Collisional shifts in optical-lattice atom clocks

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The most accurate configuration for atomic clocks may be:

• Atoms trapped in a deep magic-wavelength optical lattice at low filling.
• Fermionic atoms look very promising -- “no” collisional shift for spin-polarized atoms.
• For bosonic atoms, linewidth can be extremely narrow, but hopping of an atom into an adjacent filled site can introduce collisional shifts. We discuss the many-body interactions that can result in a shift of the clock frequency.

Band and Vardi, PRA 74, 033807 (06)
H. Katori demonstrated a far-detuned deep “magic wavelength” optical lattice to confine neutral atoms (Ye, Vernooy, Kimble)

- Load 3D optical lattice created by far-detuned light w. low filling. Atoms individually occupy lattice sites (preferably in ground motional state). \[ V(x) = \hbar |\Omega(x)|^2/(4\Delta) \] where \[ \Omega(x) = 2\mu \cdot E(x)/\hbar \] e.g., 1D optical potential \( V(x) \)

- **Lamb-Dicke regime** – deep lattice. Atoms stay in the same trap-state even after absorbing a photon, hence there is no Doppler shift.

- **Light-shift difference can be cancelled** by properly choosing intensity and frequency (magic wavelength) of lattice light.

- The number of 3D optical lattice sites occupied may be > 10^6.
Optical Lattice potential
Fermion filled optical lattice

- For $T = 0$, fermionic atoms fill bands up to the Fermi energy
- If only one spin component is present then -- only one atom per state due to Pauli exclusion principle
- For unit filling, only one atom per lattice site (at tight binding level of approximation), hence no atom-atom interaction
  - $s$-wave excluded, $p$-wave frozen out

Bosonic filled optical lattice

- $\Delta \nu$ can be made extremely small of $I = 0$ isotopes
- For $T = 0$, only lowest state of lowest state filled
- If more than one atom is present per site -- large collisional clock frequency shift of occurs.
- Low filling and small $J_{\text{hop}}$ will reduce but not eliminate the collisional shift problem.
Neutral atoms in an optical lattice (cont)


The precision measurement of time and frequency is a prerequisite not only for fundamental science but also for technologies that support broadband communication networks and navigation with global positioning systems (GPS). The SI second is currently realized by the microwave transition of Cs atoms with a fractional uncertainty of $10^{-15}$ (ref. 1). Thanks to the optical frequency comb technique\(^2\)\(^3\), which established a coherent link between optical and radio frequencies, optical clocks\(^4\)\(^5\) have attracted increasing interest as regards future atomic clocks with superior precision. To date, single trapped ions\(^4\)\(^6\) and ultracold neutral atoms in free fall\(^7\)\(^8\) have shown record high performance that is approaching that of the best Cs fountain clocks\(^1\). Here we report a different approach, in which atoms trapped in an optical lattice serve as quantum references. The ‘optical lattice clock’\(^9\)\(^10\) demonstrates a linewidth one order of magnitude narrower than that observed for neutral-atom optical clocks\(^7\)\(^8\)\(^11\), and its stability is better than that of single-ion clocks\(^4\)\(^5\). The transition frequency for the Sr lattice clock is $429,228,004,229,952(15)$ Hz, as determined by an optical frequency comb referenced to the SI second.
Neutral atoms in an optical lattice (cont)

3D lattice - low filling
Lamb-Dicke regime
\( \Delta \alpha(\omega, e) = 0 \)

Simplified optical coupling scheme for \(^{87}\text{Sr}\).

- \( 5s5p\, ^1P_1 \)
- \( 5s6s\, ^3S_1 \)
- \( 5s4d\, ^3D_1 \)

\( \lambda_0 = 698 \text{ nm} \)

Position / \( \lambda_L \)

5s5p \(^3P_0\) (\( F = 9/2 \))

5s\(^2\) \(^1S_0\) (\( F = 9/2 \))
Simplified optical coupling scheme for $^{88}$Sr

Nuclear spin, $I = 0$

$^{3}P_1 | F = 1, M_F = 0 >$

$^{3}P_0 | F = 0, M_F = 0 >$

$^{1}S_0 | F = 0, M_F = 0 >$

$\gamma_{12} \sim \gamma \left( \frac{\Omega_L^2}{4} + \frac{\Omega_B^2}{\Delta_{32}^2} \right)$

Taichenachev, et al., PRL 96, 083001 (06)
SU(2) representation for Bosonic atoms in a single lattice site (e.g., $^{88}$Sr)

\[
\hat{H} = \sum_{i=g,e} E_i \hat{a}_i^\dagger \hat{a}_i - \frac{\hbar \Omega(t)}{2} (\hat{a}_g^\dagger \hat{a}_e + \hat{a}_e^\dagger \hat{a}_g) + \sum_{i,j=g,e} G_{ij} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i .
\]

where $E_g = \epsilon_g + \hbar \omega$ and $E_e = \epsilon_e$

\[
[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} , \quad [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0 , \quad [\hat{a}_i, \hat{a}_j] = 0 .
\]

\[
\Omega(t) = \begin{cases} 
\Omega & \text{if } 0 \leq t \leq \tau_p , \\
0 & \text{if } \tau \leq t \leq \tau + \tau_p , \\
\Omega & \text{if } \tau + \tau_p \leq t \leq \tau + 2\tau_p ,
\end{cases}
\]

\[
\int_0^{\tau_p} \Omega(t') dt' = \pi / 2
\]

\[
\Delta = \hbar \omega + \epsilon_g - \epsilon_e
\]

\[
\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i , \\
\hat{N} = \hat{n}_g + \hat{n}_e , \\
\hat{L}_z = \frac{\hat{n}_g - \hat{n}_e}{2} , \\
\hat{L}_x = \frac{\hat{a}_g^\dagger \hat{a}_e + \hat{a}_e^\dagger \hat{a}_g}{2} , \\
\hat{L}_y = \frac{\hat{a}_g^\dagger \hat{a}_e - \hat{a}_e^\dagger \hat{a}_g}{2i} ,
\]

Anglin and Vardi, PRA 64,13605 (01)
SU(2) representation (cont)

\[
\hat{H} = (E - \frac{G_{gg} + G_{ee}}{2})\hat{N} + (G_{gg} + G_{ee} + 2G_{ge})\frac{\hat{N}^2}{4} + \]
\[
(G_{gg} + G_{ee} - 2G_{ge})\hat{L}_z^2 + [\hbar\Delta + (G_{gg} - G_{ee})(\hat{N} - 1)]\hat{L}_z - \hbar\Omega(t)\hat{L}_x.
\]

Solve the density matrix equations:

\[
\frac{d}{dt}\rho = \frac{i}{\hbar} [\rho, \hat{H}] - \sum_k \Gamma_k (2\hat{O}_k \rho \hat{O}_k^\dagger - \{\hat{O}_k^\dagger \hat{O}_k, \rho\}),
\]

where \(O_k\) are Lindblad operators (e.g., \(L_z\)) and \(\Gamma_k\) are parameters.

Calculate \((u(t),v(t),w(t)) \equiv (\langle L_x \rangle(t), \langle L_y \rangle(t), \langle L_z \rangle(t))\)
Single atom in site Ramsey fringes

\begin{itemize}
\item \( w \) (population inversion)
\item Time (ms)
\item Detuning \( \Delta \)
\end{itemize}
Two-atoms per site Ramsey fringes

Interaction strength
\[ \kappa = (G_{gg} + G_{ee} - 2G_{ge}) = 100 \text{ Hz} \]
Ramsey dynamics

(a) 1-atom

\[ \Delta = 0 \]

(b) 2-atoms
\[ \kappa = 100 \text{ Hz} \]

(d) 10-atoms
\[ \kappa = 100 \text{ Hz} \]
1-atom

2-atoms, $\kappa = 100$ Hz

10-atoms, $\kappa = 100$ Hz

Ramsey fringes
Asymmetric Ramsey fringes due to interactions

Asymmetric Ramsey fringes. Numerically calculated Ramsey fringe patterns, at three different values of the nonlinear interaction: (a) \( \Delta G = 0, \kappa = 0 \), (b) \( \Delta G = 1500\text{Hz}, \kappa = 0 \), and (c) \( \Delta G = 0, \kappa = 3000 \text{Hz} \).

Even for single site occupancy, hopping of atoms from one site to another can cause a frequency shift:

\[
\Delta \nu \sim J_{\text{hop}} T(\Delta G^2 + \kappa^2)^{1/2}
\]
Two state Bose-Hubbard model

Include hopping to nearest neighbor sites

Ground state \( |a\rangle \), excited state \( |b\rangle \), sites \( \alpha, \beta \)

\[
H = \sum_{\langle \alpha, \beta \rangle} H^{(\alpha, \beta)}_{\text{hop}} + \sum_{\alpha} \left[ H^{(\alpha)}_{\text{int}} + H^{(\alpha)}_{\text{diff}} + H^{(\alpha)}_{\text{sum}} + H^{(\alpha)}_{R} \right],
\]

\[
H^{(\alpha, \beta)}_{\text{hop}} = J_{\text{hop}}[(a_{\alpha}^\dagger a_{\beta} + h.c.) + (b_{\alpha}^\dagger b_{\beta} + h.c.)],
\]

\[
H^{(\alpha)}_{\text{int}} = \frac{1}{2} U_{aa} a_{\alpha}^\dagger a_{\alpha}^\dagger a_{\alpha} a_{\alpha} + \frac{1}{2} U_{bb} b_{\alpha}^\dagger b_{\alpha}^\dagger b_{\alpha} b_{\alpha} + U_{ab} a_{\alpha}^\dagger b_{\alpha}^\dagger b_{\alpha} a_{\alpha},
\]

\[
H^{(\alpha)}_{\text{diff}} = \frac{\Delta}{2}(a_{\alpha}^\dagger a_{\alpha} - b_{\alpha}^\dagger b_{\alpha}),
\]

\[
H^{(\alpha)}_{\text{sum}} = \frac{\epsilon_a + \hbar \omega + \epsilon_b}{2}(a_{\alpha}^\dagger a_{\alpha} + b_{\alpha}^\dagger b_{\alpha}),
\]

\[
H^{(\alpha)}_{R} = \frac{\Omega(t)}{2}(a_{\alpha}^\dagger b_{\alpha} e^{i\phi_{\alpha}(t)} + b_{\alpha}^\dagger a_{\alpha} e^{-i\phi_{\alpha}(t)}),
\]

\[
\exp(ik \cdot r_{ij}) = \exp(i\theta)
\]

\[
\theta \equiv \phi_2 - \phi_1
\]
FIG. 9: (color online) Two-particle level schemes for Ramsey spectroscopy in the presence of tunneling: (a) general scheme (b) coupling between parity eigenstates for $\theta = 0$ (c) coupling between parity eigenstates for $\theta = \pi$. Rabi coupling between ground- and excited states, is denoted by solid (red) and dot-dashed (green) arrows, corresponding to different phases of the driving fields in adjacent sites. Dashed (blue) arrows denote hopping between sites. Fock states are denoted as $|n_1^g, n_1^e; n_2^g, n_2^e\rangle$, where $n_\alpha^g = \langle \hat{a}_\alpha^\dagger \hat{a}_\alpha \rangle$ and $n_\alpha^e = \langle \hat{b}_\alpha^\dagger \hat{b}_\alpha \rangle$. 

Two-state Two-site B-H model
FIG. 9: (color online) Two-particle level schemes for Ramsey spectroscopy in the presence of tunneling: (a) general scheme (b) coupling between parity eigenstates for $\theta = 0$ (c) coupling between parity eigenstates for $\theta = \pi$. Rabi coupling between ground- and excited states, is denoted by solid (red) and dot-dashed (green) arrows, corresponding to different phases of the driving fields in adjacent sites. Dashed (blue) arrows denote hopping between sites. Fock states are denoted as $|n^g_1, n^e_1; n^g_2, n^e_2\rangle$, where $n^g_1 = \langle a^\dagger_\alpha a_\alpha \rangle$ and $n^e_\alpha = \langle b^\dagger_\alpha b_\alpha \rangle$. 

2-state 2-site B-H model (cont)

(b)

\[
\begin{align*}
|10,2;0,0\rangle &\rightarrow |10,0;0,2\rangle \\
|10,1;0,1\rangle &\rightarrow |10,2;0,0\rangle + |10,0;0,2\rangle \\
|11,1;0,0\rangle &\rightarrow |10,0;1,1\rangle \\
|10,1;1,0\rangle &\rightarrow |11,0;0,1\rangle \\
|10,1;1,0\rangle &\rightarrow |11,0;0,1\rangle + |11,1;0,0\rangle \\
|10,0;2,0\rangle &\rightarrow |12,0;0,0\rangle \\
|11,0;1,0\rangle &\rightarrow |10,0;2,0\rangle + |12,0;0,0\rangle \\
\end{align*}
\]

\[\Omega\]

(c)

\[
\begin{align*}
|10,2;0,0\rangle &\rightarrow |0,0;0,2\rangle \\
|10,1;0,1\rangle &\rightarrow |10,2;0,0\rangle + |10,0;0,2\rangle \\
|10,1;1,0\rangle &\rightarrow |11,0;0,1\rangle \\
|11,1;0,0\rangle &\rightarrow |10,0;1,1\rangle \\
|11,0;1,0\rangle &\rightarrow |11,1;0,0\rangle + |10,0;1,1\rangle \\
|10,0;2,0\rangle &\rightarrow |12,0;0,0\rangle \\
|11,0;1,0\rangle &\rightarrow |10,0;2,0\rangle + |12,0;0,0\rangle \\
\end{align*}
\]

\[\Omega\]
Using control of the wavevector direction

\[ \exp(i\mathbf{k} \cdot \mathbf{r}_{ij}) = \exp(i\theta) \] controlled using direction of the \( \mathbf{k} \) vector.

A proper choice of the relative phase angle between Rabi drive fields in adjacent sites may be used to suppress the tunneling between them and thus reduce collisions.
Control of wavevector direction -- $P_2$ vs $\Delta$ and $J_{hop}$

Strongly interacting -- interaction strength $g_{11} = 1000$ (Hz)

$\theta = 0$
Control of wavevector direction -- $P_2$ vs $\Delta$ and $J_{hop}$

Strongly interacting -- interaction strength $g_{11} = 1000$ (Hz)

$\theta = \pi$
Control via wavevector direction -- $P_2$ and $P_e$

(a) $\Delta G = \kappa = 0$

(b) $\Delta G = \kappa = 500$ Hz

(c) $\Delta G = \kappa = 1000$ Hz

$J_{\text{hop}} = 100$ Hz
Conclusions

Advantages of optical-lattice optical atom-clock

- No Doppler shift – Lamb-Dicke regime
- No optical lattice potential shift – magic wavelength
- Small $\Delta \nu/\nu$ at optical wavelengths (factor of $10^5$ better than MW)
- Long-lived excited states can be used so $\Delta \nu$ is small
- For Bosonic atoms, $\Delta \nu$ can be made particularly small
  
  $^1S_0(F=0) \leftrightarrow ^3P_0(F=0)$

- Large number of occupied sites – $\Delta \nu/\nu \sim 1/N^{1/2}$
- Coherent link between optical and MW – frequency comb method
- At low filling, hopping (and therefore collisional effects) suppressed
  
  so $\Delta \nu \sim J_{\text{hop}} T(\Delta G^2 + \kappa^2)^{1/2}$ can be reduced for bosonic atoms
- Squeezed state improvement of signal to noise
- …
Thank you for your attention!