Light Shifts
From Optical Pumping to Cavity QED

Claude Cohen-Tannoudji

FRISNO 12
Ein Gedi, Israël
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Purpose of this lecture

- Describe the perturbations experienced by an atom when it interacts with an incident quasi resonant light beam

  Physical interpretation of these effects

  Experimental observation

- Point out the connections which exist between these effects and the absorption and anomalous dispersion for light

  Index of refraction

- Show that these perturbations are also useful for manipulating atoms and light and for preparing new interesting quantum states

  Atom traps, atom mirrors, optical lattices

  “Schrödinger cats”, Non destructive detection of photons
1 - Light shifts and absorption rates. Theory

2 - Experimental observations

3 - Perturbation of the field. Absorption and dispersion

4 - Using light shifts for manipulating atoms and fields
**Dressed atom approach**

Two-level atom \{g,e\} coupled to a single mode of the quantum radiation field with frequency \( \omega_L \)

- \( g, N+1 > \): Atom in g in the presence of \( N+1 \) photons
- \( e, N > \): Atom in e in the presence of \( N \) photons

\[
\begin{align*}
E_e - E_g &= \hbar \omega_0 \\
E_{g,N+1} - E_{e,N} &= \hbar \delta \\
\delta &= \omega_L - \omega_0 = \text{Detuning}
\end{align*}
\]

\[
\langle e,N|V_{AL}|g,N+1\rangle = \left(\frac{\hbar}{2}\right) \Omega_{N+1}
\]

\[
\Omega_{N+1} = \Omega_0 \sqrt{N+1}
\]

\( \Omega_0 \): Rabi frequency in the manifold \( \{g,1\},\{e,0\} \)

We keep here the N-dependence of the Rabi frequency because we will also consider the case of an atom in a real cavity.
Reduced evolution within $\mathcal{E}(N)$

$\mathcal{E}_{N+1}$: Manifold of 2 states \{ $g,N+1$ ; $e,N$ \}

$e,N$ is coupled not only to $g$, $N+1$ (by stimulated emission), but also to $g,N,k$ (by spontaneous emission of a photon $k$).

One can show that the reduced evolution of the system in this manifold is governed by an effective Hamiltonian obtained by adding an imaginary term – $i \Gamma / 2$ to the energy of $e$

$\Gamma$: Natural width of $e$, describing the radiative instability of $e$

$$H_{\text{eff}} = \hbar \begin{pmatrix} \delta & \Omega_{N+1} / 2 \\ \Omega_{N+1} / 2 & -i \Gamma / 2 \end{pmatrix}$$

In sections 1, 2, we consider an atom in free space and a field in a coherent state, so that we take all the $\Omega_N$ equal to $\Omega$ for all $N$

In sections 3.4, we consider an atom in a cavity and we keep the $N$-dependence of $\Omega_N$
Light shifts and absorption rates

Since $H_{\text{eff}}$ is not hermitian, its eigenvalues are not real. The eigenvalue which tends to $\hbar \delta$ when $\Omega \to 0$ can thus be written:

$$\hbar \left( \delta + \delta_g - i \gamma_g / 2 \right)$$

where $\delta_g$ et $\gamma_g$ are real

- $\delta_g$: Displacement of the state $g$
- $\gamma_g$: Departure rate from the state $g$. Width of $g$

- The interaction with the incident wave displaces $g$
- $\delta_g$ is called « light shift »

- This interaction also « contaminates » the stable state $g$ by the unstable state $e$ which gets a lifetime $1/\gamma_g$.
- $\gamma_g$ is the absorption rate of a photon
Weak intensity limit \( \Omega \ll |\delta| \) or \( \Gamma \)

A perturbative calculation of \( \delta_g \) et \( \gamma_g \) is then possible and gives:

\[
\delta_g = \Omega^2 \frac{\delta}{4\delta^2 + \Gamma^2} \quad \gamma_g = \Omega^2 \frac{\Gamma}{4\delta^2 + \Gamma^2}
\]

Variations with the light intensity \( I_L \)

\( \Omega^2 \) is proportional to \( \langle N \rangle \), thus to \( I_L \)

\( \gamma_g \) and \( \delta_g \) are thus proportional to \( I_L \)

Variations with the detuning \( \delta \)

Limit \( |\delta| \gg \Gamma \)

\[
|\delta_g| \propto \frac{I_L}{4\delta} \gg \gamma_g \propto \frac{I_L \Gamma}{4\delta^2} \propto \left| \delta_g \right| \frac{\Gamma}{\delta}
\]
Semiclassical interpretation

The atomic dipole is driven by the monochromatic field and has a component in phase with the field and a component in quadrature related to the field by a dynamic polarizability $\alpha(\omega)$.

The component in quadrature with the field absorbs energy. It varies with $d$ as an absorption curve. It is this component which is responsible for the absorption rate.

The component in phase with the field gives rise to a polarization energy. It varies with $d$ as a dispersion curve. It is this component which is responsible for the light shift. This effect is analogous to the Stark effect describing the interaction of a static electric field with the static dipole that it induces. The light shift $\delta_g$ is often called for that reason « dynamical Stark shift ». 
Perturbation of the excited state e

In the weak intensity limit, the same perturbative calculation as above shows that the excited state e is shifted by an amount

$$\delta_e = -\delta_g$$

and that its width $\Gamma$ changes:

$$\Gamma \rightarrow \gamma_e = \Gamma - \gamma_g$$

The light shifts of g and e are thus equal in absolute value but have opposite signs.

The unstable state e is contaminated by the stable state g and its radiative instability decreases.
Case of a degenerate ground state

For example, the angular momentum $J_g$ of $g$ is different from 0 and there are several Zeeman sublevels $M_g$ in $g$. The light shifts generally depend on the polarization of the light and vary from one Zeeman sublevel to another.

Simple example of a transition $1/2 \leftrightarrow 1/2$ (case of $^{199}$Hg)

\[
\begin{align*}
M_e = -1/2 & \quad \text{A } \sigma_- \text{ excitation only displaces the sublevel } M_g = -1/2 \\
M_e = +1/2 & \quad \text{A } \sigma_+ \text{ excitation only displaces the sublevel } M_g = +1/2 \\
\end{align*}
\]

For more complicated transitions, see:
High intensity limit

With laser sources, light shifts can be much larger, in the GHz range. The Rabi frequency $\Omega$ can be much larger than $\Gamma$.

The eigenstates of $H_{\text{eff}}$ in $E(N)$ are no longer close to the unperturbed states $g,N+1$ and $e,N$. For example, if $\Omega \gg \Gamma$, they are equal to:

\[
|1(N)\rangle = \left(\frac{1}{\sqrt{2}}\right)[|g,N+1\rangle + |e,N\rangle]
\]

\[
|2(N)\rangle = \left(\frac{1}{\sqrt{2}}\right)[|g,N+1\rangle - |e,N\rangle]
\]

with eigenvalues:

\[
\pm \frac{\hbar \Omega_{N+1}}{2} - i\frac{\hbar \Gamma}{4}
\]

At resonance ($\delta = 0$), the real parts of the 2 eigenvalues of $H_{\text{eff}}$ are equal as long as $\Omega \ll \Gamma$. When $\Omega > \Gamma$, these 2 real parts are different. One can no longer describe the behavior of the system in terms of light shifts, but rather in terms of doublets of dressed states.

These doublets are at the origin of effects like the Mollow triplet or the Autler Townes doublet.
OUTLINE

1 - Light shifts and absorption rates. Theory

2 - Experimental observations

3 - Perturbation of the field. Absorption and dispersion

4 - Using light shifts for manipulating atoms and fields
First experimental studies

Optical pumping of $^{199}$Hg on the transition $1/2 \leftrightarrow 1/2$ with a discharge lamp (no laser sources at that time!) filled with the isotope $^{204}$Hg which resonantly excites the transition $F = 1/2 \leftrightarrow F = 1/2$.

Optical detection of the magnetic resonance line between the 2 Zeeman sublevels $M_g = \pm 1/2$ of the ground state $g$ of $^{199}$Hg. Very narrow line because relaxation times in $g$ are very long.

One adds a second perturbing light beam with a source filled with another isotope ($^{201}$Hg) in order to have a non resonant excitation ($\delta \neq 0$). This second beam is filtered with a cell filled with $^{204}$Hg in order to eliminate all resonant frequencies from the second beam.

The intensity of the second light beam, coming also from a discharge lamp is weak, so that one can apply the results derived above for the weak intensity limit. The light is not monochromatic, but, in a second order perturbation treatment, one can add independently the shifts produced by the various frequency components.
Depending whether the second beam has a $\sigma_+$ or $\sigma_-$ polarization, it displaces only the sublevel $M_g = -1/2$ or $M_g = +1/2$, changing in this way the Zeeman splitting between these 2 sublevels (the Zeeman is very small compared to the detuning $d$ of the light beam).

The magnetic resonance curve in $g$, optically detected with the first beam thus undergoes a light shift whose sign depends on the polarization of the second perturbing beam.

Very small light shifts can be detected if they are not too small compared to the width of the magnetic resonance curve in $g$. 
Experimental observation

First beam
Pumping beam $\sigma_+$

Atoms

Second perturbing beam
$\sigma_+$ or $\sigma_-$ polarized

Possibility to detect very small light shifts on very narrow magnetic resonance curves (relaxation times are very long in $g$)

C. Cohen-Tannoudji,
C.R.Acad.Sci. 252, 394 (1961)
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To study the perturbation of the field, we suppose now that the atom is put in a real cavity and we keep the \(N\)-dependence of the Rabi frequency.

There are actually experiments of this type in a research field called « Cavity Quantum Electrodynamics »

The field is supposed to be initially in a coherent state. One introduces in the cavity an atom whose frequency \(\omega_0\) is close to the frequency \(\omega\) of the field

How are the amplitude and the phase of the field perturbed by the atom-field interaction?

How do these effects vary with the detuning \(\delta = \omega - \omega_0\)?
N dependence of the perturbation

We keep now the N-dependent Rabi frequency appearing in the effective Hamiltonian $H_{\text{eff}}$ governing the reduced evolution in the manifold $\mathcal{E}_{N+1}$

In the perturbative limit, which is realized when $N$ is sufficiently small, the eigenstates of $H_{\text{eff}}$ are very close to $g,N+1$ and $e,N$, and the state $g,N+1$ undergoes a shift $\delta_{g}$ and a broadening $\gamma_{g}$, which depend on $N$ and which will be denoted here $\delta_{N}$ et $\gamma_{N}$.

$$\delta_{N} = (N+1) \Omega_{0}^{2} \frac{\delta}{4 \delta^{2} + \Gamma^{2}} \quad \gamma_{N} = (N+1) \Omega_{0}^{2} \frac{\Gamma}{4 \delta^{2} + \Gamma^{2}}$$

The state $e,N$ undergoes a shift $-\delta_{N}$.
Splitting between dressed states

\[ E_{g,N+1} - E_{g,N} = \hbar (\omega + \delta_0) \]

\[ E_{e,N} - E_{e,N-1} = \hbar (\omega - \delta_0) \]
The separation between the states g,N+1 (or e,N) displaced by light correspond to a new frequency of the field

Atom in state g \[ \omega \rightarrow \omega + \delta_0 \]
Atom in state e \[ \omega \rightarrow \omega - \delta_0 \]

The field-atom interaction thus displaces the frequency of the intracavity field by opposite amounts depending whether the atom is in g or in e.

If this interaction lasts for a time T, the oscillation of the field accumulates a phase shift \( \Phi \) with respect to the free oscillation in the absence of interaction

Atom in state g \( \Phi = + \delta_0 T \)
Atom in state e \( \Phi = - \delta_0 T \)
A light beam passing through a medium of length $L$ undergoes a phase shift proportional to $L$ and to the real part of the index of refraction.

Near a resonance of the atoms of the medium, the real part of the index of refraction varies like a dispersion curve as a function of the frequency of the field. This is what is called « anomalous dispersion ».

The effect studied here for an intracavity field is of the same nature. It involves a single atom. The phase shift increases with time and not in space. It varies like a dispersion curve as a function of the field frequency.

$$\phi = \Omega_0^2 T \frac{\omega - \omega_0}{4(\omega - \omega_0)^2 + \Gamma^2}$$
Damping of the field

Atom initially in g + field in a coherent state $\alpha$

$$|\psi(0)\rangle = |g\rangle \otimes |\alpha\rangle = \sum_{N=0}^{\infty} \frac{\alpha^N}{\sqrt{N!}} e^{-|\alpha|^2/2} |g,N\rangle$$

In the perturbative limit, each state $g,N$ evolves with an energy:

$$\tilde{E}_N = N\hbar\omega + \hbar\delta_N - i\hbar\gamma_N / 2 = N\hbar(\omega + \delta_0 - i\gamma_0 / 2)$$

After a time $t$, the initial state becomes:

$$|\psi(t)\rangle = \sum_{N=0}^{\infty} \frac{\alpha^N}{\sqrt{N!}} e^{-|\alpha|^2/2} \exp[-i N (\omega + \delta_0 - i\gamma_0 / 2)t] |g,N\rangle$$

$$\propto |g\rangle \otimes |\alpha \exp[-i(\omega + \delta_0 - i\gamma_0 / 2)t]\rangle$$

The field is still in a coherent state, evolving at the frequency $\omega + \delta_0$, and has an amplitude damped with a rate $\gamma_0/2$, where:

$$\gamma_0 = \Omega_0^2 \frac{\Gamma}{4(\omega - \omega_0)^2 + \Gamma^2}$$

This damping rate varies with the detuning as an absorption curve. Effect analogous to the one described by the imaginary part of the index of refraction.
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Light shifts: a tool for manipulating atoms and fields

Light shifts are a perturbation for high resolution spectroscopy since they change the atomic frequencies that one tries to measure. An extrapolation to zero light intensity of the position of the resonances has to be done for extracting the unperturbed frequency.

Light shifts are however useful for creating potential wells, potential barriers for neutral atoms, spatially periodic arrays of potential wells whose parameters can be fully controlled.

The depth of these wells, the height of these barriers are small. But they are large enough for trapping and reflecting the ultracold atoms which can now be obtained by laser cooling methods.

The field in a cavity can be also manipulated by sending atoms in this cavity.
Laser traps

Spatial gradients of light shifts

Focused laser beam with a red detuning \((\omega_L < \omega_A)\)

The light shift \(\delta E_g\) of the ground state \(g\) is negative and its absolute value is maximum at the focus

Attractive potential well in which neutral atoms can be trapped if they are slow enough

S. Chu, J. Bjorkholm, A. Ashkin, A. Cable, P.R.L. 57, 314 (1986)

Other types of traps using radiation pressure forces of polarized waves and magnetic field gradients will be described in subsequent lectures (magneto-optical traps).
Mirrors for atoms

Total reflection of a laser beam giving rise to an evanescent wave

If the laser is detuned to the blue, the light shift is positive and creates a potential barrier $U(z)$ near the surface

If the total energy $E$ of an atom falling on the surface is smaller than the height $U_0$ of the barrier, the atom bounces on the surface

R.J. Cook, R.K. Hill, 1982
Several ground state sublevels

In a laser standing wave, spatial modulation of the laser intensity and of the laser polarization

- Spatially modulated light shifts of $g_{\downarrow}$ and $g_{\uparrow}$ due to the laser light
- Correlated spatial modulations of optical pumping rates $g_{\downarrow} \leftrightarrow g_{\uparrow}$

The atom is always running up potential hills (like Sisyphus)!
Very efficient cooling leading to temperatures in the mK range
Optical lattices for neutral atoms

Periodic array of potential wells created by the light shifts associated with a detuned laser standing wave. Analogy with a box for eggs.

Lattice of atoms trapped in a periodic potential. Analogy with a crystal.

Important differences with a true crystal

- The spatial order does not result from interactions between atoms but from a light external potential
- Orders of magnitude are completely different for the spatial period: Å for the crystal, µm for the optical lattice.
- Possibility to vary the lattice parameters by varying the parameters of the laser standing wave.
A few other applications of optical lattices

Two counterpropagating waves with different frequencies $\omega_1$ and $\omega_2$

Give rise to a standing wave moving with a velocity

$$v = \frac{\omega_1 - \omega_2}{k_1 + k_2}$$

Useful for launching atoms with a velocity while keeping them cold by Sisyphus cooling

Frequency difference $\omega_1 - \omega_2$ increasing linearly with time

Give rise to an accelerated standing wave. In the rest frame of this wave atoms feel a constant inertial force

Useful for observing Bloch oscillations with neutral atoms

Varying the intensity of the standing wave

Provides a control of the tunneling rate between adjacent potential wells

Useful for observing the superfluid – Mott insulator transition
An application of the phase shift of the field produced by its interaction with an atom

We have seen above that the interaction of an intracavity field with an atom produces a phase shift of the field equal to $+\Phi$ if the atom is in $g$, $-\Phi$ if the atom is in $e$.

If the atom enters the cavity in the state
\[
\frac{1}{\sqrt{2}} [ |g\rangle + |e\rangle ]
\]

The state of the total system atom+field at the end of the interaction (when the atom leaves the cavity) is:
\[
\frac{1}{\sqrt{2}} [ |g,+\Phi\rangle + |e,-\Phi\rangle ]
\]

The 2 states $+\Phi$ et $-\Phi$ can be « mésoscopically » different.
Superposition of mesoscopically different states

Applying a $\pi / 2$ pulse on the atom transforms $e$ and $g$ into linear superpositions of $e$ and $g$, and the detection of the atom in $e$ or $g$ then prepares the field into a linear superposition of 2 states with opposite phases:

$$\frac{1}{\sqrt{2}} [ |+\Phi\rangle \pm |-\Phi\rangle ]$$

One can then study the evolution of this superposition state under the effect of dissipation (essentially due to the cavity losses and not from spontaneous emission from $e$).

How does the damping rate of the coherence between the states $+\Phi$ and $-\Phi$ vary as a function of their « distance »?

Problem of the decoherence

Atom in a linear superposition of 2 Rydberg states e and g ($\pi/2$ pulse) sent through a high Q cavity

Non resonant case (photons are not absorbed)

Strong coupling: the light shift created by a single photon is detectable)

The 2 light shifts of e and g are different

When the atom leaves the cavity, the linear superposition of e and g has undergone a light induced phase shift which depends on the number n of photons in the cavity, since light shifts are proportional to n

By measuring this phase shift with a second $\pi/2$ pulse, one can thus detect the presence of a single photon in the cavity without destroying it since there is no absorption

2012 Nobel Prize in Physics

For groundbreaking experimental methods that enable measuring and manipulation of individual quantum systems

Serge Haroche  David Wineland

Jean-Michel Raimond  Michel Brune
Kastler’s Nobel Prize in 1966
Conclusion

Importance of a long-term research driven by curiosity and by a desire to get a deep understanding of the physical phenomena
  Increase of the background of knowledge which can be at the origin of new fruitful ideas

Importance of keeping talented and experienced people a long enough time in a group in order to allow them to transmit their expertise to younger people

Importance of a strong coupling between research and teaching at a high level
  - Essential for stimulating and attracting young bright students
  - Teaching a subject provides a better understanding of the topics that one teaches and can suggest new ideas