Insulating Phases of a BEC in Novel Optical Lattices

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Ultracold atoms in optical lattices

Adding disorder

Noise interferometry

“Ideal” BEC
Why disorder?
- Disorder is a key ingredient of the microscopic (and macroscopic) world
- Fundamental element for the physics of conduction
- Superfluid-insulator transition in condensed-matter systems

Why cold atoms?
- Ultracold atoms are a versatile tool to study disorder-related phenomena
- Precise control on the kind and amount of disorder in the system
- Quantum simulation

Localization effects
- Bose glasses, spin glasses (strongly interacting systems)
- Anderson localization (weakly interacting systems)
At zero temperature the state of the system is determined by the competition between two energy scales: the hopping energy $J$ and the on-site interaction energy $U$. 

$$\hat{H} = -J \sum_{\langle i, j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$
At zero temperature the state of the system is determined by the competition between two energy scales: the hopping energy $J$ and the on-site interaction energy $U$.

### Bose-Hubbard model for interacting bosons in a lattice:

- **SUPERFLUID** ($J \gg U$):
  - Long-range phase coherence
  - High number fluctuation
  - Gapless excitation spectrum
  - Compressible

- **MOTT INSULATOR** ($J \ll U$):
  - No phase coherence
  - Zero number fluctuation (Fock states)
  - Gap in the excitation spectrum
  - Not compressible
Experimental scheme

strongly interacting regime:

3D optical lattice

Mott insulator phase first realized in
Superfluid to Mott Insulator transition at LENS


momentum distribution of the atomic sample after expansion

test of phase coherence

Increasing the lattice height $s$ increases $U/J$.
Measuring the excitation spectrum

Mott Insulator spectrum

rms width [px]

modulation frequency [kHz]

U

2U
Measuring the excitation spectrum
Ultrasound atoms in optical lattices

Adding disorder

Noise interferometry

“Ideal” BEC
The random potential is produced by shining an off-resonant laser beam onto a diffusive plate and imaging the resulting speckle pattern on the BEC.

\[ V(x, y) = \frac{3\pi c^2}{2\omega_0^3} \frac{\Gamma}{\Delta} I(x, y) \]  

optical dipole potential

stationary in time randomly varying in space
BEC expansion in a disordered waveguide

*In-situ* images of the BEC expanding in the disordered waveguide:

Experiments also in Orsay (Aspect)
issue #1: producing “dense” disorder

issue #2: role of interactions!
The bichromatic lattice

\[ V(x) = s_1 E_{R1} \cos^2(k_1 x) + s_2 E_{R2} \cos^2(k_2 x) \]

\[ \lambda = 830 \text{ nm} \]

\[ \lambda = 1076 \text{ nm} \]

\[ \lambda = 830 \text{ nm} + \lambda = 1076 \text{ nm} \]
The bichromatic lattice

Energy minima of the lattice potential along $y$ direction

Non-periodic modulation of the energy minima with length scale

$$d = \left( \frac{2}{\lambda_1} - \frac{2}{\lambda_2} \right)^{-1} = 1.8 \ \mu m = 4.3 \ sites$$
Localization in a quasi-periodic lattice

\[
\begin{pmatrix}
\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + s_1 E_{R_1} \cos^2(k_1 x) + s_2 E_{R_2} \cos^2(k_2 x)
\end{pmatrix} \psi(x) = E \psi(x)
\]

\[\lambda_1 = 830 \text{ nm} \quad s_1 = 10 \quad / \quad \lambda_2 = 1076 \text{ nm}\]

square modulus of the ground state wavefunction

localized states

extended states

localization transition in 1D (Aubry-André)
Localization in a quasi-periodic lattice + harmonic trap (no interactions)

\[ s_1 = 10 \quad s_2 = 0.1 \]

\[ s_2 = 0.25 \]

\[ s_2 = 1 \]

Localization in a quasi-periodic lattice + harmonic trap adding interactions

Increasing interactions the “size” of the ground state progressively increases going from a localized state to an extended state...characterization through transport properties at low atom number
Colloquium on Group Theoretical Methods in Physics
Kiriat Anavim (Israel) 25-30 Mars 1979

Analyticity Breaking and Anderson Localization in Incommensurate Lattices

Thanks to Nir Davidson
Disordered systems: Role of interactions

- Anderson insulator
- Superfluid
- Bose-Glass insulator
- Mott insulator

- noninteracting
- weakly interacting ($U<J$)
- strongly interacting ($U>J$)
- interactions
Bose-Hubbard model with bounded disorder in the external potential $\epsilon_j \in [-\Delta/2, \Delta/2]$

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i \epsilon_j \hat{n}_j$$

In the presence of disorder an additional energy scale $\Delta$ enters the description of the system. The interplay between these energy terms may induce new quantum phase transitions.

- hopping energy $J$
- interaction energy $U$
- disorder $\Delta$
Qualitative phase-diagram for a system of interacting bosons in a disordered lattice
from M. P. A. Fisher et al., PRB 40, 546 (1989)

for ultracold atoms see B. Damski et al., PRL 91, 080403 (2003); R. Roth et al., PRA 68, 023604 (2003).
Phase diagrams

**BOSE-GLASS**
(T. Giamarchi and H. J. Schulz, PRB 37, 325 (1988))

- No long-range phase coherence
- **Gapless excitation spectrum**
- Finite compressibility

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for ultracold atoms see B. Damski et al., PRL 91, 080403 (2003); R. Roth et al., PRA 68, 023604 (2003).
Starting from a Mott Insulator and adding disorder, the energy required for the hopping of a boson from a site to a neighboring one becomes a function of position. When $\Delta_j = U$ the excitation energy goes to zero and the gap disappears.
Experimental geometry

1D atomic systems + 1D disorder

\[ \lambda_1 = 830 \text{ nm} \quad s_1 = 40 \]

\[ \lambda_2 = 1076 \text{ nm} \quad s_2 < 3 \]
Excitation spectrum for $s_1=16$ and increasing disorder strength from $s_2=0$ to $s_2=2.5$:

\[\Delta = 0 \quad \Delta < U \quad \Delta > U\]
MI spectral broadening

Excitation maximum at $U$
as a function of disorder strength:

Good agreement with the MI broadening for weak disorder $\Delta < U$

No agreement for strong disorder $\Delta > U$ when the gap goes to zero
Phase coherence

Insulating state (no long-range phase coherence) with broad excitation spectrum

Bose-Glass

$s_2=0$

$s_1=4$  $s_1=8$  $s_1=12$  $s_1=16$

$s_2=2.5$

L. Fallani et al., PRL 98, 130404 (2007)
Ultracold atoms in optical lattices

Adding disorder

Noise interferometry

“Ideal” BEC
HB&T noise interferometry in quantum gases

absorption image of a Mott Insulator state

in a single image we have approx. 30000 detectors!
Noise correlations

**Spatial quantum noise interferometry in expanding ultracold atom clouds**

Simon Fölling, Fabrice Gerbier, Artur Widera, Olaf Mandel, Tatjana Gericke & Immanuel Bloch
Nature 434, 481 (2005)

\[
C(d) = \frac{\int \langle n(x + d/2) \cdot n(x - d/2) \rangle \, d^2x}{\int \langle n(x + d/2) \rangle \langle n(x - d/2) \rangle \, d^2x}
\]

Probing many-body states of ultracold atoms via noise correlations

Ehud Altman, Eugene Demler, and Mikhail D. Lukin
Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 10 June 2003; published 6 July 2004)

\[ G_{\alpha,\beta}(\mathbf{r}, \mathbf{r}') \sim \frac{1}{W^{2d}} \sum_{ii'jj'} e^{i \mathbf{R}_{ii'} \cdot \mathbf{Q}(\mathbf{r}) + i \mathbf{R}_{jj'} \cdot \mathbf{Q}(\mathbf{r}')} \langle a_{i\alpha}^\dagger a_{j\beta}^\dagger a_{j'} a_{i'} \rangle \]

\[ + \delta_{\alpha\beta} \delta(\mathbf{r} - \mathbf{r}') \langle n_\alpha(\mathbf{r}) \rangle \langle n_\beta(\mathbf{r}') \rangle - \langle n_\alpha(\mathbf{r}) \rangle \langle n_\beta(\mathbf{r}') \rangle. \]

Experiments: Mainz (Nature 2005)

JILA (PRL 2005) – pairs from molecules

NIST- Maryland (PRL 2007)

LENS (2007) – **Breaking of Mott order**
Noise correlations (Mott phase)

Breaking the MI order

1 optical lattice

uniform filling of regularly-spaced lattice
Breaking the MI order

2 optical lattices

controlled creation of particle / holes

inhomogeneous filling of (almost) regularly-spaced lattice
Breaking the MI order

one-color lattice

two-color lattice
First and second order correlations reveal the inhomogeneous filling of the lattice when first-order coherence does not provide any spatial information.
1st: Harmonic potential

- **site filling**
  - A horizontal line at zero, indicating a constant value.

- **site energy**
  - A graph showing a parabolic curve with a minimum at the origin, typical of a harmonic potential.
2\textsuperscript{nd}: Harmonic potential + incommensurate lattice

site filling

site energy
Destroying the MI domains

the disordering lattice destroys the “wedding cake”
Calculating noise correlations

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$G_{\alpha,\beta}(r,r') \sim \frac{1}{W}$

\[ \hat{H} = -J \sum_{\langle i,j \rangle} \hat{n}_i \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i \epsilon_j \hat{n}_j \]

Calculation of the ground state for $J=0$

The ground state is a disordered Mott insulator with the site filling minimizing the total energy

$|\Psi\rangle = |n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle \otimes \ldots$
Controlled breaking of Mott insulator order detected via noise correlation interferometry

Noise correlations in the bichromatic lattice

Observable:

the ratio between the heights of the $k_2$ peak and the $k_1$ peak along the horizontal median line

\[
\frac{P(k_2)}{P(k_1)}
\]
Quantitative analysis of the $k_2$ peak growth
Disordered systems: Role of interactions

CONTROLLED BREAKING of MOTT DOMAINS
Ultracold atoms in optical lattices

Adding disorder

Noise interferometry

“Ideal” BEC
Science 2001
Modugno et al

$^{87}\text{Rb}$

$^{41}\text{K}$ boson
...the last stable alkali isotope!

$^{39}$K boson
**BEC of $^{39}\text{K}$**


Feshbach assisted sympathetic cooling in a mixture with $^{87}\text{Rb}$

\[ B_0 = 317.9 \text{ G} \]
\[ B_0 = 402.4 \text{ G} \]

$N_K = 7 \times 10^4$
$T_c \sim 100 \text{ nK}$
$^{39}$K BEC with tunable interactions

Collisional physics: see also D'Errico et al N.J.P. 9, 223 (2007)
Interference with degenerate trapped atoms

- Confined-atom interferometers are promising in terms of compactness and portability.

- They offer the possibility of extending interrogation times beyond the typical 0.5 s achievable in atomic fountains.

- They offer high spatial resolution for study of atom surface interaction, Casimir Polder forces, deviations from Newtonian gravitational law at small distances.

- Main problems: interaction induced shift, decoherence induced by the trapping potential.
Interferometry with a vertical optical lattice

- High vacuum cell
- Mirror
- Ultracold atoms
- Non resonant lattice
- CCD
- Resonant laser (imaging)
A multiple well interferometer

\[ \lambda = \Delta \]

Wannier-Stark states

\[ \Delta E = \frac{F \lambda}{2} \]

Interference of Wannier-Stark states: Bloch oscillations

\[ \omega_B = \frac{F \lambda}{2\hbar} \]
Combination of high brightness and ultralow momentum spread: in principle ideal sources for matter wave interferometry ...

... however

\[ i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + sE_R \cos^2(kx) + g|\psi|^2 \right) \psi \]
Bloch oscillations with fermions

Bloch oscillations of $^{40}$K fermions trapped in a vertical optical lattice in presence of gravity


Lattice parameters: $\lambda = 1032$ nm, $s_{\text{lattice}} = 6$, $v_{\text{radial}} = 40$ Hz

100 a$_0$ vs 1 a$_0$

Widths of central peak!
Towards a high sensitivity local sensing of forces

Sensitivity: $\Delta g/g = \left[ \Delta x / (x_{\text{bragg}}) / N_{\text{osc}} \right]$

$6 \times 10^{-5} g$ per shot ($10^{-6} g$ achievable)

also Innsbruck with cesium
Atom-atom interaction in $^{39}$K Bose condensate can be accurately tuned over a large range and controlled around zero.

This is a very interesting system to investigate phenomena that can be destroyed by atom-atom interactions:

- BEC-based interferometry – forces at short distance
- disorder (Anderson localization, tuning $U$ versus $J$)
- Vortices, with IDEAL BEC (J. Dalibard talk)
- 2D physics (Z. Hadzibabic talk)