Current survival under a Rosen-Zener Quench of Hard Core Bosons

Israel Klich, KITP

Collaborators:
Gil Refael (Caltech)
Courtney Lannert (Wellesley)
Outline

• Quantum Quenches
• Hard Core Bosons on 1d lattice
• Surviving Current
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• Summary
First: Classical Quenches

System has critical temperature $T_c$

Start with $T > T_c$ then lower temperature to $T < T_c$

Order parameter may relax to different values at different spatial points

$\rightarrow$ create topological defects (Kibble-Zurek)

(Experimental observation: A. Maniv et al, PRL. 03)
Quantum Quenches:

• Start with a given state (usually ground state of a Hamiltonian $H_0$) of an extended quantum system such as a spin system or a field theory

• At time $t=0$ change the Hamiltonian to $H$, usually describing a different phase, evolve unitarily with $t$, and study the new state

Note that the system doesn’t relax since $\langle H \rangle$ is conserved

NOW: may be observable in cold atomic systems where coupling to environment is weak
Quantum Quenches

Just a few samples:
Quench dynamics across quantum critical points:
K. Sengupta, S. Powell and S. Sachdev, PRA (04);

Oscillating Superfluidity of Bosons in Optical Lattices
E. Altman and A. Auerbach, PRL (02), A. Polkovnikov, et al., PRA (02).
Numerical quench of hard core bosons M. Rigol and A. Muramatsu, Phys.Rev.A 04,
Quenches to critical systems: P. Calabrese and J. Cardy, PRL(06).
The Luttinger model interaction switch-on: M. A. Cazalilla PRL(06)
Quantum Ising model, relation to Kibble-Zurek: Zurek, Dorner &Zoller PRL(06)
Quantum quenches in a spinor condensate: A. Lamacraft PRL(07)
Quantum field theory picture:

Calabrese&Cardy:
If $H$ is CFT, one assumes that the Initial state flows into a CFT boundary condition, then correlation functions may be evaluated at long time and separations.

More Generic:
One goes off the critical point, so CFT treatment is not valid. Correlations decay with the gap.

Few exact solutions: Integrable boundary field theories (Ghoshal&Zamolodchikov)
Such as Quantum Sine Gordon with certain boundary conditions.
Excitation survival?

• If the initial state contains excitations of $H$ that become massive ($H$ is a massive (gapped) theory) we expect decay with time. However there may be lots of these excitations.

• Question: a current carrying system is subject to a quench to an insulating phase, does anything remain? (Dissipative regime has been discussed by many authors e.g. A. Polkovnikov, et al. PRA05)
Hard Core Bosons

Bosons on a lattice with the condition: \[(b_n^+)^2 = 0\]

\[H = -w \sum b_i^+ b_{i+1} + h.c.\]

Also called Tonks Girardeau gas.

Experimental access: B. Paredes et al., Nature 04

Can be mapped into a Fermi gas using Jordan Wigner transformation:

\[b_n^+ = e^{i\pi \sum_{m<n} a_m^+ a_m} a_n^+\]
The correlations are very different from fermion correlations due to appearance of a “Jordan Wigner string”:

\[ < b_i^+ b_j > = < a_i^+ a_j e^{i\pi \sum_{i<m<j} a_m^+ a_m} > \]

Known results:

\[ < b_0^+ b_x > \rightarrow C \sqrt{\frac{\rho}{\pi \chi}} \quad \text{(Lenard)} \quad C = 0.92 \quad \text{(Vaidya & Tracy)} \]

\[ n(k) \sim \frac{1}{\sqrt{k}} \]

Condensate fraction:

\[ < b_x^+ b_{x'} > = \sum n_j \phi_j^*(x) \phi_j(x') \]

Largest eigenvalue scales as

\[ \text{Max}(n_j) \approx \sqrt{N} \]

Partial order, but no true BEC
We write these in a way convenient to describe time evolution:

\[
\langle b_i^+ b_j \rangle = \partial_s \left< e^{s a_i^+ a_j} e^{i \pi \sum_{i<m<j} a_m^+ a_m} \right|_{s=0} \\
= \frac{1}{2} \det(1 - 2 PnP) \left< j \left| \frac{1}{1 - 2 PnP} \right| i \right>
\]

\[n = \text{Fermi Projection} \qquad \langle k|n|k'\rangle = \theta(k - k_f)\]

\[\langle 1|P|m\rangle = \begin{cases} 
\delta_{lm} & l, m \in [i, j] \\
0 & \text{otherwise}
\end{cases}\]
Correlations:

If the system Hamiltonian is time dependent but can be mapped to fermions:

\[ \langle b_i^+(t)b_j(t) \rangle = \frac{1}{2} \det(1 - 2Pn(t)P) \langle j | \frac{1}{1 - 2Pn(t)P} | i \rangle \]

\[ n(t) = e^{iHt} n(0) e^{-iHt} \]
Introduce a current:

We choose the initial state to be current carrying, equivalently the ground state of an accelerated Hamiltonian:

\[ -w \sum (e^{iy} b_i^+ b_{i+1} + h.c.) \]

In the Fermi language this is simply a shift in Fermi function:

\[ n_s(k) = \Theta(-k_F + \nu < k < k_F + \nu) \]

The initial current is:

\[ J(t = -\infty) = \frac{2w}{\pi} \sin(\nu) \]
The Quench:

\[ H = -w \sum b_i^+ b_{i+1} + V(t) \sum (-1)^n b_n^+ b_n + h.c. \]

\[ V(t) = \begin{cases} 0 & t < 0 \\ V_0 & t \geq 0 \end{cases} \]

Can be produced by a second laser beam with twice the period

At half filling the system is a Mott insulator
Evolution of the momentum distribution in the system:

\[ n_k(t) = \sum_l e^{ikl} \langle b_l^+(t)b_0(t) \rangle \]

Momentum oscillations, exact numerical results (150 sites)
Rosen-Zener profile

In order to explain a Stern Gerlach experiment, where deviation from a rectangular pulse shape was found and considered anomalous, Rosen and Zener considered the Hamiltonian:

\[ H(t) = E\sigma_z - V(t)\sigma_x \]

With

\[ V(t) = \frac{V_0}{\cosh(\pi t/T)} \]
Rosen-Zener profile

\[ H = -w \sum b_i^+ b_{i+1} + V(t) \sum (-1)^n b_n^+ b_n + h.c. \]

\[ V(t) = V_0 \begin{cases} 
Cosh(\pi t/T)^{-1} & t < 0 \\
1 & t > 0 
\end{cases} \]
We explore the evolution of the current density:

\[ J = \frac{iw}{L} \sum <b_i^+ b_{i+1} - h.c.> \]

After the Jordan Wigner, the hamiltonian separates into:

\[ H = H_0 + H_d(t) \]

where:

\[ H_0 = -2w \sum_{k<|\pi|} \cos(k) a_k^+ a_k \]

\[ H_d(t) = \frac{V(t)}{2} \sum_{k<|\pi|} (a_k^+ a_{k+\pi} + a_{k+\pi}^+ a_k) \]
• Thus the problem separates into a set of two level problems

\[ H(t) = \bigotimes_{|k|<\pi/2} H_k(t) \]

\[ H_k(t) = 2w \cos(k)\sigma_z - V(t)\sigma_x \]

\( H_k \) acts on the \((a_k^+, a_{k+\pi}^+)\) mode space

\[ a_k(t) = U^+(t)a_k^+U(t) = s(t)a_k^+ + p(t)a_{k+\pi}^+ \]
\[ \ddot{S} = -V_0^2 \cosh\left(\frac{\pi t}{T}\right)^2 S + (4iwn \cos(k) - \frac{\pi}{T} \tanh\left(\frac{\pi t}{T}\right)) \dot{S} \]

for

\[ S(t) = e^{2iwn \cos(k)t} s(t) \]

with the substitution \( z = \frac{1}{2} (\tanh\left(\frac{t \pi}{T}\right) + 1) \)

\[ S(z) = \sum_{n=0}^{\infty} \binom{p}{q} \frac{(p)_n}{(q)_n} \frac{z^n}{n!} \]

\[ 2F_1(p, q, r; z) = \sum_{n=0}^{\infty} (p)_n (q)_n \frac{z^n}{n!} \]
In the limit $wT << 1$

$s(0) = \cos\left(\frac{1}{2} V_0 T\right); \quad p(0) = -i \sin\left(\frac{1}{2} V_0 T\right)$

$$J(t) = w \int_{-\pi/2}^{\pi/2} dk \sin(k) (2 |s_o A_k - p(0) B_k|^2 - 1)(n_k - n_{k+\pi})$$

$$J = \frac{2w \nu}{\pi} \left( \cos((2t-T)V_0) C(\kappa \nu \sqrt{t}) - \sin((2t-T)V_0) S(\kappa \nu \sqrt{t}) \right)$$

Where $C(z) = \int_{0}^{z} du \cos\left(\frac{1}{2} \pi u^2\right)$

$$\kappa = \sqrt{\frac{8w^2}{\pi V_0}}$$
Current Oscillations:

Envelope period: Extrema of $C(z)$ are at $z = \sqrt{2n}$

$$\tau_e = 4\kappa^{-2}v^{-2}$$
Surviving Current

Exact: \[ J_\infty = \frac{2w\sin(\nu)\cos(V_0T)}{\pi} \left( 1 - \frac{V_0\arctan\left(\frac{2w}{V_0}\sin(\nu)\right)}{2w\sin(\nu)} \right) \]

Small Current limit: \[ \frac{8w^2V^3}{3\pi V_0^2} \cos(V_0T) \]

Conductivity of integrable systems: N. Andrei A. Rosch
Summary

Studied the behavior of hard-core bosons under a Rosen-Zener quench in the presence of a supercurrent.

Described the evolution of the supercurrent and survival fraction,

Numerical study of the momentum distribution evolution.

Power law dependence of the surviving current

Open Questions:
Existence of a limiting steady state? Can it be considered “thermal”? What is the power-law governing softer systems? Any other solvable examples? Higher dimensions?

Other quench problems: Correlations? Entropy generation?