Nuclear Structure Theory for the Rapid Proton Capture Process (A=20-36)
NUCLEAR UNCERTAINTIES IN THE NeNa-MgAl CYCLES AND PRODUCTION OF $^{22}$Na AND $^{26}$Al DURING NOVA OUTBURSTS

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$^{25}\text{Al}(p,\gamma)^{26}\text{Si}$
\[ ^{25}\text{Al}(p,\gamma)^{26}\text{Si} \]

\[ ^{25}\text{Al} \rightarrow ^{26}\text{Si} \]

\[ Q = 5.512 \text{ MeV} \]
\[ {^{25}\text{Al}}(p,\gamma){^{26}\text{Si}} N_A \langle \sigma v \rangle \text{ ratio:} \]

This work / Wiescher et al. 1986

\[ {^{25}\text{Al}}(p,\gamma){^{26}\text{Si}} \]

A. Coc et al, 1995
Charged-particle thermonuclear reaction rates: III. Nuclear physics input

C. Iliadis\(^{a,b,*}\), R. Longland\(^{a,b}\), A.E. Champagne\(^{a,b}\), A. Coc\(^{c}\)
$^{25}\text{Al}(p,\gamma)^{26}\text{Si}$

$^{25}\text{Al} \rightarrow ^{26}\text{Si}$

$Q = 5.512 \text{ MeV}$
\[ N_A < \sigma v >_{res} = 1.540 \times 10^{11} (\mu T_9)^{-3/2} \]
\[ \times \sum_f \omega \gamma_{if} \ e^{-E_{res}/(kT)} \ \text{cm}^3 \text{s}^{-1} \text{mole}^{-1} \]

\[ T_9 \] temperature in GigaK,
\[ E_{res} = E_f - E_i \] resonance energy in the center of mass system.
Resonance strengths (in MeV) for proton capture are

\[
\omega \gamma_{if} = \frac{(2J_f + 1)}{2(2J_i + 1)} \frac{\Gamma_{pi f} \Gamma_{\gamma f}}{\Gamma_{pi f} + \Gamma_{\gamma f}}
\quad (4)
\]

\( J_i \) target spin, \( J_f \) final nucleus spin
\( \Gamma_p \) is the proton decay width
\( \Gamma_\gamma \) is the gamma decay width
\[ \Gamma_p \ll \Gamma_\gamma \text{ (smallest } E_{res}) \]

\[ \frac{\Gamma_p \Gamma_\gamma}{\Gamma_p + \Gamma_\gamma} \rightarrow \Gamma_p \quad (5) \]

\[ \Gamma_p \gg \Gamma_\gamma \text{ (usual for higher } E_{res}) \]

\[ \frac{\Gamma_p \Gamma_\gamma}{\Gamma_p + \Gamma_\gamma} \rightarrow \Gamma_\gamma \quad (6) \]
We need

1) $J_f$

2) $E_{res}$ (Q value and excitation energy)

3) $C^2 S$ (spectroscopic factors to get $\Gamma_p = C^2 S \Gamma_{sp}$)

4) $\Gamma_\gamma$ ($\gamma$ decay width or $\gamma$ decay lifetime, $\Gamma_\gamma \tau = \hbar$)
From theory we get (with uncertainties)

1) $J_f$

2) $E_{res}$ (150 keV down to 50 keV sometimes)

3) $C^2 S$ (on the order of 30% uncertainties sometimes)

4) $\Gamma_\gamma$ (on the order of 30% sometimes)
From experiment we get

0) \(\omega \gamma\) (directly in a few cases)

1) \(J_f\) (sometimes)

2) \(E_{res}\) (to 5 keV in many cases)

3) \(C^2 S\) (usually not)

4) \(\Gamma_\gamma\) (sometimes from the mirror decay)
• Theory based on full mixing of configurations within the sd \((0d_{5/2}, 0d_{3/2}, 1s_{1/2})\) set of orbitals
• Hamiltonian - for almost 30 years based upon the USD (Wildenthal-Brown) Hamiltonian
• 2005 – New Hamiltonians USDA and USDB based on an updated set of experimental constraints
• Provides first estimate of theoretical errors
• I will show you some of our theoretical apparatus

• A) Determination of the Hamiltonian
• B) Comparison to all observables
• C) Results for \(^{26}\text{Mg}\)
• D) Results for \(^{25}\text{Al}(p,\gamma)^{26}\text{Si}\)
\[ \hat{H} = \sum_{\alpha\beta} <\alpha | U | \beta> a^+_{\alpha} a_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} <\alpha\beta | \tilde{G} | \gamma\delta> a^+_{\alpha} a^+_{\beta} a_{\delta} a_{\gamma} \]

- Start with one of the doubly-magic nuclei as the vacuum
- Exact solution of \( H \) within the model space for the orbits inside the shell gaps
- 3 Single-particle energies \( U \) from experiment
- Two-body matrix elements (G) from NN interaction renormalized to the model space
\[ \hat{H} = \sum_{\alpha\beta} <\alpha | U | \beta> a^+_\alpha a_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} <\alpha\beta | \tilde{G} | \gamma\delta> a^+_\alpha a^+_\beta a_\delta a_\gamma \]

- Start with one of the doubly-magic nuclei as the vacuum
- Exact solution of \( H \) within the model space for the orbits inside the shell gaps
- Single-particle energies \( U \) from experiment
- 63 Two-body matrix elements (\( G \)) adjusted to obtain a “best fit” to known data
608 levels in 77 nuclei, 137 keV rms

Single-value decomposition fit from a diagonalization of the error matrix

USDA 30  USDB 56

rms for the 608 levels

170 keV USDA
145 keV USDB
USDA ground state energy differences

theory under bound
“island of inversion due to pf shell intruded
oxygen beyond N=16 all unbound

A5 Eilat April 3-8 2011
USDA 170 keV rms for 608 levels
290 keV rms for tbme (4.1% of largest)
A tour of the sd shell on the web

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$^{26}\text{Al}$

The diagram shows the energy levels (Ex in MeV) for $^{26}\text{Al}$, with experimental and theoretical values. The labels indicate specific levels, such as 67, 4, 3, and 2, along with energy values like -105.76 MeV and -105.74 MeV.
Observables – all magnetic moments

$g_p^s = 5.127$  $g_n^s = -3.543$

$g_p^l = 1.147$  $g_n^l = -0.090$
Observables – all quadrupole moments

Q USDB

experiment vs. theory-fit
Observables – all M1 gamma decay

\[ g_p^s = 5.127 \quad g_n^s = -3.543 \]
\[ g_p^l = 1.147 \quad g_n^l = -0.090 \]
Observables – all E2 gamma decay

M(E2) USDB

experiment

theory-fit

2011
Spectroscopic factors

Experiment vs theory

Theory (USDA) vs Theory (USDB)

Theory (USD) vs Theory (USDB)
$^{26}$Mg Comparison of excitation energies

![Graph showing excitation energies for $^{26}$Mg with experimental (exp) and theoretical (theory) data points. The graph plots $E_x$ (MeV) against J. Notable levels marked include $1^+$ and $3^+$.]
$^{26}\text{Mg}$ Comparison of level lifetimes
Experiment divided by theory

![Graph showing comparison of level lifetimes for USDB and USDA](image)
\[ ^{25}\text{Al}(p,\gamma)^{26}\text{Si} \]

\[ ^{25}\text{Al} \rightarrow ^{26}\text{Si} \]

\[ Q = 5.512 \text{ MeV} \]

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Diagram showing the reaction \( ^{25}\text{Al}(p,\gamma)^{26}\text{Si} \) with contributions for states with angular momenta \( 1^+ \) and \( 3^+ \). The horizontal axis represents \( \log_{10}(T9) \) and the vertical axis shows the \( \log_{10}(\text{rate}) \) and contribution (%) scales.
Energies in proton-rich nuclei based on those observed in neutron-rich nuclei together with theoretical models for the $b$ or $c$ coefficients of the IMME

Isobaric mass multiplet equation

\[ B(T_z) = a + bT_z + cT_z^2, \]  

where $B$ is the binding energy of a state.

\[ B({}^{26}\text{Si}) = 2B({}^{26}\text{Al}) - B({}^{26}\text{Mg}) + 2c. \]

Theory can give $c$ to within an rms of about 25 keV. So levels in $^{26}\text{Si}$ can be predicted to within 50 keV if the energies of the isobars in $^{26}\text{Mg}$ and $^{26}\text{Al}$ are known.

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\[ \text{Novae.} \]

\[ \log_{10}(\text{T9}) \]

\[ \text{log}_{10}(\text{rate}) \]

\[ \text{contribution (\%)} \]

\[ ^{25}\text{Al(p,}\gamma)^{26}\text{Si} \]

\[ ^{25}\text{Al} \rightarrow ^{26}\text{Si} \]

\[ Q = 5.512 \text{ MeV} \]
Change due to 5 keV increase in Q
At this level the value must come from experiment
$1^+ \text{ state} - \text{rate determined by } \Gamma_p$

$$\Gamma_p = C^2 S \Gamma_{sp}$$

$$C^2 S = 0.0047 \text{ (USDB), } 0.0027 \text{ (USDA), } 0.0035 \text{ (USD)}$$

It is difficult to get reliable spectroscopic factors for such small values from transfer reactions.

The best place to test theory is in comparison to measured proton decay widths in proton-rich nuclei.
3+ state – rate determined by $\Gamma_\gamma$

Some groups use the value from measured from an old DSAM lifetime measurement in $^{26}\text{Mg}$

$\Gamma_\gamma = 120 \text{ meV from theory}$

$\Gamma_\gamma = 33^{+24}_{-10} \text{ meV from } ^{26}\text{Mg}$
Rate (theoretical $3^+$ lifetime in $^{26}$Si)
Rate (experimental $3^+$ lifetime in $^{26}$Mg)
Is this factor of three difference important for astrophysics?

Results obtained last week by Jordi Jose

Nova model (1.25 Msun O-Ne White-Dwarfs, accreting mass at a rate $2 \times 10^{-10}$ Msun yr$^{-1}$)

For the relative production of isomeric state in $^{26}\text{Al}$ there is just a few percent difference.
Rate (theoretical $3^+$ lifetime in $^{26}$Si)
Rate (experimental $3^+$ lifetime in $^{26}$Mg)
Experiment needed to get energy of states in $^{33}\text{Ar}$ to 5 keV accuracy. Theory needed to get proton decay widths to ground and excited states of $^{32}\text{Cl}$ and gamma widths for $^{33}\text{Ar}$

$^{32}\text{Cl}(p,\gamma)^{33}\text{Ar}$

Stellar enhancement factor due to low-lying state in the parent

![Graph showing the log(ratio) vs. log(T9) relationship for $^{32}\text{Cl} \rightarrow ^{33}\text{Ar}$ reaction with $Q = 3.343$ MeV, $J_g = 1$, $J_e = 2$, and $E = 0.089$ MeV.](image-url)
The Future

- We need to do all A=20-36 cases with USD, USDA and USDB to get theoretical input with errors.
- We need to test small spectroscopic factors from comparison to experiments in proton-rich nuclei.
- Sd shell model is useful up to about 8 MeV in excitation. Above this use statistical models. If a specific level is important it may be hopeless.
- Pf shell is slowly reaching the level of precision obtained for the sd-shell.
- But for going to up A=100, g9/2 and other orbitals must be included - this will take time – 10 years.
collaborations

Werner Richter
iThemba, South Africa

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