Entangled lithography and microscopy

FRISNO8
Ein Bokek, Israel

February 21-25, 2005

Lorenzo Maccone
QUIT - Quantum Information Theory Group,
Dip. di Fisica "A. Volta", PAVIA-ITALY

Vittorio Giovannetti
NEST-INFM & Scuola Normale Superiore,
PISA-ITALY

check out www.qubit.it

maccone@unipv.it

Rayleigh diffraction bound

The resolution achievable is ~ the wavelength of the light employed

The problem in high resolution imaging:

- We want to use high frequency photons to achieve high res.

- High freq photons behave badly: lenses not good and high resolution lithographic masks are needed

One would like to do
IMAGING with LOW FREQUENCY PHOTONS and
LITHOGRAPHIC ETCHING (or microscopy scanning) with
HIGH FREQ.

Is it possible?

Basic idea: frequency entanglement

YES!!

Use unbalanced SPDC.

The frequency entanglement allows to perform high resolution tasks, in the high frequency signal beam controlled by a low resolution idler!

However:

We need to use pulsed pumping (for high repetition rate) and avoid spatial phase-matching difficulties!

Conventional SPDC: no good!

... now it doesn't sound all that easy, does it?

Solution

We can use **extended phase matching** conditions (well suited for pulsed pumping)

and

transform frequency correlations into spatial correlations (to avoid spatial phase-matching problems).

The input state is $\int d\omega\, \mathcal{E}(\omega)|\Omega_s + \alpha\omega\rangle_s|\Omega_i + \omega\rangle_i$ with $\Omega_s \gg \Omega_i\ ,\quad \alpha \sim 1$

...more about the source later.

Previous schemes to employ entanglement

Quantum lithography and multi-photon imaging

Their basic idea:

Entangle N-photons ⇒ Reduce the de Broglie wavelength ⇒ increase the resolution N-fold.

Problems with these aproaches:
1. Difficult to entangle even N=2 photons

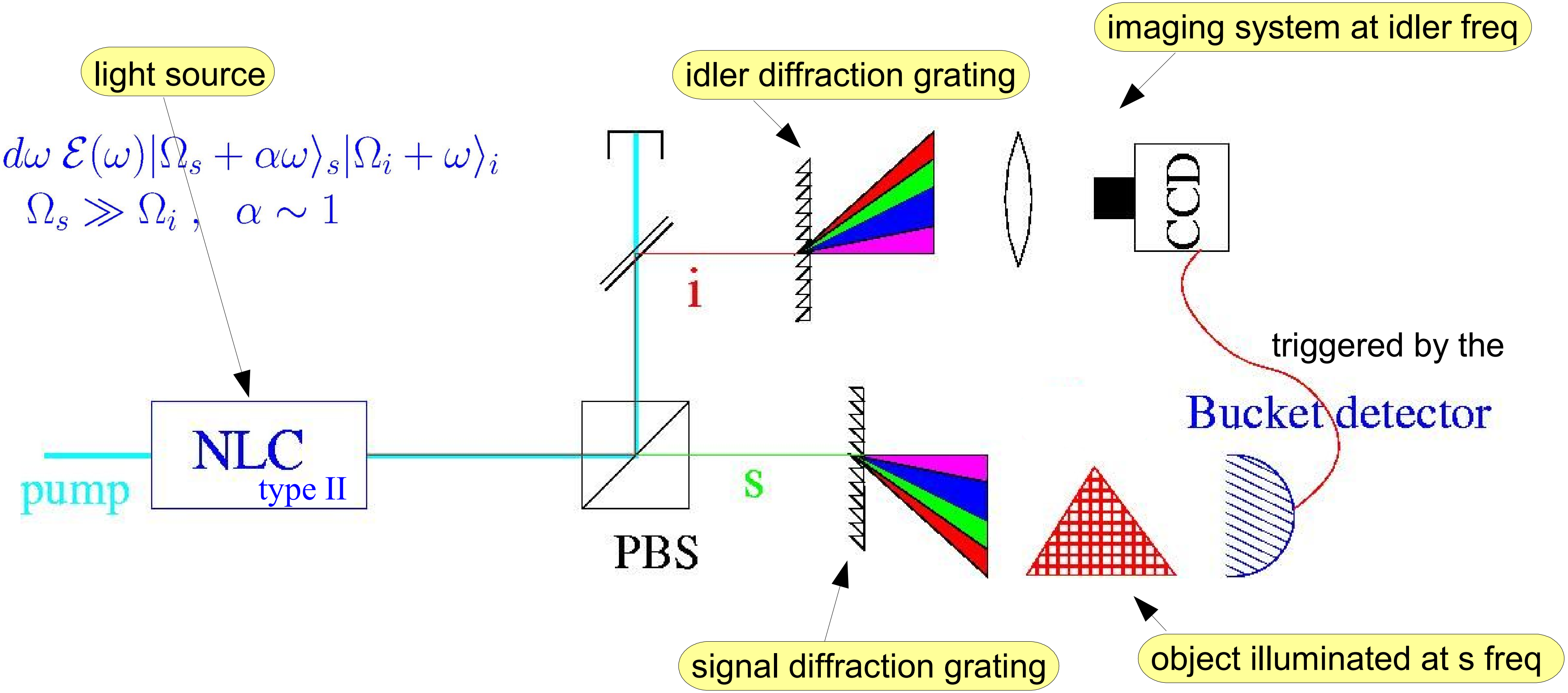
2. Can be used only with samples and photolithographic resists sensitive to N-photon (and NO less!) absorption.

3. For lithography it may be difficult to create arbitrary patterns (complex interference schemes needed).

Entangled microscopy

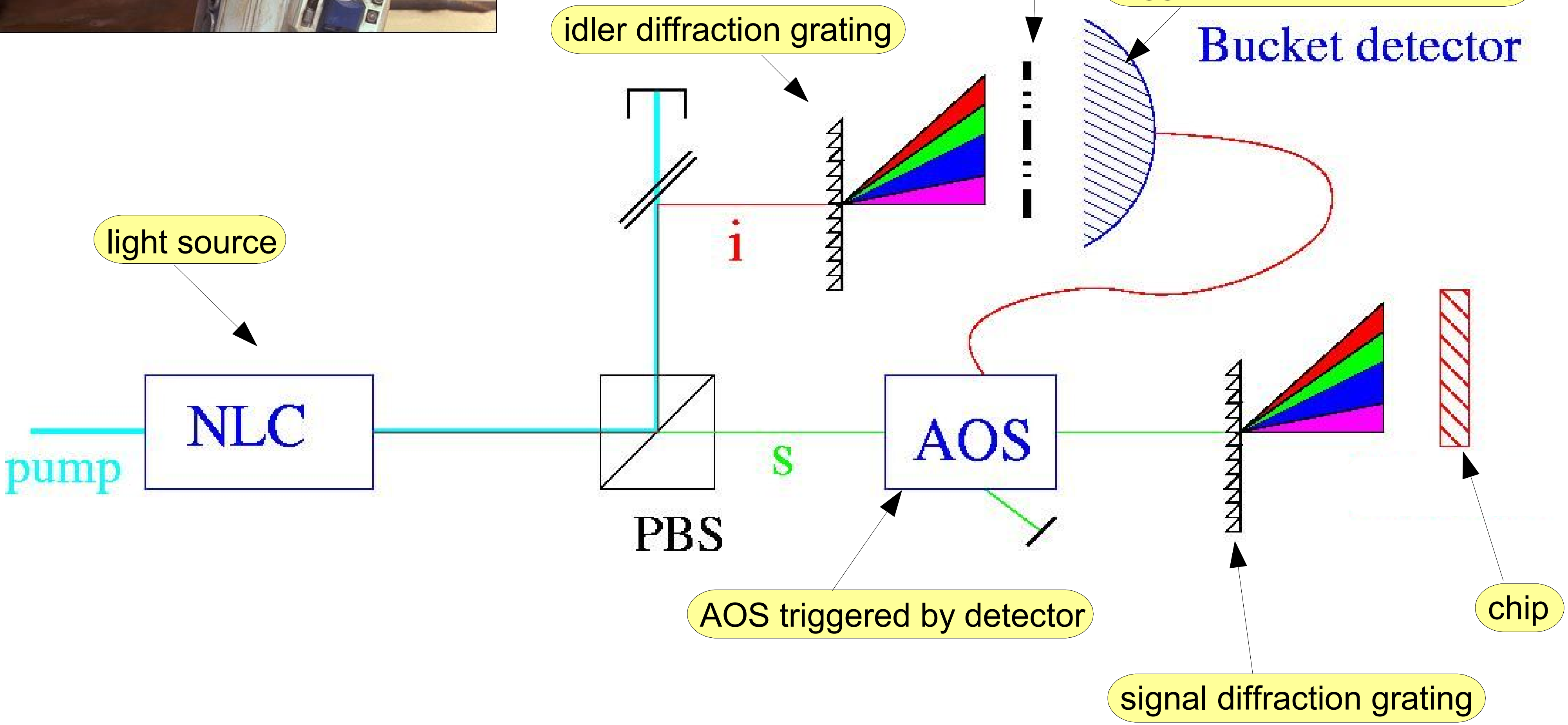
$$\int d\omega\, \mathcal{E}(\omega) |\Omega_s + \alpha\omega\rangle_s |\Omega_i + \omega\rangle_i$$

$$\Omega_s \gg \Omega_i\ , \quad \alpha \sim 1$$



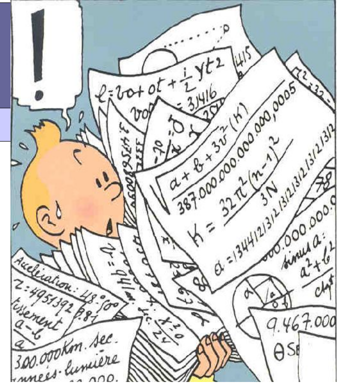
Repeat the procedure many times to accumulate photon statistics,
create a 'ghost image' with different frequencies

Entangled lithography



Repeat the procedure many times to accumulate photon statistics.

Entangled light source



Hamiltonian of SPDC $$H_I(t) = S \int_0^L dz\, \chi^{(2)} E_p^{(+)}(z,t) E_s^{(-)}(z,t) E_i^{(-)}(z,t) + \text{h.c.},$$

$\Downarrow$

Output state $$-\frac{i}{\hbar}\int_{t_0}^{t} dt' H_I(t)|0\rangle \propto \int_{t_0}^{t} dt' \int_0^L dz \int d\omega_p d\omega_s d\omega_i \mathcal{E}(\omega_p) e^{i(t\Delta\omega - z\Delta k)} a_s^\dagger(\omega_s) a_i^\dagger(\omega_i)|0\rangle$$

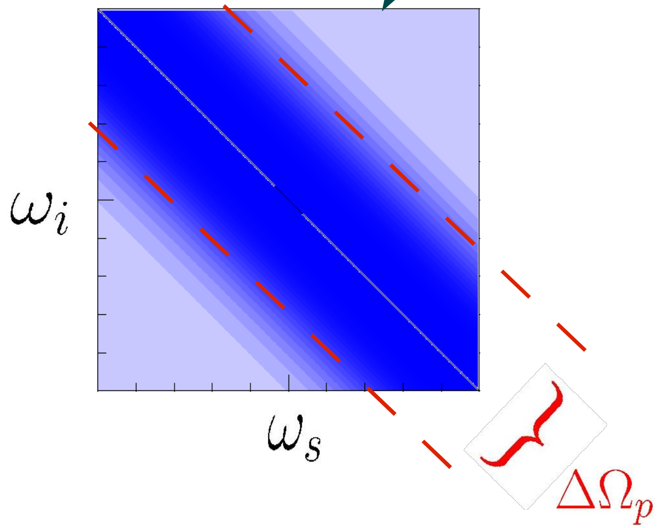
$$\Delta\omega \equiv \omega_p - \omega_i - \omega_s \ , \ \Delta k \equiv k_p(\omega_p) - k_s(\omega_s) - k_i(\omega_i)$$

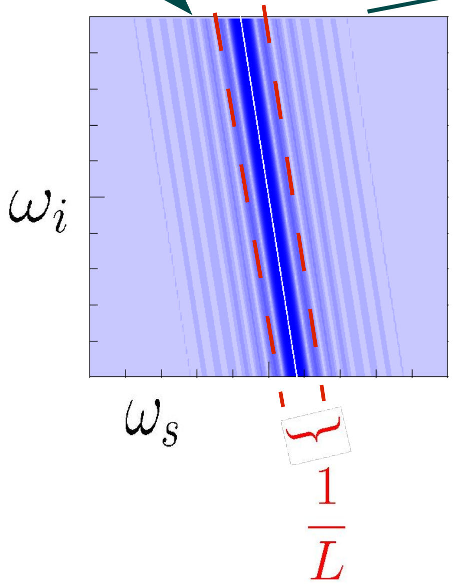
i.e. for $t \to \infty$

$$|\Psi\rangle = \int d\omega_s d\omega_i \underbrace{\mathcal{E}(\omega_s + \omega_i)}_{\text{pump spectral fn}} \underbrace{\frac{\sin(\Delta k\, L/2)}{\Delta k}}_{\text{phase matching fn}} |\omega_i\rangle_i |\omega_s\rangle_s$$

i.e., graphically...

$$|\Psi\rangle = \int d\omega_s d\omega_i \, \mathcal{E}(\omega_s+\omega_i) \frac{\sin(\Delta k\, L/2)}{\Delta k} |\omega_i\rangle_i |\omega_s\rangle_s$$





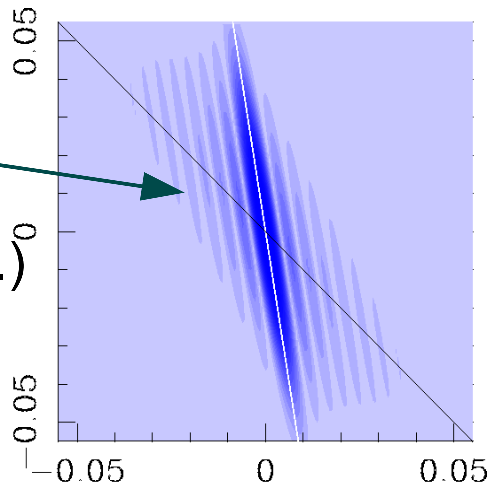
as can be seen by expanding $k_s(\omega)$ and $k_i(\omega)$:

$$\Delta k \simeq \gamma_s \omega_1 + \gamma_i \omega_2$$

$$\gamma_j \equiv \frac{\partial}{\partial \omega} k_p \Big|_{\omega=\Omega_s+\Omega_i} - \frac{\partial}{\partial \omega} k_j \Big|_{\omega=\Omega_j}$$

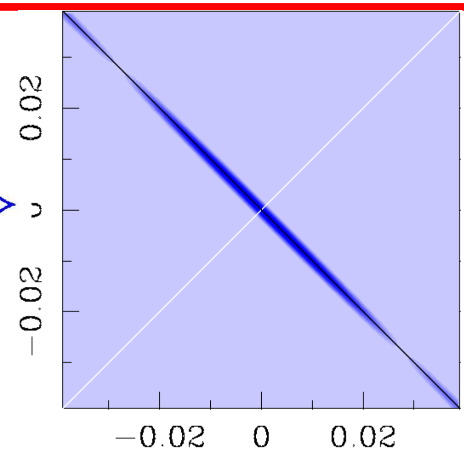
[$k_p(\Omega_p) = k_s(\Omega_s) + k_i(\Omega_i)$ enf. by p.m. cond.]

so that the product (i.e. the biphoton sp fn.)



Typically $\Delta\Omega_p \rightarrow 0$ $\Rightarrow$

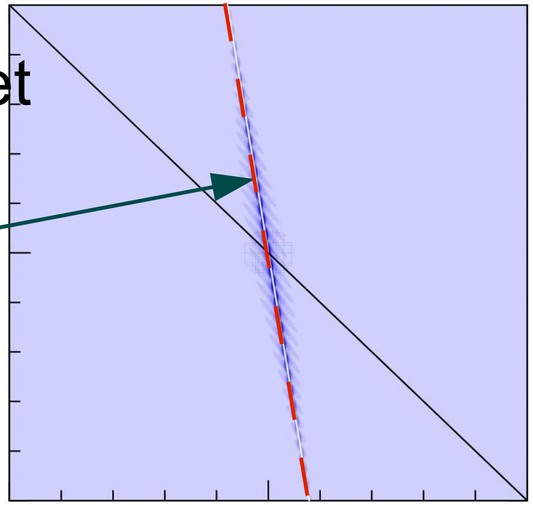
(i.e. the Gaussian pump $\longrightarrow \delta$)



f. ANTI-correlation

DB-STATE (cfr. PRL 88, 183602)

In contrast, by taking $L \longrightarrow \infty$ we get (the sinc $\longrightarrow$ δ)



where the symmetry axis can be tuned with the extended p.m. conditions.

$\Rightarrow$ for $L \rightarrow \infty$ we get a $\delta(\gamma_s\omega_1 + \gamma_i\omega_2)$ and perfect freq. entanglement:

an idler of freq. $\Omega_i + \omega_2$ corresponds to a signal $\Omega_s - \gamma_i\omega_2/\gamma_s$

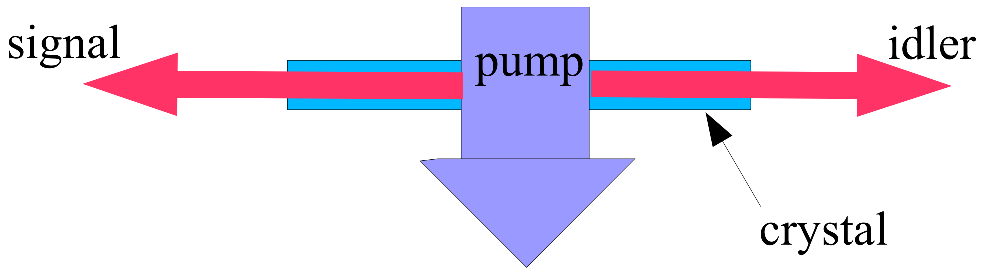
(tuning the ratio γ_i/γ_s , we can change the tilt of the symmetry axis, $\gamma_i/\gamma_s = -1$ corresponds to DB-state: perfect freq CORRELATION)

Extended P.M.

How to engineer the source crystals?

1) Periodic poling to customize the k-vector characteristics and 1^ derivatives of the crystals
   [at MIT, PP crystals are used to create DB state]

2) Counter-propagating pump, signal and idler (method proposed in PRA 67, 053810)



Angular resolution achievable

Diffraction gratings: freq. ent. ⟶ spatial ent.:

the diffraction angle: $\sin\theta = D/\omega$ ⇒ the state can be rewritten as

$$\int d\theta_s d\theta_i \frac{\cos\theta_s \cos\theta_i}{\sin^2\theta_s \sin^2\theta_i} A(\omega_s(\theta_s), \omega_i(\theta_i))\ |\theta_s\rangle_s |\theta_i\rangle_i$$

⇒ for perfect correlation ($L \to \infty$), an idler angle θ_i determined with resolution Δ_i corresponds to

a signal angle $\theta_s = \arcsin \dfrac{D_s}{\Omega_s - \alpha[\Omega_i - D_i/\sin(\theta_i)]}$, with resolution $\Delta_s \simeq 2\alpha \dfrac{\Omega_i}{\Omega_s} \Delta_i$

(calculated for small $\triangle$s around $\theta_i \simeq \pi/4$)

ie, an increase in angular resolution of the order of the freq ratio $\dfrac{\Omega_i}{\Omega_s}$
(for $\alpha \simeq 1$)

Discussion and problems

As a practical example, consider LBO pumped at 174nm. From the Sellmeier eqs, we find p.m. for signal at 198nm and idler at 1.41μm. In this case α=-3.3

Can we go to higher signal freq? It appears there are presently no non-linear crystal which work well for λ<150nm!

Is entanglement needed? NO! For our scheme (as for many other q. imaging schemes), only spatial correlations are necessary!

HOWEVER, entanglement IS useful:

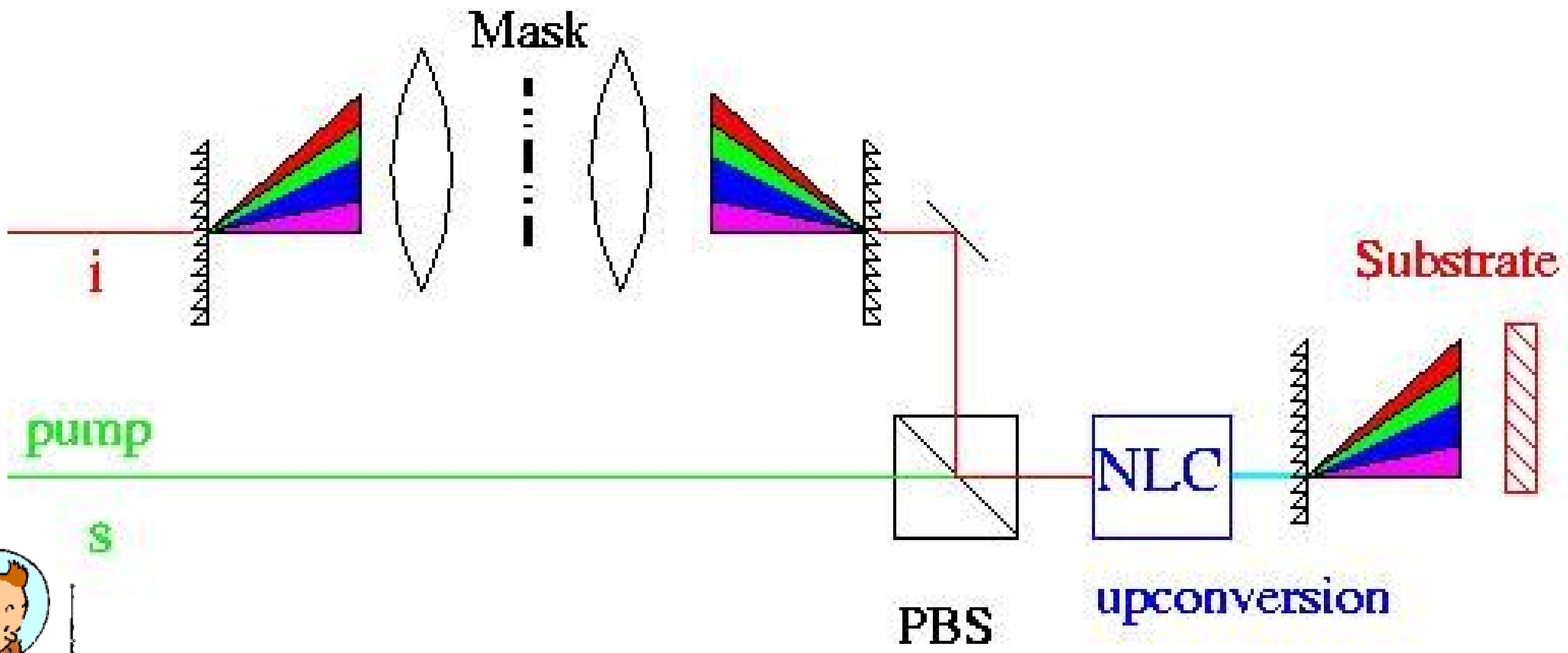
1. high resolution and high contrast at any distance from the source.

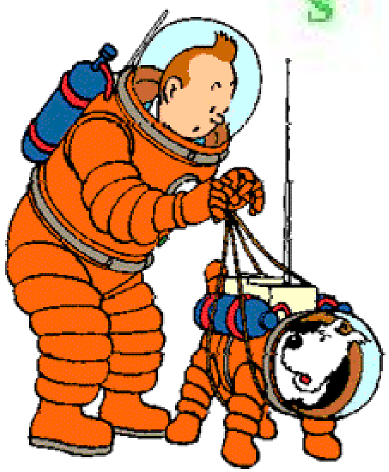
2. dispersion cancellation of intervening media

Even though entanglement is not needed, the method described appears to be the most easy method to create freq.correlated photons.

Beyond entanglement and conditioned photodetection

Simply encode the spatial info in the idler freq, and then do UPCONVERSION (using monochrom pump):





Low resolution mask → high resolution image on the substrate

TAKE HOME MESSAGE

Use entangled light to perform microscopy and lithography