central field region (fig. 3a), since closer to  $H_{DT}^H$  and  $H_{DT}^L$  the integrated  $I_c$  of the strip is larger than 15 mA due to the large  $L_r$ . As a result, the excess noise in fig. 3b is present only in the central part. At 18 mA, the field range of the observable vortex motion and noise expands, and around 23 mA, two noise peaks become apparent. In the central-field region the DP is less stable,  $L_r(H, I)$  drops with  $I$ , and hence the noise decreases rapidly with the current. Closer to the transition fields, however, the metastable DP is much more stable so that  $L_r$  remains large and  $S$  still increases with current. At 36 mA most of the sample is in the OP and the noise has accordingly dropped by two orders of magnitude. The strong excess noise is restricted now only to the narrow regions near  $H_{DT}$  fields where the metastable DP survives even at high vortex velocities.

In summary, the comparative study of excess noise generation in Corbino and strip configurations shows that the flow of the vortex lattice in Corbino does not generate an excess voltage noise. In contrast, very strong noise enhancement is found in the same samples measured in a strip-like geometry. The excess noise is found on the Bragg glass side of the disorder-driven transition both along the high-field and the re-entrant transition lines, and results from random generation of a metastable disordered vortex phase at the sample edges and its subsequent dynamic annealing in the bulk.

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#### REFERENCES

- [1] BHATTACHARYA S. *et al.*, *Phys. Rev. Lett.*, **54** (1985) 2453.
- [2] LI Y. P. *et al.*, *Phys. Rev. Lett.*, **67** (1991) 1630.
- [3] CLEM J. R., *Phys. Rep.*, **75** (1981) 1.
- [4] YEH W. J. and KAO Y. H., *Phys. Rev. B*, **44** (1991) 360; ASHKENAZY V. D., JUNG G. and SHAPIRO B. YA., *Physica C*, **254** (1995) 77; GRAY K. E., *Phys. Rev. B*, **57** (1998) 5524; TSUBOI T., HANAGURI T. and MAEDA A., *Phys. Rev. Lett.*, **80** (1998) 4550; TOGAWA Y. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 3716.
- [5] BLACAIS B., MATHIEU P. and SIMON Y., *Phys. Rev. Lett.*, **70** (1993) 1521; *Phys. Rev. B*, **49** (1994) 15813.
- [6] WOLTGENS P. J. M. *et al.*, *Physica C*, **247** (1995) 67.
- [7] MARLEY A. C., HIGGINS M. J. and BHATTACHARYA S., *Phys. Rev. Lett.*, **74** (1995) 3029.
- [8] MERITHEW R. D. *et al.*, *Phys. Rev. Lett.*, **77** (1996) 3197.
- [9] RABIN M. W. *et al.*, *Phys. Rev. B*, **57** (1998) R720.
- [10] D'ANNA G. *et al.*, *Phys. Rev. Lett.*, **75** (1995) 3521.
- [11] SAFAR H. *et al.*, *Phys. Rev. B*, **52** (1995) 6211.
- [12] OKUMA S. and KOKUBO N., *Phys. Rev. B*, **61** (2000) 671.
- [13] ARANSON I. and VINOKUR V., *Phys. Rev. Lett.*, **77** (1996) 3208; OLSON C. J., REICHHARDT C. and NORI F., *Phys. Rev. Lett.*, **80** (1998) 2197; KOLTON A. B. *et al.*, *Phys. Rev. Lett.*, **83** (1999) 3061; MARCHETTI M. C. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 1104.
- [14] KWOK W. K. *et al.*, *Physica C*, **293** (1997) 111.
- [15] GORDEEV S. N. *et al.*, *Nature*, **385** (1997) 324.
- [16] PALTIEL Y. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 3712.
- [17] MARCHEVSKY M., HIGGINS M. J. and BHATTACHARYA S., *Nature*, **409** (2001) 591.
- [18] AVRAHAM N. *et al.*, *Nature*, **411** (2001) 451.
- [19] KIERFELD J. and VINOKUR V., *Phys. Rev. B*, **61** (2000) R14928.
- [20] GIAMARCHI T. and LE DOUSSAL P., *Phys. Rev. Lett.*, **72** (1994) 1530; NATTERMANN T. and SCHEIDL S., *Adv. Phys.*, **49** (2000) 607.
- [21] PALTIEL Y. *et al.*, *Nature*, **403** (2000) 398.
- [22] HENDERSON W. *et al.*, *Phys. Rev. Lett.*, **77** (1996) 2077.
- [23] HIGGINS M. J. and BHATTACHARYA S., *Physica C*, **257** (1996) 232.
- [24] GHOSH K. *et al.*, *Phys. Rev. Lett.*, **76** (1996) 4600; BANERJEE S. S. *et al.*, *Europhys. Lett.*, **44** (1998) 91.
- [25] MASLOV S., PACZUSKI M. and BAK P., *Phys. Rev. Lett.*, **73** (1994) 2162.
- [26] SOIBEL A. *et al.*, *Nature*, **406** (2000) 282.
- [27]  $\delta J_c^{\text{dis}}$  and the resulting  $\delta V$  also have a nontrivial  $1/f$ -like spectral form which will be addressed in detail elsewhere.