Anderson localization of ultracold atoms in a (laser speckle) disordered potential: a quantum simulator

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Anderson localization of ultra cold atoms in a laser speckle disordered potential

1. Anderson localization: the naïve view of an AMO experimentalist: 1 particle quantum interference effect

2. Anderson localization with cold atoms in laser speckle: A well controlled system

3. 1D Anderson localization: An energy mobility edge?

4. 1D Anderson localization of ultra cold atoms in a speckle disordered potential: the experimental answer

5. 2D and 3D experiments: in progress…
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Anderson localization: a model for metal/insulator transition induced by disorder

Classical model of metal: disorder hinders, does not cancel, ohmic conduction

Classical particles bouncing on impurities \( \Rightarrow \) diffusive transport (Drude)

Matter waves scattered on impurities \( \Rightarrow \) incoherent addition

\( \Rightarrow \) delocalized (extended) states (cf. radiative transfer): conductor

Anderson L. (1958): disorder can totally cancel ohmic conduction

Tight binding model of electrons on a 3D lattice with disorder large enough:

exponentially localized states: insulator

Quantum effect: addition of quantum amplitudes of hopping

3D mobility edge (Ioffe Regel, Mott) \( E \leq \frac{\hbar^2}{2m\ell^2} \)
Tight binding model vs. wave model
Condensed matter vs. AMO physics

Bloch wave in a perfect crystal ⇔ Freely propagating wave

Disordered crystal ⇔ Scattering from impurities
Anderson localization: the point of view of an AMO physicist

Coherent addition of waves scattered on impurities. If mean free path $\ell$ smaller than de Broglie wavelength:
- coherent addition of trajectories returning to origin
- destructive interference in forward scattering, then in any direction

⇒ Localized states: insulator

R. Maynard, E. Akkermans, B. Van Tiggelen (Les Houches 1999)

Main features:
- Interference of many scattered wavelets ⇒ localization
- Single particle quantum effect (no interaction)
- Role of dimensionality (probability of return to origin)

3D mobility edge (Ioffe Regel)

$\ell \leq \frac{\lambda}{2\pi}$
The experimental quest of AL in AMO physics

Electromagnetic waves scattering on non absorptive impurities: not easy to discriminate from ordinary absorption

Microwaves (cm) on dielectric spheres:
  • discriminating localization from absorption by study of statistical fluctuations of transmission Chabonov et al., Nature 404, 850 (2000)

Light on dielectric microparticles (TiO₂):
  • Exponential transmission observed; questions about role of absorption Wiersma et al., Nature 390, 671 (1997)

Difficult to obtain \( \ell < \lambda / 2 \pi \) (Ioffe-Regel mobility edge)
No direct observation of the exponential profile in 3D

Most of these limitations do not apply to the 2D or 1D localization of light observed in disordered 2D or 1D photonic lattices: T. Schwartz et al. (M. Segev), Nature 446, 52 (2007). Lahini et al. (Silberberg), PRL 100, 013906 (2008).
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Ultra cold atoms (matter waves) Good candidate to observe AL

Good features

• Controllable dimensionality (1D, 2D, 3D)
• Wavelength $\lambda_{dB}$ “easily” controllable over many orders of magnitude (1 nm to 10 µm)
• Pure potentials (no absorption), with “easily” controllable amplitude and statistical properties
• Many observation tools: light scattering or absorption, Bragg spectroscopy, …

A new feature: interactions between atoms

• A hindrance to observe AL (pure wave effect for single particle)
• New interesting problems, many-body physics (T. Giamarchi, B. Altshuler, S. Skipetrov, D. Shepelyansky…)
Optical dipole potential

Inhomogeneous light field: \( \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}) \cos(\omega t - \varphi(\mathbf{r})) \)

Induced atomic dipole: \( \langle \mathbf{D}_{at}(t) \rangle_{\mathbf{r}_{at}} = \alpha \mathbf{E}(\mathbf{r}_{at}, t) \)

Far from atomic resonance, \( \alpha \) real
- \( \alpha < 0 \) above resonance
- \( \alpha > 0 \) below resonance

Interaction energy: \( W = -\mathbf{E}(\mathbf{r}_{at}, t)\langle \mathbf{D}_{at}(t) \rangle_{\mathbf{r}_{at}} = -\alpha \left[ \frac{\mathbf{E}_0(\mathbf{r}_{at})}{2} \right]^2 \)

Atoms experience a (mechanical) potential proportional to light intensity

\( U_{\text{dip}}(\mathbf{r}) = -\alpha I(\mathbf{r}) \)

• Attracted towards large intensity regions below resonance
• Repelled out of large intensity regions above resonance
Laser speckle disordered potential

Blue detuned light creates a repulsive potential for atoms proportional to light intensity

\[ V \propto \frac{I}{\delta} \propto \frac{|E|^2}{\delta} \]

Laser speckle: very well controlled random pattern
(Complex electric field = Gaussian random process, central limit theorem)

Intensity (i.e. disordered potential) is NOT Gaussian:

\[ P(I) = \frac{1}{I} \exp\left\{ -\frac{I}{\bar{I}} \right\} \]

Calibrated by RF spectroscopy of cold atoms (light shifts distribution)

Calibrated for \( \sigma_R > 1 \mu m \)

Extrapolated at \( \sigma_R < 1 \mu m \)

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1D Anderson localization?

Theorist answer: all states localized in 1D

Experimentalist question: Anderson like localization? AL: Interference effect between many scattered wavelets ⇒ exponential wave function

Localization in a strong disorder: particle trapped between two large peaks
Classical localization, not Anderson

Localization in weak disorder (numerics): interference of many scattered wavelets
Looks like Anderson localization
1D localization in a weak disorder as Bragg reflection

Periodic potential \[ V = V_0 \cos k z \]

No propagation of matter wave
\[ \psi = A \exp \left( \pm i \frac{p}{\hbar} z \right) \text{ if } p \sim \hbar k/2 \]

even in the case of \( E = \frac{p^2}{2M} >> V_0 \) (weak disorder)

- Bragg reflection of \( p \sim \hbar k/2 \) on \( \cos k z \)
- No propagation in a gap around \[ E = \frac{\hbar^2 k^2}{8M} \]

although \( E >> V_0 \)
1D localization in a weak disorder as Bragg reflection

Disordered potential: many independents \( k \) components, acting separately on various \( p \) components \( (\text{Born approximation}) \).

Anderson localization: all \( p \) components Bragg reflected.
Demands broad spectrum of disordered potential
1D Anderson localization in a weak uncorrelated disorder

Disordered potential with a white spectrum of $k$ vectors

Anderson L.: all $p$ components Bragg reflected
1D Anderson localization in a weak uncorrelated disorder

Disordered potential with a white spectrum of $k$ vectors

Anderson L.: all $p$ components Bragg reflected

What happens for a correlated potential (finite spectrum)?
Case of a speckle disorder: cut off in the spatial frequency spectrum

Speckle potential, created by diffraction from a scattering plate: no \( k \) component beyond a cut off

\[
\frac{2}{\sigma_R} = \frac{\sin \theta}{k_{\text{light}}}
\]

Only matter waves with \( E < \hbar^2 / 2m\sigma_R^2 \) (\( p < \hbar / \sigma_R \)) localize

Effective (Born approx.) mobility edge

First order perturbative calculation (second order in \( V \))

Lyapunov coefficient

\[
\gamma(p) = \frac{1}{L_{\text{loc}}(p)} \propto \hat{c}(2 \frac{p}{\hbar}) \implies \gamma(p) = 0 \text{ for } p > \frac{\hbar}{\sigma_R}
\]

L. Sanchez-Palencia et al., PRL 98, 210401 (2007)
1D Anderson localization in a weak speckle potential?

First order calculations*: exponentially localized wave functions provided that $p < \hbar / \sigma_R$

- Localization results from interference of many scattered wavelets (not a classical localization, weak disorder)
- Effective mobility edge

Same features as genuine (3D) Anderson localization

Worth testing it

* What happens beyond Born approximation? Ask question!
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A 1D random potential for 1D guided atoms

Atoms tightly confined in x-y plane, free along z: 1D matterwaves

BEC elongated along z and confined (focussed laser) transversely to z

Imaged with resonant light

1 D situation for the elongated BEC.

Many speckle grains covered (self averaging system = ergodic)

1D situation: invariant transversely to z
Ballistic expansion of a 1D BEC

Cloud of trapped ultracold atoms (dilute BEC) observable on a single shot: $N$ atoms with the same confined wave function.

Release of trapping potential along $z$: expansion in the 1D atom guide

Initial interaction energy $\mu_{\text{in}}$ converted into kinetic energy

$\Rightarrow$ After a while, interaction free ballistic expansion

$\Rightarrow$ Superposition of plane waves with $p \leq p_{\text{max}} = \sqrt{2M \mu_{\text{in}}}$
Search for Anderson localization in a weak speckle potential: a demanding experiment

Requirements:

• Good optical access for fine speckle \((\sigma_R = 0.26 \ \mu m)\)

• Initial density small enough for max velocity well below the effective mobility edge
  \[ p_{\text{max}} = 0.65 \ \hbar / \sigma_R \]
  (fluorescence imaging: 1 at / \mu m)

• Deep in weak disorder regime
  \[ V_R = 0.1 \ \mu_{\text{ini}} \]

Residual longitudinal potential well compensated: ballistic expansion over 4 mm
Anderson localization in a weak speckle: below the effective mobility edge

\[ p_{\text{max}} \sigma_R = 0.65 \hbar \]


Expansion stops. Exponential localization?
Anderson localization in a weak speckle below the effective mobility edge

Direct observation of the wave function (squared modulus)

\[ p_{\text{max}} \sigma_R = 0.65 \hbar \]

Is that measured localization length meaningful?

Exponential localization in the wings

Exponential fit \(\Rightarrow\) Localization length
Anderson localization in a weak speckle potential below the effective mobility edge

Profile stops evolving: wings well fitted by an exponential

Fitted localization length stationary: meaningful
Comparison to perturbative calculation

Magnitude and general shape well reproduced by perturbative calculation without any adjustable parameter.

\[ L_{\text{loc}} \text{ vs } V_R \]

\[ L_{\text{loc}} = \frac{2\hbar^4 k_{\text{max}}^2}{\pi m^2 V_R^2 \sigma_R (1 - k_{\text{max}} \sigma_R)} \]

\[ k_{\text{max}} \sigma_R = 0.65 \hbar \]

\[ \mu_{\text{ini}} = 220 \text{ Hz} \]
What happens beyond the mobility edge?

Theoretical prediction (1\textsuperscript{st} order Born approximation):

BEC with large initial interaction energy

$\Rightarrow$ Waves with $p$ values between $\frac{\hbar}{\sigma_R}$ and $p_{\text{max}} = \sqrt{2M\mu_\text{in}}$ do not localize

$\Rightarrow$ Waves with $p$ values below $\frac{\hbar}{\sigma_R}$ localize with different $L_{\text{loc}}$

$\Rightarrow$ power law wings $\sim z^{-2}$

L. Sanchez-Palencia et al., PRL 98, 210401 (2007)
What happens beyond the mobility edge?

Theoretical prediction (1\textsuperscript{rst} order):
BEC with large initial interaction energy

\[ p_{\text{max}} > \frac{\hbar}{\sigma_R} \Rightarrow \text{power law wings} \sim z^{-2} \]

L. Sanchez-Palencia et al., PRL 98, 210401 (2007)

\begin{align*}
\text{Experiment at} & \quad p_{\text{max}} \sigma_R = 1.15 \hbar \\
\text{Not exponential wings} & \quad \text{Power law wings} \sim z^{-2}
\end{align*}
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First results on 2D diffusion of ultra-cold atoms in a disordered potential

200 nK thermal sample released in a speckle potential with average 50 mK

Classical anisotropic diffusion

Anisotropic 2D Diffusive Expansion of Ultracold Atoms in a Disordered Potential

First results on 2D diffusion of ultra-cold atoms in a disordered potential

Ballistic vs. diffusive 
2D expansion

M. Robert-de-Saint-Vincent et al., PRL 104, 220602 (2010) 
Bouyer-Bourdel team

Determination of 2D 
(energy depending) 
diffusion coefficients by 
fits of profiles at various 
expansion times
Anderson localisation in 2D is going to be hard to observe unambiguously

Theoretical predictions (scaling th.)

In a finite size system $k l_B$ not bigger than 1: cf Ioffe-Regel

Observation in a speckle demands

- Close to the percolation threshold: how to distinguish AL from classical trapping?
- Classical diffusion quite slow (several seconds): how to distinguish stationary situation from very slow diffusion?

Still open questions
Preliminary results on 3D diffusion of ultra-cold atoms in a disordered potential

Ultra cold atoms at 10 nK, released in a suspending magnetic gradient, with a 3D speckle

A. Bernard, F. Jendrzejewski, P. Cheinet, K. Muller, Bouyer – Josse team
Preliminary results on 3D diffusion of ultra-cold atoms in a disordered potential

Ultra cold atoms at 10 nK, released in a suspending magnetic gradient, without or with a 3D speckle

A. Bernard, F. Jendrzejewski, P. Cheinet, K. Muller
Bouyer –Josse team

The disordered potential freezes the expansion
Observation of AL in 3D?

Classical trapping should not be a problem
(percolation threshold $\ll V_R$)

Here again, observing a steady state will demand long observation times (10 s): for parameters favourable to AL (according to scaling theory), classical diffusion coefficients quite small

Easier than in 2D?

In all cases, it would be crucial to have an unambiguous signature of AL: a method for cancelling AL without affecting classical diffusion, *ie* suppressing coherence between the various loops involved in AL process

- Shaking or scrambling the disorder?
- Breaking time-reversal invariance?
Conclusion: 1D localization

Evidence of Anderson localization of Bosons in 1D laser speckle disordered potential

• Crossover from exponential to algebraic profiles (effective mobility edge)
• Good agreement with perturbative ab initio calculations (no adjustable parameter)

Related results
Florence (Inguscio): Localization in a bichromatic potential with incommensurate periods (Aubry-André model), interaction control
Austin (Raizen), Lille (Garreau): Dynamical localization in momentum space for a kicked rotor
Hannover (Ertmer): lattice plus speckle
Rice (Hulet): Localization in speckle with controlled interactions
Urbana-Champaign (DiMarco): 3D lattice plus speckle, interactions
Outlook

Assets of our system

• Well controlled and well understood disordered potential (laser speckle = gaussian process)
• Cold atoms with controllable kinetic and interaction energy
• Direct imaging of atomic density (~wave function)
• Unambiguous distinction between algebraic and exponential

Future plans:

• more 1D studies (tailored disorder)
• control of interactions
• 2D & 3D studies
• fermions and bosons

Theory far from complete. 
A quantum simulator!
Anderson localisation in the Atom Optics group at Institut d’Optique

Experimental teams (Philippe Bouyer)
1. David Clément, A. Varon, Jocelyn Retter
2. Vincent Josse, Juliette Billy, Alain Bernard, Patrick Cheinet, Fred J., S. Seidel
3. Thomas Bourdel, J. P. Brantut, M. Robert dSV, B. Allard, T. Plisson
and our electronic wizards: André Villing and Frédéric Moron

Theory team (Laurent Sanchez Palencia): P. Lugan, M. Piraud, L. Pezze, L. Dao

Collaborations: Dima Gangardt, Gora Shlyapnikov, Maciej Lewenstein
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No localization beyond the effective 1D mobility edge?
Localization beyond the effective 1D mobility edge

Calculations (P. Lugan, L. Sanchez-Palencia) beyond the Born approximation (4th order) (agreement with numerics, D. Delande, and diagrams, C. Müller)

Lyapunov coefficient $\gamma$ not exactly zero but crossover to a much smaller value at effective mobility edge

Sharper crossover for weaker disorder

Effective transition in a finite size system

Analogous results in E. Gurevich, PRA 79, 063617
Groupe d’Optique Atomique du Laboratoire Charles Fabry de l’Institut d’Optique
Welcome to Palaiseau