Quantum micro-mechanics with ultracold atoms

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Motivations

- Force detection (Caves, Braginski, Milburn, others)
  - Apply force resonantly. Measure Z or E.
- Basics of quantum metrology: e.g. what detector to use?
  - Direct vs. post-amplification
  - Quadrature specific/non-specific
  - Destructive vs. non-destructive (QND)
- Quantum mechanics of macroscopic bodies.
Some experimental realizations

Nanofabricated cantilevers

Schwab [(Nature 443, 193 (2006))]
also Roukes, Cleland, Wineland

Bouwmeester [(Nature 444, 75 (2006))]
also Aspelmayer/Zeilinger, Rugar, Pinard/Heidman, Harris
Some experimental realizations

Micro-toroidal resonators

Ultracold atoms in a cavity

Vahala, Kippenberg

[(PRL 97, 243905 (2006))]

this talk
Optical (bosonic) detection

\[ H = H_{osc} + H_{cav} + H_{in/out} - f Z n \]

Detection limits:

\[ (\Delta Z)^2 \approx \frac{1}{N_{ph}} \left( \frac{\hbar \kappa}{f} \right)^2 \]

\[ (\Delta P)^2 \approx N_{ph} \left( \frac{f}{2\kappa} \right)^2 \]

- position measurement is photon-shot-noise limited: want more photons
- quantum back-action = radiation pressure fluctuations from shot noise: want fewer photons

Force per photon:

\[ \phi = \frac{f Z}{\hbar \kappa} \]
Optical (bosonic) detection

Detection

\[ \phi = \frac{fZ}{K} \]

force per photon:

\[ f = \frac{2h}{\lambda} \frac{1}{2cL} \]

Detection limits:

\[ (\Delta Z)(\Delta P) = \text{const.} \]

Uncertainty limit for quadrature specific measurement

\[ \Delta Z_{SQL} = \sqrt{\frac{\hbar}{2M\omega_z}} \]

Uncertainty limit for quadrature non-specific measurement

\[ H = H_{osc} + H_{cav} + H_{in/out} - fZn \]
Micro-mechanical state of the art

- Cannot passively cool to ground state
  - freq. = kHz – MHz < 10 mK = 200 MHz

- Cooling by active feedback or by dynamical back-action demonstrated, though not to ground state
  - closest approach: n = 59 quanta (Schwab, 2006)
  - requires resolved sidebands:
    detector bandwidth < oscillator freq.
    [PRL 99, 093901-2 (2007)]

- Measurement sensitivity approaching quantum limit
  - $\Delta Z = 6 \times \text{SQL}$ (Schwab, 2004)

- Back-action observed, but above quantum level
  - $(\Delta Z)(\Delta P) > 15 \times \text{SQL}$ (Schwab, 2006)
A cavity is mounted through here, serving as the home of ring-trap experiments. The image highlights the MOT beams, with a distance of 7.5 cm, and a motional trap. The Zeeman-slowed atomic beam is also indicated.
Stabilized to sub picometer!

Max coupling strength $g/2\pi$ 16 MHz
Cavity half-linewidth $\kappa/2\pi$ 0.66 MHz
Atomic half-linewidth $\gamma/2\pi$ 3 MHz
Finesse/Q $5.8 \times 10^5/3 \times 10^8$
Beam waist 23 μm
Critical atom number 0.02
Critical photon number 0.02
Cavity loading procedure

- Magnetic trapping outside cavity (TOP trap)
- Evaporative cooling
- Translation of the magnetic trap to within the cavity mode
- Transfer to 1D optical lattice inside cavity + turn off magnetic trap

Result: ~50,000 atoms trapped in ~200 sites of the in-cavity standing wave trap, at $T \sim 1 \, \mu\text{K (20 kHz)}$

Note: Following the lead of Vuletic, Chapman, Zimmermann, Hemmerich, Esslinger, Reichel; Walther, Blatt
Consider dispersive case of large cavity-atom detuning:

\[
H = \hbar \omega c \hat{n} + \sum_{\text{atoms}} \frac{\hbar g^2 \langle \hat{z}_i \rangle}{\Delta_{ca}} \hat{n} + H_{\text{atoms}} + H_{\text{in/out}}
\]

AC Stark shift per photon per atom  \quad \text{Cavity resonance shift per atom}
Many-atom cavity QED

Normal mode spectrum:

\[ \Delta_c = \frac{\Delta_{ca}}{2} \pm \sqrt{N \left\langle g^2 \right\rangle + \left( \frac{\Delta_{ca}}{2} \right)^2} \rightarrow \Delta_{ca} + \Delta_N \]

Dispersive regime:
Atom = index of refraction
Cavity frequency shift

\[ \Delta_N = N \frac{\left\langle g^2 \right\rangle}{\Delta_{ca}} \]

Normal mode spectrum:

\[ \sqrt{\left\langle g^2 \right\rangle} = 2\pi \times 10\text{ MHz} \]

N = 60,000 atoms
The image contains a diagram and text explaining a physics concept. The text reads:

\[
\frac{g^2(z_i)}{g_0} = g_{\text{eff}}^2(z_i) = 1
\]

The Hamiltonian is expanded to first order in atomic displacement from trap center:

\[
H = \hbar \omega_c \hat{n} + \sum_{\text{atoms}} \frac{\hbar g^2 \left( \hat{z}_i \right)}{\Delta_{\text{ca}}} \hat{n} + H_{\text{atoms}} + H_{\text{in/out}}
\]

defines collective operators relevant to the cavity probe:

\[
Z = \frac{1}{N_{\text{eff}}} \sum_{\text{atoms}} \sin(2kz_i) \Delta z_i \quad P = \sum_{\text{atoms}} \sin(2kz_i) p_i \quad N_{\text{eff}} = \sum_{\text{atoms}} \sin^2(2kz_i)
\]
Optomechanical bistability

\[ g(z) = g_0 \sin(k_p z) \]

"Coherent" effect: Optical forces in the cavity will displace the trapped atoms

- Probing the cavity shifts the cavity resonance \( \Rightarrow \) nonlinear cavity optics

"Cavity nonlinear optics at low photon numbers from collective atomic motion,"

PRL 99, 213601 (2007)
Transmission spectrum of nonlinear cavity

Cavity shift for undriven cavity

$\Delta_{ca} > 0$

Probe light repels atoms

Cavity transmission

Frequency

Bare cavity

Probe probe probe probe probe probe
Nonlinear optics with $n < 1$ photons

Requirements for nonlinear optics at $n \leq 1$
- medium whose optical properties are altered by single photons
  - many atoms held in loose trap; displaced by single photon optical force
- medium for which this alteration persists longer than photon lifetime
  - cold atoms with long motional coherence times

\[
\tau_{obs} = 1\text{ms} \quad \tau_{ph} = \frac{1}{K} = 100\text{ns}
\]

![Graph 1](image1.png)

**triggered sweep, $\Delta a/2\pi = -30\text{GHz}$**

![Graph 2](image2.png)

**back+forth sweep, $\Delta a/2\pi = -100\text{GHz}$**
Optomechanical potential

\[ H = \hbar (\omega_c + \Delta_N) \hat{n} - N_{\text{eff}} f_0 \hat{Z} \hat{n} + \hbar \omega_z \hat{a} \hat{a}^\dagger + H_{\text{else}} + H_{\text{in/out}} \]

Consider force on collective variable:

\[ F_Z = N_{\text{eff}} f_0 n(Z) - M \omega_z^2 Z \]

"Optical spring"
Lorenzian vs. position

some effects:

- Optomechanical frequency shift
- Optomechanical bistability
- Dynamical back-action (non-adiabatic) – heating, cooling, amplification
Bistability and atomic motion

Spectrum of driven cavity

Potential seen by atoms

Cavity transmission

bare cavity

drive frequency

position Z

$\Delta_{ca} > 0$

$\sim$ nm
Exciting collective motion from bistable switching
Direct observation of collective atomic motion

excite collective motion diabatically and observe

wait for cavity to come into resonance

single-shot detection

$\Delta Z(\text{measured, rms}) = 1.1 \text{nm}$

$\Delta Z(\text{SQL, rms}) = \sqrt{\frac{\hbar}{N m \omega}} = 0.22 \text{ nm}$
“Incoherent” effect: (quantum) fluctuations of the intracavity intensity cause momentum diffusion / diffusive heating

- Atoms serve as “spectators,” i.e. non-destructive detectors, of quantum fluctuations of an intracavity field
- Force fluctuations (→ momentum diffusion) represent backaction of position measurement
Atom-cavity dynamics and "granularity"

Equations of motion

Collective atomic field

Cavity optical field

Transient driven by "force history"

Bare-cavity response

Modulations (mixing) due to atomic motion

Generally: a hard problem! Some insights from Ritsch, Vuletic, Rempe, etc.

Look for small parameters (weak optomechanical coupling):

- Granularity parameter
- Scaled temperature
Atom-cavity dynamics and “granularity”

**Granularity parameter:**

\[
\varepsilon = \frac{N_{\text{eff}} f_0 \times \frac{1}{\kappa}}{\frac{\hbar}{Z_{\text{SQL}}}}
\]

(line single-photon force) x (residence time of photon) zero-point momentum spread

Does a single photon’s kick significantly perturb the trapped atom?

**Scaled temperature:**

\[
\varepsilon^2 = \left( \frac{N_{\text{eff}} f_0}{\kappa} \times \frac{1}{M \omega_z} \right) \times \left( N_{\text{eff}} f_0 \right)
\]

(line shift from atomic displacement cavity linewidth)

Does a single photon’s kick significantly affect the cavity?

**Scaled temperature:**

\[
\frac{\omega_z a^\dagger a}{\kappa}
\]

(atom temperature cavity-cooling equilibrium temperature)

Is cavity cooling/anti-cooling significant compared to diffusive heating
Fluctuation and dissipation

... one obtains a general result

\[ \frac{d}{dt} \langle \hat{a}^\dagger \hat{a} \rangle = \varepsilon^2 \kappa^2 \left[ S_{nn}^- + (S_{nn}^- - S_{nn}^+) \langle \hat{a}^\dagger \hat{a} \rangle \right] \]

with spectral density of photon number fluctuations

\[ S_{nn}^{\pm} = \int_{-\infty}^{\infty} dt \, e^{i(\Delta \pm \omega_\varepsilon)t} \left( \langle \hat{n}(t)\hat{n}(0) \rangle - \langle \hat{n}(t) \rangle \langle \hat{n}(0) \rangle \right) \]

- **Diffusive heating + coherent amplification/cooling:**
  - heating by red-detuned noise (promotes oscillator)
  - amplification/cooling = cavity anti-cooling/cooling (Ritsch, Vuletic, Rempe)

- **Resolved sidebands:**
  - blue-detuned cavity: cooling to ground state (Zwerger/Kippenberg, Girvin/Harris)
  - red-detuned cavity: quantum-limited quadrature-non-specific amplifier
Quantum fluctuations of light within a cavity

For our system (non-granular, low temperature):

\[
\frac{dE}{dt} = \frac{D_{\text{freespace}}}{m} \times C \times \frac{1}{1 + (\Delta - \omega_z)^2 / \kappa^2}
\]

Proportional to intensity

Note: At constant intensity, shot noise fluctuations of photon number in a resonant cavity are enhanced w.r.t. those in free space!

single-atom cooperativity

\[
C = \frac{g_0^2}{2\kappa \Gamma}
\]
Cavity-induced heating: measured by atom loss

\[ N \frac{dE}{dt} = (U - k_B T) \frac{-dN}{dt} \]

\[ k_B T \approx \frac{1}{10} U \]
wait and see

$\Delta_N$

bare cavity

probe

Prepares atoms in well defined initial state
Cavity-induced heating: measured by atom loss

![Graph showing D/Dfs vs. \(\delta/2\pi\) (MHz)](image)

What this means:

- Quantum metrology: back-action heating of macroscopic object at level prescribed by quantum measurement limits
- Quantum optics: Shot-noise spectrum is enhanced/colored by a cavity
- Cavity-light+atom interactions: “theory” looks good
- Quantum measurement of atom number or spin: spells trouble for some non-destructive measurements
Cavity mode structure selects a single collective mode in a large N gas of atoms.

Evaporative cooling cools this mode to the ground state

Detection: Atomic motion affects cavity properties

Actuation: Cavity light exerts concerted forces on trapped ultracold atoms

Back-action: Probe-light fluctuations cause momentum diffusion

Ahead:

Measurement: optimal method? fundamental limit?

Bistability: Quantum effects? Use as quadrature-specific amplifier? (cf Josephson bistability amplifier)

Amplification/cooling: Observe directly and test theories. Quantum amplifier?

Granular regime: quantum optics. Can we detect a single photon by radiation pressure?
E1: Spinor BEC
Jennie Guzman
Sabrina Leslie
Christopher Smallwood
Mukund Vengalattore
(James Higbie)
(Lorraine Sadler)

E2, E3: Cavity QED
Thierry Botter
Daniel Brooks
Joseph Lowney
Zhao-Yuan Ma
Kater Murch
Tom Purdy
(Kevin Moore)
(Subhadeep Gupta)

E4: Ring-trap interferometry
Joanne Daniels
Ed Marti
Ryan Olf
Tony Oettl
Enrico Vogt
Tiger Wu

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