Multiple filamentation of intense laser beams

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Laser propagation in Kerr medium
Vector NL Helmholtz model (cw)

\[ \Delta E - \nabla(\nabla \cdot E) + k_0^2 E = -\frac{k_0^2}{\varepsilon_0 n_0} P_{NL} \]

\[ \nabla \cdot E = -\frac{1}{2} \nabla \cdot P_{NL} \]

\[ P_{NL} = \frac{4 \varepsilon_0 n_0 n_2}{1 + \gamma} \left[ (E \cdot E^*) E + \gamma (E \cdot E) E^* \right] \]

\[ E = (E_1, E_2, E_3) \]

<table>
<thead>
<tr>
<th>Kerr Mechanism</th>
<th>( \gamma )</th>
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</thead>
<tbody>
<tr>
<td>electrostriction</td>
<td>0</td>
</tr>
<tr>
<td>non-resonant electrons</td>
<td>0.5</td>
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<tr>
<td>molecular orientation</td>
<td>3</td>
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</tbody>
</table>
Simplifying assumptions

- Beam remains linearly polarized
  \[ \mathbf{E} = (E_1(x,y,z), 0, 0) \]
- Slowly varying envelope
  \[ E_1 = \psi(x,y,z) \exp(i k_0 z) \]
- Paraxial approximation
2D cubic NLS

\[ i \psi_z (z, x, y) + \Delta \psi + |\psi|^2 \psi = 0, \quad \Delta = \partial_{xx} + \partial_{yy} \]

\[ \psi (z = 0, x, y) = \psi_0 (x, y) \]

- Initial value problem in \( z \)
- Competition: \textbf{self-focusing} nonlinearity versus diffraction
- Solutions can become singular (\textbf{collapse}) in finite propagation distance \( z = Z_{cr} \) if their input power \( P \) is above a critical power \( P_{cr} \) (Kelley, 1965), i.e.,

\[ P = \int \left| \psi_0 \right|^2 dxdy \geq P_{cr} \]
Multiple filamentation (MF)

- If $P > 10P_{cr}$, a single input beam can break into several long and narrow filaments
- Complete breakup of cylindrical symmetry
Breakup of cylindrical symmetry

- Assume input beam is cylindrically-symmetric
  \[ \psi_0(x, y) = \psi_0(r), \quad r = \sqrt{x^2 + y^2} \]
- Since NLS preserves cylindrical symmetry
  \[ \psi_0(x, y) = \psi_0(r) \quad \Rightarrow \quad \psi(z, x, y) = \psi(z, r) \]
- But, cylindrical symmetry does break down in MF
- Which mechanism leads to breakup of cylindrical symmetry in MF?
Standard explanation (Bespalov and Talanov, 1966)

- Physical input beam has noise
  \[ \psi_0(x, y) = c \exp(-r^2)[1 + \text{noise}(x, y)] \]
- Noise breaks up cylindrical symmetry
- Plane waves solutions

\[ \psi = a \exp(i \alpha^2 z) \]

of the NLS are linearly unstable (MI)
- Conclusion: MF is caused by noise in input beam
Weakness of MI analysis

- Unlike plane waves, laser beams have finite power as well as transverse dependence
- Numerical support?
- Not possible in the sixties
G. Fibich, and B. Ilan
Optics Letters, 2001
and
Physica D, 2001
Testing the Bespalov-Talanov model

- Solve the NLS

\[ i \psi_z(z, x, y) + \Delta \psi + \left| \psi \right|^2 \psi = 0 \]

- Input beam is Gaussian with 10% noise and \( P = 15P_{cr} \)

\[ \psi_0(x, y) = c \exp(-\rho^2)[1 + 0.1 \cdot \text{rand}(x, y)] \]
- Blowup while approaching a symmetric profile
- Even at $P=15P_{cr}$, noise does not lead to MF in the NLS
Model for noise-induced MF

NLS with saturating nonlinearity (accounts for plasma defocusing)

\[ i\psi_z(z, x, y) + \Delta \psi + \frac{|\psi|^2}{1 + \epsilon|\psi|^2} \psi = 0, \]

Initial condition: cylindrically-symmetric Gaussian profile + noise

\[ \psi_0(x, y) = c \exp(-\rho^2)[1 + noise(x, y)] \]
Typical simulation (P=15Pcr)

Ring/crater is unstable
Noise-induced MF

- MF pattern is random
- No control over number and location of filaments
- Disadvantage in applications (e.g., eye surgery, remote sensing)
Can we have a deterministic MF?
Vectorial effects and MF

- NLS is a scalar model for linearly-polarized beams
- More comprehensive model – vector nonlinear Helmholtz equations for $\mathbf{E} = (E_1, E_2, E_3)$
- Linear polarization state $\mathbf{E} = (E_1, 0, 0)$ at $z=0$ leads to breakup of cylindrical symmetry
- Preferred direction
- Can this lead to a deterministic MF?
Linear polarization - analysis

- vector Helmholtz equations for \( E = (E_1, E_2, E_3) \)

\[
\Delta E - \nabla(\nabla \cdot E) + k_0^2 E = -\frac{k_0^2}{\varepsilon_0 n_0^2} P_{NL}
\]

\[
\nabla \cdot E = -\frac{1}{\varepsilon_0 n_0^2} \nabla \cdot P_{NL}
\]

\[
P_{NL} = \frac{4 \varepsilon_0 n_0 n_2}{1+\gamma} \left( (E \cdot E^*) E + \gamma (E \cdot E) E^* \right)
\]
Derivation of scalar model

- Small nonparaxiality parameter

\[ f = \frac{1}{k_0 r_0} = \frac{\lambda}{2\pi r_0} \ll 1 \]

- Linearly-polarized input beam

\[ E = (E_1,0,0) \text{ at } z=0 \]
NLS with vectorial effects

- Can reduce vector Helmholtz equations to a scalar perturbed NLS for $\psi = |E_1|$: 

$$i \psi_z + \Delta \psi + \left| \psi \right|^2 \psi =$$

$$- f^2 \frac{1}{4} \psi_{zz}$$

$$- f^2 \left[ \frac{1+6\gamma}{1+\gamma} \left| \psi_x \right|^2 \psi + \left( \psi_x \right)^2 \psi^* + \frac{1+2\gamma}{1+\gamma} \left( \left| \psi \right|^2 \psi_{xx} + \psi^2 \psi_{xx} \right) \right]$$
Vectorial effects and MF

- Vectorial effects lead to a deterministic breakup of cylindrical symmetry with a preferred direction
- Can it lead to a deterministic MF?
Simulations

- Cylindrically-symmetric linearly-polarized Gaussian beams \( \psi_0 = c \exp(-r^2) \)
- \( f = 0.05, \ \gamma = 0.5 \)
- No noise!
\[ P = 4P_{cr} \]

Ring/crater is unstable

Splitting along x-axis
$P = 10P_{cr}$
$P = 20P_{cr}$

more than two filaments
What about circular polarization?
G. Fibich, and B. Ilan
Physical Review Letters, 2002
and
Physical Review E, 2003
Circular polarization and MF

- Circular polarization has no preferred direction
- If input beam is cylindrically-symmetric, it will remain so during propagation (i.e., no MF)
- Can small deviations from circular polarization lead to MF?

\[ E = (E_+, E_-, E_3), \quad e = \frac{E_-}{E_+} << 1 \]
Standard model (Close et al., 66)

\[ E_+ = \exp(i k_0 z) y_+ , \quad E_- = \exp(i k_0 z) y_- \]

\[
i (y_+) + D y_+ + \frac{1}{1 + g} y_+ ^2 + (1 + 2g) | y_- |^2 y_+ = 0
\]

\[
i (y_-) + D y_- + \frac{1}{1 + g} y_- ^2 + (1 + 2g) | y_+ |^2 y_- = 0
\]

- Neglects \( E_3 \) while keeping the coupling to the weaker \( E_- \) component
Circular polarization - analysis

- vector Helmholtz equations for $E = (E_+, E_-, E_3)$

\[
\Delta E - \nabla (\nabla \cdot E) + k_0^2 E = -\frac{k_0^2}{\varepsilon_0 n_0^2} P_{NL}
\]

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\nabla \cdot E = -\frac{1}{\varepsilon_0 n_0^2} \nabla \cdot P_{NL}
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\[
P_{NL} = \frac{4 \varepsilon_0 n_0 n_2}{1 + \gamma} \left( (E \cdot E^*) E + \gamma (E \cdot E) E^* \right)
\]
Derivation of scalar model

- Small nonparaxiality parameter

\[ f = \frac{1}{k_0 r_0} = \frac{\lambda}{2\pi r_0} \ll 1 \]

- Nearly circularly-polarized input beam

\[ e = \frac{E_0}{E_+} \ll 1 \quad \text{at } z=0 \]
NLS for nearly circularly-polarized beams

- Can reduce vector Helmholtz equations to the new model:

\[
i(y^+)_z + D_y^+ + \frac{1}{1+g} |y^+|^2 y^+ = - \frac{1+2g}{1+g} |y^-|^2 y^- - \frac{f^2}{4} (y^+)_z^2
\]

\[
- \frac{f^2}{2(1+g)^{3/2}} \cdot \hat{y} + |y^+|^2 y^+ + (y^+)^2 y^+ + |y^-|^2 D_y^+ + (y^+)^2 D_y^+
\]

\[
i(y^-)_z + D_y^- + \frac{1+2g}{1+g} |y^-|^2 y^- = 0
\]

- Isotropic to O(f^2)
Circular polarization and MF

- New model is isotropic to $O(f^2)$
- Neglected symmetry-breaking terms are $O(\varepsilon f^2)$
- Conclusion – small deviations from circular polarization unlikely to lead to MF
Back to Linear Polarization
Testing the vectorial explanation

- Vectorial effects breaks up cylindrical symmetry while inducing a preferred direction of input beam polarization.
- If MF pattern is caused by vectorial effects, it should be deterministic and rotate with direction of input beam polarization.
A. Dubietis, G. Tamosauskas, G. Fibich, and B. Ilan, Optics Letters, 2004
First experimental test of vectorial explanation for MF

- Observe a deterministic MF pattern
- MF pattern does not rotate with direction of input beam polarization
- MF not caused by vectorial effects
- Possible explanation: collapse is arrested by plasma defocusing when vectorial effects are still too small to cause MF
So, how can we have a deterministic MF?
Ellipticity and MF

- Use elliptic input beams
  \[ \psi_0 = c \exp\left(-\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) \]
- Deterministic breakup of cylindrical symmetry with a preferred direction
- Can it lead to deterministic MF?
Possible MF patterns

- Solution preserves the symmetries $x \rightarrow -x$ and $y \rightarrow -y$
Simulations with elliptic input beams

- NLS with NL saturation

\[ i \psi_z (z, x, y) + \Delta \psi + \frac{|\psi|^2}{1 + 0.005|\psi|^2} \psi = 0, \]

- elliptic initial conditions \( \psi_0 = c \exp(-x^2/a^2-y^2/b^2) \)
- \( P = 66P_{cr} \)
- No noise!
- $e = 1.09$
- central filament
- filament pair along minor axis
- filament pair along major axis
- $e = 2.2$
- central filament
- quadruple of filaments
- very weak filament pair along major axis
All 4 filament types observed numerically
Experiments

- Ultrashort (170 fs) laser pulses
- Input beam ellipticity is $b/a = 2.2$
- "Clean" input beam
- Measure MF pattern after propagation of 3.1cm in water
P = 4.8P_{cr}

- Single filament
$P = 7P_{cr}$

- Additional filament pair along major axis
P=18Pcr

- Additional filament pair along minor axis
$P=23 \, P_{cr}$

- Additional quadruple of filaments
All 4 filament types observed experimentally
Rotation Experiment

MF pattern rotates with orientation of ellipticity
2nd Rotation Experiment

MF pattern does not rotate with direction of input beam polarization
Dynamics in z
G. Fibich, S. Eisenmann, B. Ilan, and A. Zigler,
Optics Letters, 2004
Control of MF in atmospheric propagation

- Standard approach: produce a clean(er) input beam
- New approach: Rather than fight noise, simply add large ellipticity
- Advantage: easier to implement, especially at power levels needed for atmospheric propagation (>10GW)
Ellipticity-induced MF in air

- Input power 65.5GW (~20P_{cr})
- Noisy, elliptic input beam

Typical

Average over 100 shots
Experimental setup

- Control astigmatism through lens rotation
MF pattern after 5 meters in air

- Strong central filament
- Filament pair along minor axis
- Central and lower filaments are stable
- Despite high noise level, MF pattern is quite stable
- Ellipticity dominates noise
Typical average over 1000 shots.
Control of MF - position

- $\varphi = 0^0$ : one direction (input beam ellipticity)
- $\varphi = 20^0$ : all directions (input beam ellipticity + rotation lens)
Control of MF – number of filaments

- $\varphi = 0^\circ$ 2-3 filaments
- $\varphi = 20^\circ$ single filament
Ellipticity-induced MF is generic

- cw in sodium vapor (Grantham et al., 91)
- 170fs in water (Dubietis, Tamosauskas, Fibich, Ilan, 04)
- 200fs in air (Fibich, Eisenmann, Ilan, Zigler, 04)
- 130fs in air (Mechain et al., 04)
- Quadratic nonlinearity (Carrasco et al., 03)
Summary - MF

- Input beam ellipticity can lead to deterministic MF
- Observed in simulations
- Observed for clean input beams in water
- Observed for noisy input beams in air
- Ellipticity can be "stronger" than noise
Theory needed

- Currently, no theory for this high power strongly nonlinear regime (P >> P_cr)
- In contrast, fairly developed theory when P = O(P_cr)
- Why? the "Townes profile" attractor

\[
|\psi(z, x, y)| \sim \frac{1}{L(z)} R \left( \frac{r}{L(z)} \right) \quad \text{as} \quad z \to Z_{cr}
\]