מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Manipulating System-Bath Correlations

Universal control of Open Quantum Systems

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Weizmann Institute of Science
Israel

FRISNO 12, March 2011

Synopsis

Goals

Purification (cooling) $\rightarrow$ initialization
Protection / preparation $\rightarrow$ transfer $\rightarrow$ storage
Intrication (entanglement) $\rightarrow$ computation, teleportation
Challenge: Optimization (adapt to bath, save power)

Condition

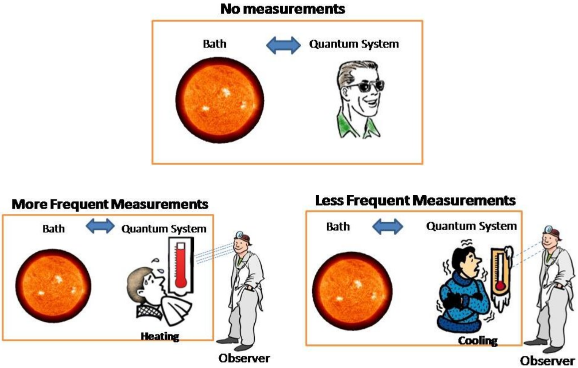
Act within bath memory time (reversibility)

References

Nature **452**, 724 (2008)
Phys. Rev. Lett. **102**, 080405 (2009)
Physica E **42**, 477(2010)
New J. Phys. **11** , 123025 (2009)
Phys. Rev. Lett. **104**, 040401 (2010)
New J. Phys. **12**, 053033 (2010)
Phys. Rev. Lett. **105**, 160401 (2010)

“Observations turn up the heat” News & views, *Nature* **452**, 705 (2008)

Zeno heating



Anti-Zeno cooling

These effects defy thermodynamic equilibrium: Role of Zeno and anti-Zeno?

N. Erez, G. Gordon, M. Nest & G. K., Nature **452***, 724 (2008)*

Qubit Evolution

$$H_{SB} = \sum_k \kappa_k \left(\underbrace{(b_k\sigma_+ + b_k^\dagger\sigma_-)}_{RW\text{(Freq. difference)}} + \underbrace{(b_k\sigma_- + b_k^\dagger\sigma_+)}_{CR\text{(Freq. sum)}} \right)$$

AZE

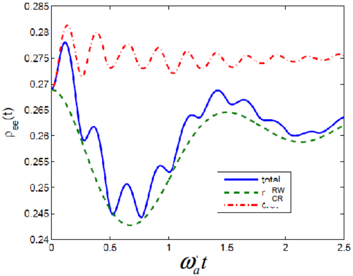
Long times

Resolved energy levels

Nature **405** 546 (2000)

$\dot{\rho}_{ee} = R_e(t)\rho_{ee} + R_g(t)\rho_{gg} < 0$

May yield cooling



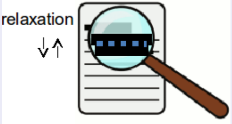
Nature **452**, 724 (2008)

New J. Phys. **11** 123025 (2009)

QZE

Ultrashort times

Unresolved energy levels



$\dot{\rho}_{ee} \xrightarrow[t\to 0]{} R(t)(\rho_{gg} - \rho_{ee}) > 0$

Always yields heating

Universal cooling bound

New J. Phys. **11** 123025 (2009)
New J. Phys. **12** 053033 (2010)

Master Eq. $\dot{\rho}_{ee} = R_G(t)\rho_{gg} - R_e(t)\rho_{ee}$

Solution for n measurements (QND disturbances)

$$\rho_{ee}(n\tau) = e^{-nJ(\tau)}\rho_{ee}(0) + \left(1 - e^{-nJ(\tau)}\right)\chi(\tau)$$

fixed point

$$\chi(\tau) = \frac{\int_0^\tau dt e^{J(t)} R_g(t)}{\int_0^\tau dt e^{J(t)} (R_g(t) + R_e(t))}$$

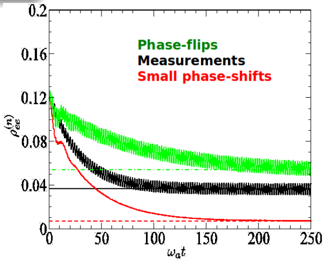
Relax. integral

$$J(t) = \int_0^t dt' (R_g(t') + R_e(t'))$$

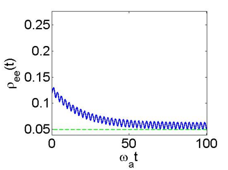
$$\tau \gtrsim \omega_a^{-1} \Rightarrow$$

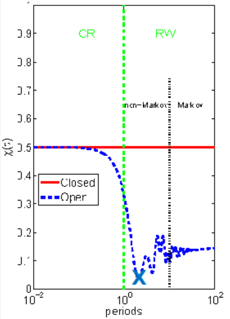
$$\rho_{ee} \approx \chi \ll \rho_{ee}(0) :$$

AZE cooling



Cooling





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Relax. integral

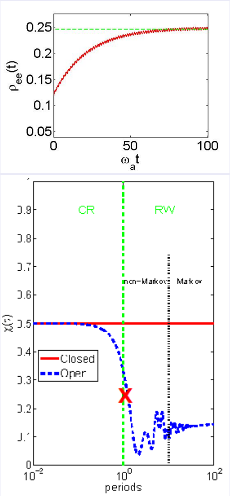
$$J(t) = \int_0^t dt'(R_g(t') + R_e(t'))$$

After $n > t_c/\tau^2\kappa$,

$\tau \ll \omega_a^{-1} \Rightarrow \rho_{ee} \approx \chi \approx 1/2$ — fully mixed

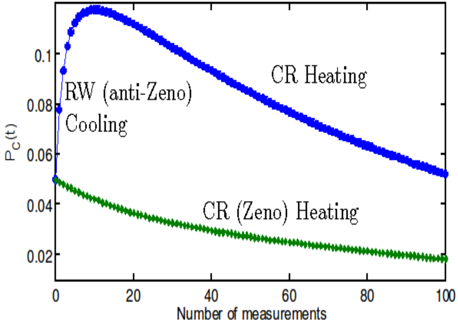
Zeno= "heating"

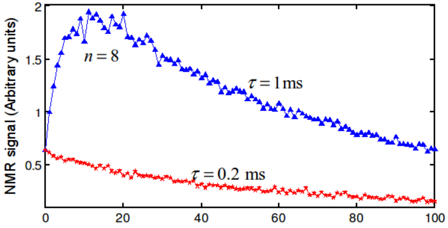
Heating



Measurement-driven control of quantum bits in a spin-bath

G. Alvarez, D. Dasari, L. Frydman and G. K., *PRL* **105**, 160401 (2010)



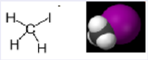


Interaction

$H_{SB} = J_{CH} \sum_k \hat{S}^x \hat{I}_k^x$ (CR+RW)

$\left.\begin{array}{l} P_C(0) = 0.05 \\ P_H(0) = 0.2 \end{array}\right\}$ non-equil

Experimental parameters



13C-methyl iodide (Iodomethane)

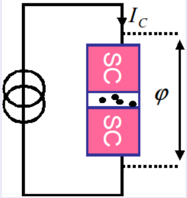
$J_{CH} = 150$Hz; $\frac{\gamma_H \omega_H}{\gamma_C \omega_C} = 2$ (off-resonant)

Induced Dephasings amplify the polarization transfer

No Born: bath changes till $[\rho_{eq}, H_{tot}^{RW}] \approx 0$

II. Protection: Quantum memory: storage & transfer to/from spin ensembles

Two-level fluctuators (TLF) coupled to JJ phase qubit



P. Bushev, D. D. B. Rao, G. K., A. Ustinov

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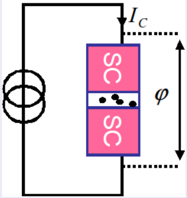

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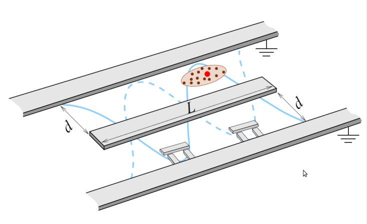
Electron spin coupled to nuclear spins in quantum dots



D. Petrosyan *et al.* *PRA* **79**, 040304(R) (2009)

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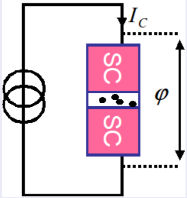

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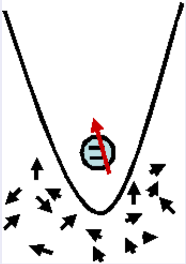
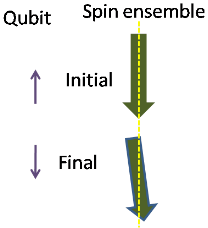


D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998)

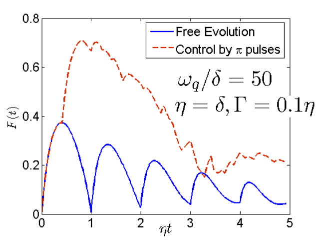
Dynamic Control of Spin/TLF Ensembles

Durga Dasari & GK

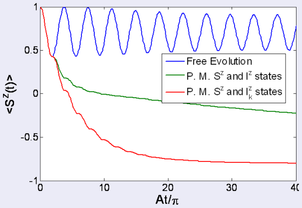
- Long coherence time of TLF useful for quantum memory.
- Perfect SWAP possible only for resonant fields.
- Non-selective measurements can achieve state (polarization) transfer even for *inhomogeneous* ensembles: AZE (π-pulses on write-in qubit)



Uniform coupling, $N = 100$

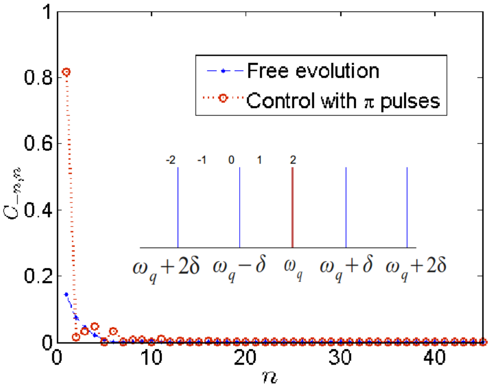


Gaussian distributed coupling, $\delta = 0$, $N = 100$



Entanglement of TLFs

AZE control via write-in qubit



Bath-optimized minimal-energy control (BOMEC)

Protection of quantum operations from decoherence & leakage

- Dynamic control that ensures **bath-optimized** fidelity of *any* quantum operation in general non-Markovian baths and noises.
- Benefits from the vast freedom of **arbitrary**, not just pulsed, time-dependent control.

Phys. Rev. Lett. **104** 040401 (2010); *Phys. Rev. Lett.* **102** 080405 (2009); *Phys. Rev. Lett.* **101** 010403 (2008)

Bath-optimized minimal-energy control (BOMEC)

General equation

Universal formula

$$\text{Error} \propto \int_{-\infty}^{\infty} d\omega F_t(\omega) G(\omega),$$

$F_t(\omega)$ depends only on the gate/control modulation (power spectrum), $G(\omega)$ is the bath spectrum.

Goal

Find a system Hamiltonian $Hs(t)$, $0 < t < T$, implementing unitary gate $U(T)$ while minimizing the bath-induced error at given energy.

Unified theory

Phys. Rev. Lett. **93** 130406 (2004); *Phys. Rev. Lett.* **97** 110503 (2006); *Phys. Rev. Lett.* **101** 010403 (2008); *Phys. Rev. Lett.* **104** 040401 (2010);

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Bath-optimized minimal-energy control (BOMEC)

Euler-Lagrange **Optimal Modulation**

PRL **104** 040401 (2010)

- Minimizes error (in any bath) under energy constraint $E = \int_0^T |\Omega(t)|^2 dt$, or action constraint $A = \int_0^T |\Omega(t)| dt$, etc.
- Not restricted to π-pulses: smooth, energy efficient modulations.
- Takes advantage of bath spectrum dips.

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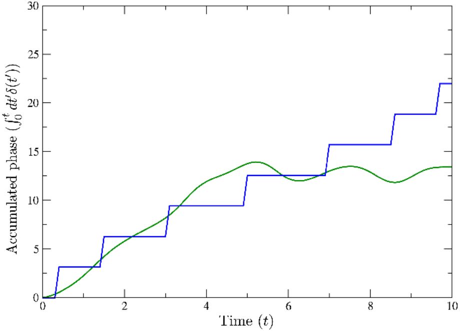

BOMEC vs. Uhrig, for given total action and bath.

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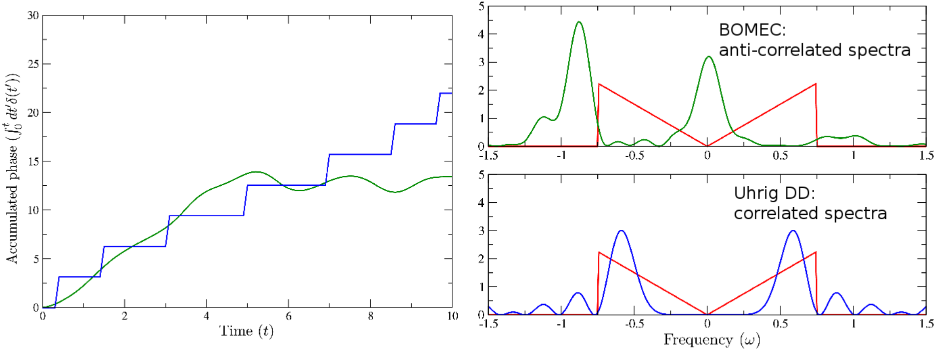

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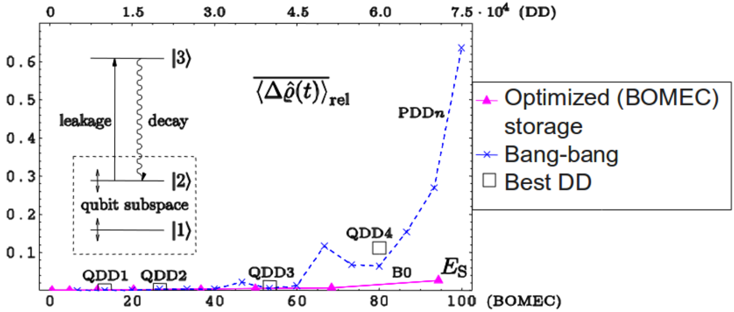
Bath-optimized minimal-energy control (BOMEC)

Qubit — With leakage
PRL **104** 040401 (2010)



- Qubit gate error scaled to unmodulated error, as a function of the energy constraint
- Dramatic reduction of the invested energy and the error compared to π-pulsed dynamical-decoupling (DD) control
- Error with allowance for leakage compared to error without leakage. Dynamical control between $|1\rangle$ and $|2\rangle$ causes leakage to $|3\rangle$.

Bath-optimized minimal-energy control (BOMEC)

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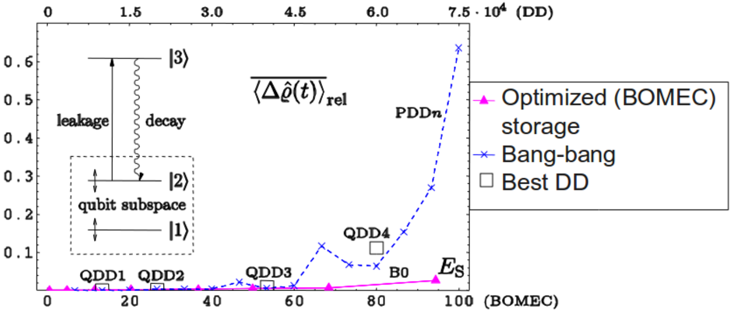
PRL **104** 040401 (2010)



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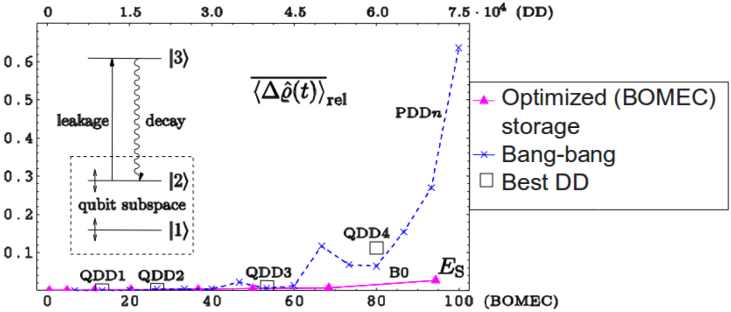

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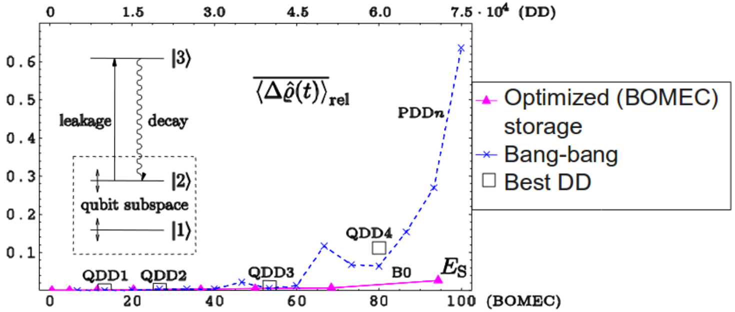

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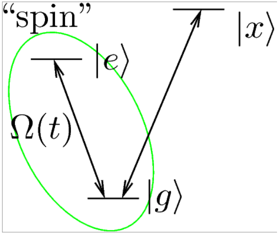
Optimizing Inhomogeneous Atomic Ensembles for Quantum Information Storage

G. Bensky, D. Petrosyan, J. Schmiedmayer, G.K.



1. Starting with all atoms in $|g\rangle$, excite atoms via $\Omega(t)$ to $|e\rangle$
2. Atoms remaining in $|g\rangle$ go to $|x\rangle$ (adiabatic sweep)
3. Return selected atoms from $|e\rangle$ to $|g\rangle$ (adiabatic sweep)

The chosen subensemble

$$P(\omega) = \left| \langle e | \, T_+ \, e^{-i \int_0^T H(t) dt} \, | g \rangle \right|^2 \approx \left| \int_0^T \Omega(t) e^{-i(\omega - \omega_0)t} dt \right|^2$$

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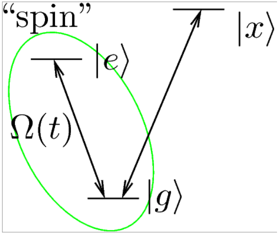

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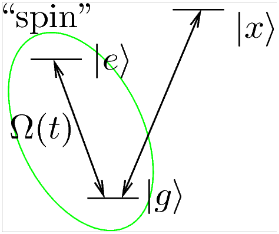

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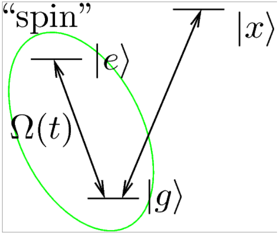

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Optimizing Inhomogeneous Atomic Ensembles for Quantum Information Storage

G. Bensky, D. Petrosyan, J. Schmiedmayer, G.K.

If $P(\omega)$ is much narrower than the initial spectrum:

$$\mathcal{F}(\tau) \approx \frac{4\pi^2 n^2(\omega_0)}{N^2} \left| \int_0^T dt\, \Omega(t+\tau)\Omega(t) \right|^2$$

Use Euler-Lagrange to find optimal $\Omega(t)$

$$\Omega(t) = \Omega_0 \sin(\pi t/T)$$

$$\mathcal{F}(\tau) \approx 1 - \frac{\pi^2 \tau^2}{T^2}$$

$$N = \pi n(\omega_0)\Omega_0^2 T$$

$$\alpha(t) \approx \left[1 - \frac{t_{\rm tr}^2}{T^2}\right] \cos(\pi t/t_{\rm tr}) + \frac{t_{\rm tr}^2}{T^2}$$

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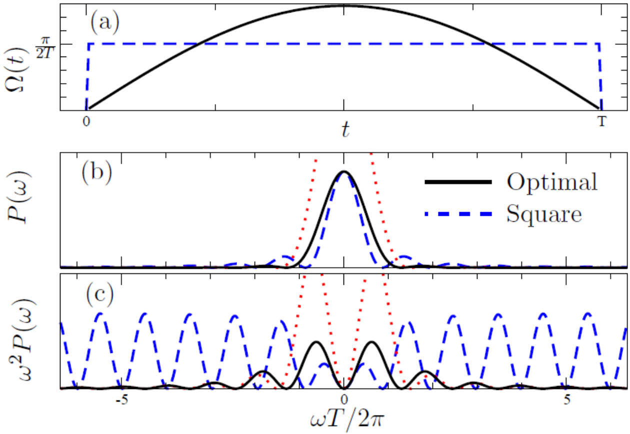

The spectral variance $\langle \omega^2 \rangle \sim T^{-2}$ for the optimal preparation pulse, but does not converge for the square preparation pulse.

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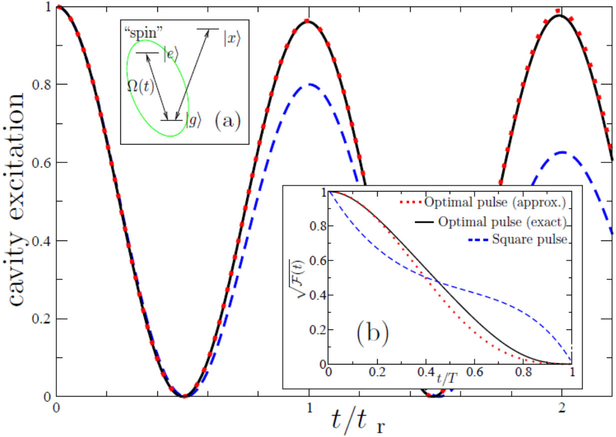
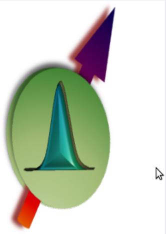


Results for preparation pulse of $T = 10t_{\text{tr}}$.

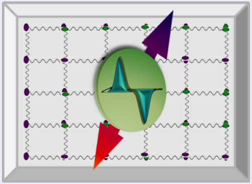
Schrödinger's cat states generated by the environment

Durga Dasari, Nir Bar-Gill & G. K. *PRL* **106**, 010404 (2011)

Noninteracting spin/atomic ensemble



Entangled spin/atomic GHZ state



All spins identically coupled to bath

$$|\uparrow\uparrow\uparrow\cdots\uparrow\rangle \Rightarrow p\underbrace{\left[\frac{|\uparrow\uparrow\cdots\uparrow\rangle + e^{\imath\frac{\pi}{2}}|\downarrow\downarrow\cdots\downarrow\rangle}{\sqrt{2}}\right]\left[\frac{\langle\uparrow\uparrow\cdots\uparrow| + e^{-\imath\frac{\pi}{2}}\langle\downarrow\downarrow\cdots\downarrow|}{\sqrt{2}}\right]}_{GHZ}$$

$$+(1-p)\rho_S$$

$$p \simeq 1$$

Schrödinger's cat states generated by the environment

Durga Dasari, Nir Bar-Gill & G. K. *PRL* **106**, 010404 (2011)

Collective spin $\hat{L}_z = \sum_j \hat{\sigma}_{zj}$

$$H = \omega_0 \hat{L}_z + \sum_k \omega_k b_k^\dagger b_k + \underbrace{\hat{L}_z \sum_k \eta_k (b_k + b_k^\dagger)}_{H_I = \hat{L}_z \hat{B}\text{: collective coupling to bath}}$$

Magnus expantion

$$U \equiv T_{\leftarrow} e^{-\imath \int_0^t H(t')dt'} = \prod_n e^{-\imath A_n(t)}$$

$$A_1(t) \equiv \int_0^t H(t')dt' = \hat{L}_z \hat{B}(t) \quad A_2(t) \equiv \int_0^t \int_0^{t'} [H(t'), H(t'')] dt' dt'' = \hat{L}_Z^2 (c\text{-number})$$

$$A_3(t) = \int_0^t \int_0^{t'} \int_0^{t''} \{[[H(t'), H(t'')], H(t''')] + [H(t'), [H(t''), H(t''')]]\} dt' dt'' dt''' = 0$$

Phase-damped spin/atomic ensembles

$$U(t) = \exp\left[-\imath \left\{ \omega_0 t \hat{L}_z + \underbrace{f(t)\hat{L}_z^2}_{\text{Collective Lamb shift}} \right\} + \underbrace{\hat{L}_z t \sum_k (\alpha_k b_k + \alpha_k^* b_k^\dagger)}_{\text{Decoherence (damping)}} \right]$$

Single spin-1/2 particle

$$\hat{L}_z \rightarrow \hat{\sigma}_z, \hat{\sigma}_z^2 = \hat{I}$$

Linear Evolution

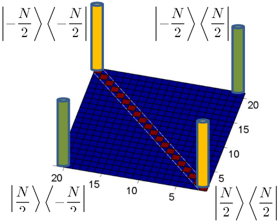
'N' spin-1/2 particles

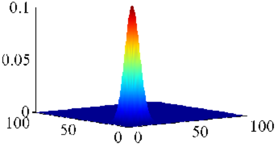
$$\hat{L}_z \rightarrow \sum_k \hat{\sigma}_z^k, \hat{L}_z^2 = \hat{I} + \sum_{j,k} \hat{\sigma}_z^j \hat{\sigma}_z^k$$

Nonlinear Evolution

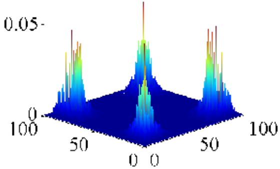
Natural formation of macroscopic superposition states in thermal bath

Schrödinger's cat generated by an Ohmic Bath





$|\rho(t=0)|$



$|\rho(t=\tau_{\text{cat}})|$

More than 100 spins in high-fidelity *GHZ* state for
$f_M \sim \omega_D \gtrsim 100\,GHz,\ T \lesssim 1^\circ K.$

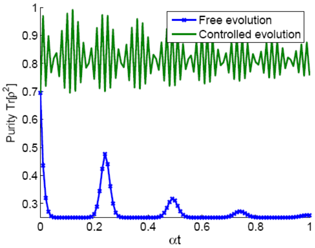
$$\tau_{\text{cat}}\Gamma_M N^2 < 1$$
$$\tau_{\text{cat}} \sim \frac{\pi}{2f_M}$$

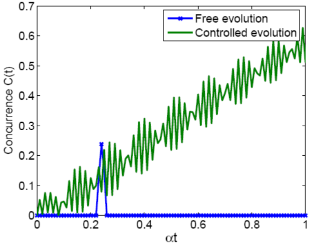
π-pulse control to keep high purity BIE

$$H_\pm = \sum_k \omega_k b_k^\dagger b_{jk} \pm \underbrace{\sum_{k,j} \beta_{kj}\sigma_{zj}(b_k + b_k^\dagger)}_{\hat{L}_z\hat{B}-\text{odd}}$$

$$U(t) = [\dots U_-(4\tau)U_+(3\tau)U_-(2\tau)U_+(\tau)] \exp\left(\imath t\kappa(t) \underbrace{\sum_{j,j'} \hat{\sigma}_{zj}\hat{\sigma}_{zj'}}_{\hat{L}_z^2-\text{even}}\right)$$

Optimal pulse sequence $R \to 0$, $f(t) \to f_M$



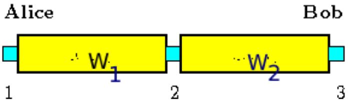


Lorentzian bath, $N = 100$ modes

$\alpha = \int G_T(\omega)d\omega$, PDD, $\alpha\tau = 0.01$

Bath prepares remote parties in an entangled state

- Interesting:
  - no approximations involved
  - independent of bath (temperature!)
  - $\hat{U} = \prod_{jj'}^{N} \hat{U}_{jj'}$ with $\left[\hat{U}_{jj'}, \hat{U}_{mm'}\right] = 0$: Strictly pairwise entanglement
    - ⋆ $\hat{U}_{12}$ unaffected by trace: $\mathrm{Tr}_{3,\ldots,N}(\hat{U}\rho\hat{U}^{\dagger}) = \hat{U}_{12}\rho_{12}\hat{U}_{12}^{\dagger}$
  - universal quantum operations:
    - ⋆ local unitaries + nearest neighbor $\hat{S}_1 \otimes \hat{S}_2 = \hat{\sigma}_{z1} \otimes \hat{\sigma}_{z2}$
- Challenge:
  - maximize relative weight of non-local bath modes
- Solution: confined geometries (waveguides)



Conclusions I — *Non-Markov Purification*

- Frequent measurements at non-Markov τ effect anti-Zeno cooling (purification) or Zeno heating (mixing) of open quantum systems.
- **Fundamental message**: Simple oscillatory dynamics governs thermal macrosystems (system+bath), if monitored frequently enough $\rightarrow$ thermodynamics fails.

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- Know thine enemy: measure noise / bath spectra
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Conclusions III — *Entanglement Control*

1. In contrast to single-qubit/spin, non-linear (L_z^2 or L_x^2) collective shifts are naturally imposed by the bath on N-qubit/spin systems with $N \geq 2$.
2. Collective coupling to bath can yield high purity cat (GHZ) state on Markovian time-scales.
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General quantum task optimization

$P(\rho_f)$ is a measure (score) of how well the task was completed in the presence of a bath/noise. Examples:

Maximize avr. fidelity: $P(\rho_f) = \overline{\langle\psi_0|\, \rho_f \,|\psi_0\rangle}$
Minimize entropy: $P(\rho_f) = \mathrm{Tr}\{\rho_f^2\}$
Maximize entanglement (concurrence): $P(\rho_f) = C(\rho_f)$

$$H = H_0 + H_c(t) + \sum_k S_k \otimes B_k$$

In the interaction picture:

$$H = \sum_k S_k(t) \otimes B_k(t)$$

General function, averaged over initial states

$\Delta P(\rho_f) \approx \partial_\rho P \cdot \Delta\rho_f$ (Linear approximation)
$\hat{\Gamma}_{ij} = \partial_\rho P \cdot [\sigma_i, \sigma_j \rho_0]$ — the change in task score after operations σ_i, σ_j
$\Delta P \approx \int_{-\infty}^{\infty} d\omega \hat{G}(\omega) \hat{F}_t(\omega)$
Control spectrum $\hat{F}_t(\omega) = t^{-1}\hat{\varepsilon}_t(\omega)\hat{\Gamma}\hat{\varepsilon}_t^\dagger(\omega)$ depends on task

$$\hat{G}_{ij}(\omega) = \mathrm{FT}\underbrace{\{\langle B_i(0)B_j(t)\rangle\}}_{\text{bath correlation}}; \quad \hat{\varepsilon}_{t,ij}(\omega) = \mathrm{FT}_t\{\varepsilon_{ij}(t)\}; \quad S_i(t) = \sum_j \underbrace{\varepsilon_{ij}(t)}_{\text{rotation}} \sigma_j$$

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Optimizing Inhomogeneous Atomic Ensembles for Quantum Information Storage

G. Bensky, D. Petrosyan, J. Schmiedmayer, G.K.

If all the spins had the same frequency ω_0: $|\psi_1(\tau)\rangle = N^{-1/2}\sum_j e^{-i\omega_0\tau}|j\rangle$.
Inhomogeneous broadening: $\left|\tilde{\psi}_1(\tau)\right\rangle = N^{-1/2}\sum_j e^{-i\omega_j\tau}|j\rangle$

storage fidelity

$$\mathcal{F}(\tau) \equiv \left|\left\langle \psi_1(\tau)|\tilde{\psi}_1(\tau)\right\rangle\right|^2 = \left|\frac{1}{N}\int n(\omega)e^{-i(\omega-\omega_0)\tau}d\omega\right|^2$$

$$N = \int n(\omega)d\omega$$

In the rotating wave approximation and neglecting the cavity relaxation:
$|\Psi(t)\rangle = \alpha(t)|1,\psi_0\rangle + \sum_j \beta_j(t)|0,j\rangle$ where $\dot{\alpha}(t) = -N\bar{\eta}^2\int_0^t dt'\alpha(t')\sqrt{\mathcal{F}(t-t')}$

Optimized transfer to storage

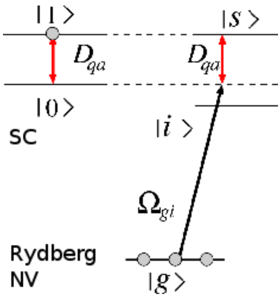
Given energy constraint

Goal: Transfer high-decoherence qubit 1 (write-in) to stable (storage) qubit 2

Model:

$H = H_0 + B \otimes \sigma_{z1} + V(t) \cdot \sigma_{x1} \otimes \sigma_{x2}$

For given energy $E = \int_0^T |V(t)|^2 dt$, find best coupling $V(t)$ to maximize fidelity.



Rydberg: Petrosyan *et al.*, *PRA* **79** 040304 (2009);
NV: Kubo *et al.*, *arXiv*:1006.0251v1;
Hyperfine: Verdu *et al.*, *PRL* **103**, 043603 (2009)

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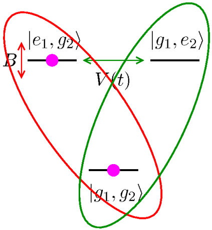

de Moura-Escher *et al.*

Counterintuitively, fastest transfer isn't best!

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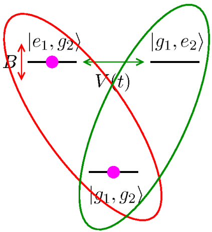

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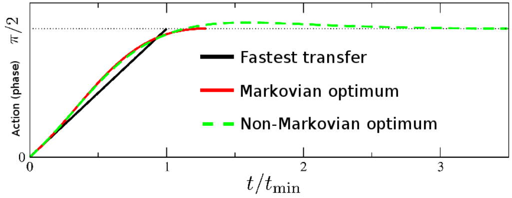
RWA — $\omega_0 t_{min} \gg 1$

No leakage during the transfer.



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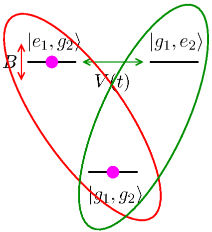

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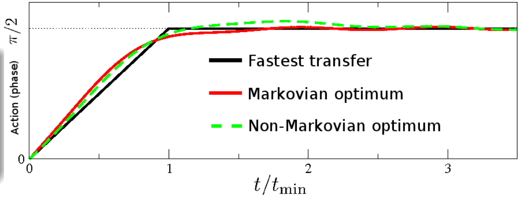

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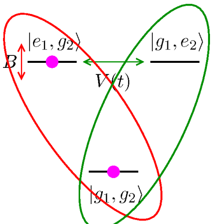

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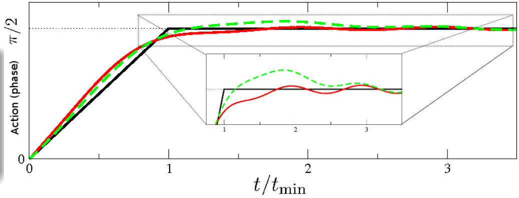
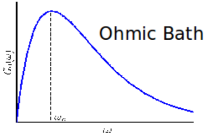


Unitary (entangling) vs non-unitary (decoherence) rates

$$\text{Entangling rate } f(t) = \underbrace{\frac{1}{t}\int_0^\infty G_0(\omega)\left[\frac{\omega t - \sin\omega t}{\omega^2}\right]d\omega}_{\beta-\text{indep.}}$$

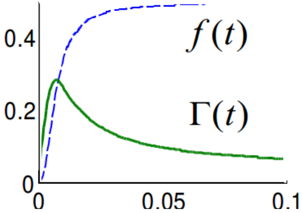


$$\underset{\text{(per particle)}}{\text{Decoherence rate}}\ \Gamma(t) = \underbrace{\frac{1}{t}\int_0^\infty G_0(\omega)\coth\beta\omega\left[\frac{1-\cos\omega t}{\omega^2}\right]d\omega}_{\beta-\text{dep., sym. different}}$$

$G_0(\omega)$-bath ($T = 0$) autocorrelation (memory spectrum); $\beta = \frac{1}{\kappa_B T}$

$f(t) = \omega_c - \frac{1}{t}\tan^{-1}(\omega_c t)$ [β-independent]

$\Gamma(t) \xrightarrow{\beta\to 0} \frac{1}{\beta}\tan^{-1}(\omega t) - \frac{1}{t}\frac{\log(1+\omega_c^2 t^2)}{2\beta\omega_c}$



Schrödinger's cat formed if $\omega_c \gg \kappa_B T \quad f_M \gg \gamma_M \quad f_M \gg R \sim N^2\Gamma_M$.

Bath-induced entanglement (BIE): Can serve QIP?

J. Clausen, D. D. B. Rao & G.K.

Particles *locally* interacting with $1D$ bath, $\hat{H}_{=}\sum_{j=1}^{N}\hat{S}_j\hat{B}_j$

$$\left[\hat{H}_I(t_1),\hat{H}_I(t_2)\right]=\sum_{j,j'=1}^{N}\hat{S}_j\underbrace{\left[\hat{B}_j(t_1),\hat{B}_{j'}(t_2)\right]}_{\text{c-number}}\hat{S}_{j'}\ (\hat{S}_j=\hat{\sigma}_{zj})$$

$\hat{H}_{eff}(t)=\sum_{j=1}\hat{S}_j\hat{\Gamma}_j(t)+\sum_{j,j'=1}\hat{S}_j\kappa_{jj'}(t)\hat{S}_j$ – indep. of bath, *exact*

j-th part. decoh. func.

$$\hat{\Gamma}_j(t)=\frac{1}{t}\int_0^t dt'\hat{B}_j(t')=\sum_m\underbrace{\beta_{jm}(t,\omega_m)}_{\text{Coupling to } m\text{th bath mode}}\hat{b}_m+\text{H.C.}$$

Long time: Markov dephasing

Decoh. Rate $R_j(t)=\frac{1}{t}\int_0^{\omega}d\omega G_T^j(\omega)\left[\frac{1-\cos\omega t}{\omega^2}\right]$

jth part. coupl. spect. $G_T^j(\omega)=\sum_m\beta_{jm}^2\left[1+2n_T(\omega_m)\right]\delta(\omega-\omega_m)$

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Principle value (odd sym. in ω)

$j-j'$ cross-coupling spectrum: $G_0^{jj'}=\sum_m\beta_{jm}\beta_{j'm}\delta(\omega-\omega_m)$

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J. Clausen, D. D. B. Rao & G.K.

Particles *locally* interacting with $1D$ bath, $\hat{H}_{=} \sum_{j=1}^{N} \hat{S}_j \hat{B}_j$

$$\left[\hat{H}_I(t_1), \hat{H}_I(t_2)\right] = \sum_{j,j'=1}^{N} \hat{S}_j \underbrace{\left[\hat{B}_j(t_1), \hat{B}_{j'}(t_2)\right]}_{\text{c-number}} \hat{S}_{j'} \; (\hat{S}_j = \hat{\sigma}_{zj})$$

$\hat{H}_{eff}(t) = \sum_{j=1} \hat{S}_j \hat{\Gamma}_j(t) + \sum_{j,j'=1} \hat{S}_j \kappa_{jj'}(t) \hat{S}_j$ – indep. of bath, *exact*

j-th part. decoh. func.

$$\hat{\Gamma}_j(t) = \frac{1}{t} \int_0^t dt' \hat{B}_j(t') = \sum_m \underbrace{\beta_{jm}(t, \omega_m)}_{\text{Coupling to } m\text{th bath mode}} \hat{b}_m + \text{H.C.}$$

Long time: Markov dephasing

Decoh. Rate $R_j(t) = \frac{1}{t} \int_0^\omega d\omega G_T^j(\omega) \left[\frac{1-\cos\omega t}{\omega^2}\right]$

jth part. coupl. spect. $G_T^j(\omega) = \sum_m \beta_{jm}^2 \left[1 + 2n_T(\omega_m)\right] \delta(\omega - \omega_m)$

BIE rate $\kappa_{jj'}(t) = \frac{t}{2\pi} \int_{-\infty}^{\infty} d\omega \sum_m \frac{\beta_{jm}(t,\omega)\beta^*_{j'm}(t,\omega)}{\omega - \omega_m} = \frac{1}{t} \int_0^t d\omega G_0^{jj'}(\omega) \left[\frac{\omega t - \sin\omega t}{\omega^2}\right]$ —

Principle value (odd sym. in ω)

$j - j'$ cross-coupling spectrum: $G_0^{jj'} = \sum_m \beta_{jm} \beta_{j'm} \delta(\omega - \omega_m)$

Decoherence Control by Modulation

G. Gordon & GK

Asymmetric modulation changes *GHZ* decoherence scaling from N^2 to N (*indep.* qubits).

Requirement

$t_{coh}(N) < \tau < t_{corr}$

Modulation-induced improvement of cat-state coherence time

$$\sim \left(\frac{t_{corr}}{\tau}\right)^2$$

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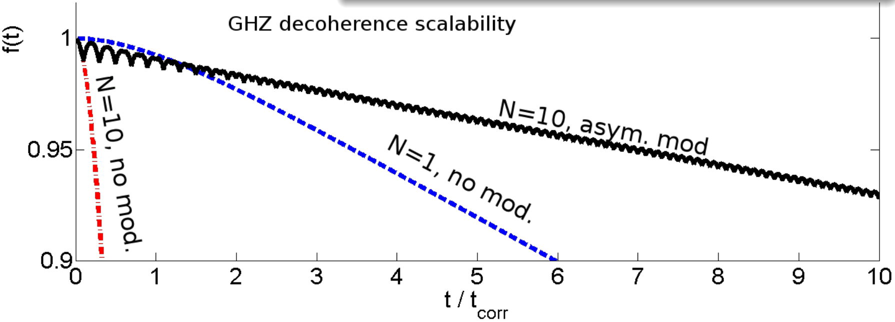
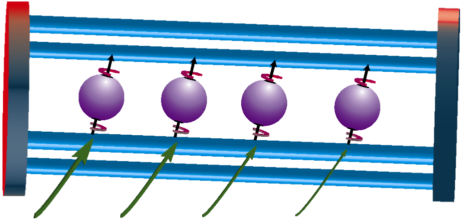


Experimental Setup: Ion traps



Eliminate cross-dephasing by different laser intensities

$$\frac{1}{N^2} t_{\text{coh}}(N=1) \rightarrow t_{\text{coh}}^{\text{mod}} \sim \frac{1}{N} t_{\text{coh}}^{\text{mod}}(N=1)$$

$$t_{\text{coh}}^{\text{mod}}(N=100) > t_{\text{coh}}(N=1)$$