Mesoscopic field state superpositions in Cavity QED: present status and perspectives

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Ein Bokek, February 21st 2005

Entangling single atoms with larger and larger fields: an exploration of the quantum classical boundary
1. A reminder about Cavity QED: a story about springs and spins

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1. A story about springs and spins
Cavity QED: a story about springs and spins

Towards Classical...

Mesoscopic field (n~10-100)

Microscopic field

2

1

n=0

The spin: 2-level atom

The spring: Cavity mode

Standard Hamiltonian

\[ H = \hbar \omega_{at} \frac{\sigma_z}{2} + \hbar \omega_c a \dagger a + \hbar \Omega \left[ \sigma_+ a + \sigma_- a \dagger \right] \]

Strong coupling

\[ \Omega \gg \frac{1}{T_c}, \frac{1}{T_a} \]

50kHz \sim 1kHz \sim 30Hz

\[ H = \hbar \omega \sigma_z + \hbar \omega_{ca} a \dagger a + \hbar \Omega \sigma \]

\[ \left[ \begin{array}{c}
    \sigma_+ \\
    \sigma_- \\
    \sigma_z \\
  \end{array} \right] \]

\[ T_c, T_a \]
Coherent field state

Superposition of photon number state with Poissonian distribution:

\[ |\alpha\rangle = \sum_n C_n |n\rangle \quad , \quad C_n = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} , \]

\[ \bar{n} = |\alpha|^2 \quad , \quad P(n) = |C_n|^2 = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \]

Phase space representation

Produced by coupling for a short time cavity to a classical source
Cavity QED set up

Oven
Circular Rydberg atom preparation (principal quantum number 51 or 50)
Ramsey pulses source
High Q Cavity
Source for photon injection in cavity
Field ionization detector
2. Microscopic atom-field entanglement in Cavity QED (atom and field behave as qubits)
Non resonant atom-field coupling: atomic light shift

Non-resonant atom-field coupling

Vacuum Rabi frequency \( \Omega \)
Atom-field detuning \( \delta \)

Detecting these shifts without « looking » inside the box:

Measure atomic coherence phase shift during the atom box crossing time by Ramsey interferometry!

Amounts to non-destructive (QND) detection of single photon
How to realize a $\pi$ atomic phase shift per photon? Dispersive and resonant methods

$\Phi = \frac{\Omega^2 (2n + 1)}{4\delta} t \rightarrow$

$\Phi = \pi$ per photon if $\frac{\Omega^2 t}{\delta} = 2\pi$

Dispersive coupling

Resonant atom-cavity interaction on e-g transition

$|g, 1\rangle \xrightarrow{\Omega t = 2\pi} e^{i\pi} |g, 1\rangle$

$2\pi$ Rabi rotation in 1 photon induces

$|i, 1\rangle \xrightarrow{\Omega t = 2\pi} |i, 1\rangle$

$\pi$ phase shift of atom state (like spin $1/2$)

Works only in the 0-1 photon subspace
Quantum Non Demolition measurement of a photon


Phases adjusted so that atom exits in one state if there is 0 photon in C, in the other if there is 1 photon (π phase shift in a one photon field)

QND method with atom acting as a meter measuring the field: quantum gate operation

If field is in a superposition of photon states, entanglement created:

$$|e\rangle \otimes \left[ c_0 |0\rangle + c_1 |1\rangle \right] \rightarrow c_0 |e,0\rangle + c_1 |g,1\rangle$$
3. Mesoscopic atom field entanglement in dispersive regime
Exchanging the roles of matter and field: Use mesoscopic coherent field for a QND measurement of atoms with 100% efficiency

An atom in level $e$ or $g$ dephases coherent field by

$$\Phi = \pm \frac{\Omega^2}{4 \delta} t$$

(single atom index)

Measuring the phase of the field after the atom has crossed the cavity reveals the state of the atom (or the absence of atom) in a non-destructive way.

In order to measure the field phase distribution, use an homodyne method: mix field with a reference with variable phase and measure resulting field intensity.
Field phase distribution measurement

How to measure a coherent field phase-shift?

Cavity QED Homodyne method

Injection of a coherent field $|\alpha\rangle$

Second injection $|-\alpha e^{i\phi_S}\rangle$

Resulting field $|\alpha(1-e^{i\phi_S})\rangle$

Back to the vacuum state $\phi_S = 0$

A probe atom is sent in $|g\rangle$

- Field in the vacuum state $P_g \approx 1$
- Field in an excited state $P_g \approx 1/2$

$P_g(\phi_S) = a$ signal to measure the field phase distribution

Field phase-shifted by $\phi$ $\rightarrow$ Maximum displaced by $\phi$
Phase-shifting by single index atom

Signals conditioned to detection of index atom in e or g

Macroscopic single atom effect
Phase-shifts with opposite signs for index atom in e and g

Probe atom
Coherent field source (used twice)
Index atom
Atom detector

P. Maioli et al, PRL to be published

\[
v = 200 \text{ m/s} \\
\delta = 50 \text{ kHz} \\
n = 29 \text{ photons} \\
\phi_0 = 39^\circ
\]
Coupling the field to atom in state superposition: dispersive atom-field entanglement

Atom prepared in symmetric superposition of $|e\rangle$ and $|g\rangle$ before crossing the cavity storing $|\alpha\rangle$.

The two components of the atomic state rotate the phase of the field by two opposite angles:

$$\chi = \frac{1}{4\delta} \int_{t_i}^{t_f} dt \, \Omega^2(t)$$

The final atom-cavity state is entangled: the system evolves towards two states with different phases, correlated to the two atomic states. Ideal example of pre-measurement: the field is a meter whose state «points towards» the atomic energy: a Schrödinger cat state situation.

$$|e\rangle|\alpha\rangle \rightarrow \frac{1}{\sqrt{2}} (|e\rangle + |g\rangle)|\alpha\rangle \rightarrow \frac{e^{-i\chi}}{\sqrt{2}} |e\rangle|\alpha\rangle e^{-i\chi} + \frac{1}{\sqrt{2}} |g\rangle|\alpha\rangle e^{i\chi}$$
4. Mesoscopic atom field entanglement in resonant regime
Resonant atom-field coupling: atomic eigenstates at classical limit

Two-level system \(|e\rangle;|g\rangle\) interacting with a resonant field

Rotating frame
Rotating wave approximation

Eigenstates of the Hamiltonian

\[
\begin{align*}
|+\rangle &= \frac{1}{\sqrt{2}} (|e\rangle + |g\rangle) \\
|\rangle &= \frac{1}{\sqrt{2}} (-|e\rangle + |g\rangle)
\end{align*}
\]

\[<\sigma_z>\]

Rabi oscillation at frequency \(\Omega_{cl}\)

Field in rotating frame \((\propto \Omega_{cl})\)
Classical Rabi oscillation: an interference effect

Evolution

\[ |e\rangle \rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\Omega_{cl}t/2} |+\rangle + e^{i\Omega_{cl}t/2} |-\rangle \right) \]

Detection in \{|e\>, |g\>\} basis
Coupling atomic eigenstates to mesoscopic coherent field (1st order in $n-n$)

$$\left(\frac{|e\rangle + |g\rangle}{\sqrt{2}}\right) \otimes |\alpha\rangle \rightarrow e^{-2i\bar{\varphi}(t)} |\psi_{+}^{at}(t)\rangle \otimes |\alpha_{+}(t)\rangle$$

$$\frac{1}{\sqrt{2}} \left(e^{-i\varphi(t)} |e\rangle + |g\rangle\right) \sim |\alpha e^{-i\varphi(t)}\rangle$$

$$\left(\frac{|e\rangle - |g\rangle}{\sqrt{2}}\right) \otimes |\alpha\rangle \rightarrow e^{2i\bar{\varphi}(t)} |\psi_{-}^{at}(t)\rangle \otimes |\alpha_{-}(t)\rangle$$

$$\frac{1}{\sqrt{2}} \left(e^{i\varphi(t)} |e\rangle - |g\rangle\right) \sim |\alpha e^{i\varphi(t)}\rangle$$

As in dispersive case, the atomic eigenstates induce opposite phase rotations on coherent fields. Field reacts back on atom and induces a correlated atomic dipole phase shift. Effect vanishes at classical limit ($n \rightarrow \infty$)

$$\varphi(t) = \frac{\Omega t}{4\sqrt{n}}$$

Resonant phase shift
Representation in phase space

Atomic state in the equatorial plane of the Bloch sphere

Coherent field in the Fresnel plane

Equatorial plane of the Bloch sphere

Phase correlation

Atomic dipole and field « aligned »
\[ |e, \alpha\rangle = \left( \frac{1}{\sqrt{2}} \right) (|+, \alpha\rangle + |-, \alpha\rangle) \rightarrow \]

\[ \frac{1}{\sqrt{2}} \left( e^{2i\phi(t)} |\alpha\rangle \hat{\psi}_{+}^{at}(t) \hat{\psi}_{+}^{at}(t), \alpha^{+}(t) \rangle + e^{2i\phi(t)} |\alpha\rangle \hat{\psi}_{-}^{at}(t), \alpha^{-}(t) \rangle \right) \]

\[ A \text{ microscopic object leaves its imprint on a mesoscopic one} \]

\[ \text{Schrödinger-cat situation} \]

\[ \text{"Size" of the cat} = D \]

\[ D = 2\sqrt{n} \sin \left( \frac{\Omega_0 t}{4\sqrt{n}} \right) \]

\[ \text{The field acts as a Which-Path detector} \]

\[ \text{Contrast of the Rabi oscillation} \]

\[ C(t) = |< \alpha_{+}(t) | \alpha_{-}(t) >| = e^{-D^2(t)} \]
Rabi oscillation in mesoscopic field collapses and revives as field components separate and recombine.

Atom and field disentangle: Rabi oscillation revives

Atom and field entangled: Rabi oscillation collapses

Complementarity in action!

At classical limit, collapse and revival times rejected to $t = \infty$
Evidence of phase splitting

\[ v = 335 \text{ m/s} \]
\[ t_{\text{int}} = 32 \mu\text{s} \]
\[ \bar{n} = 36 \]

Measured phase \( \varphi = 23^\circ \)

Expected value \( \varphi = \frac{\Omega_0 t_{\text{int}}}{4\sqrt{\bar{n}}} = 23^\circ \)

Experiment and theory in very good agreement.
5. Probing decoherence of mesoscopic superpositions
Fast decoherence of Schrödinger cat state

Entanglement with environment: complementarity again...

\[
\left| \alpha e^{-i\chi} \right\rangle \pm \left| \alpha e^{i\chi} \right\rangle \left| E_0 \right\rangle \xrightarrow{\text{entanglement with environment}} \left| \alpha e^{-i\chi} \right\rangle \left| E_{-\chi} \right\rangle \pm \left| \alpha e^{i\chi} \right\rangle \left| E_{\chi} \right\rangle
\]

Very quickly state superposition is turned into statistical mixture:

\[
\left| \alpha e^{-i\chi} \right\rangle \pm \left| \alpha e^{i\chi} \right\rangle \xrightarrow{\text{tracing over environment}} \rho = \left| \alpha e^{-i\chi} \right\rangle \langle \alpha e^{-i\chi} \rangle + \left| \alpha e^{i\chi} \right\rangle \langle \alpha e^{i\chi} \rangle
\]

Interference terms in photon probability distribution...

\[P^\text{coh}_\pm(n) = \frac{1}{2} \left| \langle n | \alpha e^{-i\chi} \rangle \pm \langle n | \alpha e^{i\chi} \rangle \right|^2 = e^{-n} \frac{n}{n!} \left( 1 \pm \cos 2n\chi \right)\]

...are washed out by decoherence

\[P^\text{Dec}_\pm(n) = \frac{1}{2} \left| \langle n | \alpha e^{-i\chi} \rangle \right|^2 + \left| \langle n | \alpha e^{i\chi} \rangle \right|^2 = e^{-n} \frac{n}{n!}\]

Experiment: Brune et al, P.R.L 77, 4887 (1996)

Mixture of even and odd cats

\[P^\text{Decoh}_+(n)\]
Demonstrating the cat state coherence with a single atom by observing Rabi oscillation revival

Observing the revival of Rabi oscillation would prove that the cat state produced during the collapse time was coherent...

...but the revival time is, at present time, too long compared to the cavity damping time...

Solution: induce revival at earlier time by echo technique
Test of coherence: quantum revivals induced by time reversal

Initial Rabi rotation, Collapse, and slow phase rotation

Apply short Stark pulse equivalent to $\pi$ rotation around Oz ($|+\rangle$ into $|->$)

Reversal of field

Recombination of field components induces revival of Rabi oscillation

A spin echo experiment

Observation of induced Rabi revival signals

Revivals up to $2T= 50 \mu s$ with $n=13$ photons

Evidence of « long lived » coherent mesoscopic superpositions of field states

T. Meunier et al, PRL, December 2004
How big a Schrödinger cat?

Competition between preparation time $t_{\text{int}}$ (time of flight of atom across cavity) and decoherence time $T_c/n$:

In early work (1996), $T_c \sim 150\mu s$ restricted $n$ to $\sim 3-5$ (dispersive experiment)

In more recent work (2000-2003), $T_c \sim 1\text{ms}$ and $n \sim 30$

Very recently (December 2004), we have obtained $T_c \sim 14\text{ms}$, opening the possibility to realize cat states with $n \sim 200-300$...
Conclusions and perspectives

Larger and longer lived cats (n in the hundreds) with better cavities

Non local field states in two cavities

Prepare and detect $|\alpha,0\rangle + |0,\alpha\rangle$
(similar to $|n,0\rangle + |0,n\rangle$)

Marry the strangeness of Schrödinger cats with non locality: an exploration of the quantum-classical boundary

Similar cat experiments with matter waves (BEC) instead of fields?
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