Probing QCD Phase Diagram in Heavy Ion Collisions

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- QCD Phase Diagram from LQCD
- Fluctuations of conserved charges as probe of thermalization and QCD phase boundary
- Linking LQCD results to HIC data at the LHC
- STAR data on net-proton number fluctuations and chiral criticality
Deconfinement and chiral symmetry restoration in QCD

- The QCD chiral transition is **crossover** Y. Aoki, et al. Nature (2006) and appears in the O(4) critical region

- Chiral transition temperature
  \[ T_c = 155(1)(8) \text{ MeV} \]
  - T. Bhattacharyya et al.

- Deconfinement of quarks sets in at the chiral crossover

- The shift of \( T_c \) with chemical potential
  \[ T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2] \]
  - Ch. Schmidt Phys. Rev. D83 (2011) 014504
  - O. Kaczmarek QM 2017

See also:
JHEP, 0906 (2009)

Pisarski & Wilczek;
- 2nd order, O(4)
- 2nd order, Z(2)
- 1st order
- crossover

Asakawa-Yazaki
Rajagopal, Schuryak
Stephanov; Hatta, et al.
O(4) scaling and magnetic equation of state

\[ P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t, b^{\beta \delta / \nu} h) \]

- Phase transition encoded in the magnetic equation of state

\[ <qq> = \frac{-\partial P}{\partial m} \Rightarrow \text{pseudo-critical line} \quad t = (T - T_c) / T_c \]

\[ \frac{<qq>}{m^{1/\delta}} = f_s(z) \quad , \quad z = tm^{-1/\beta \delta} \]

universal scaling function common for all models belonging to the O(4) universality class: known from spin models

J. Engels & F. Karsch (2012)

QCD chiral crossover transition in the critical region of the O(4) 2\textsuperscript{nd} order
Due to expected $O(4)$ scaling in QCD the free energy:

$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta \delta/v} h)$$

Generalized susceptibilities of net baryon number

$$\chi_B^{(n)} = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n} = \chi_R^{(n)} + \chi_S^{(n)}$$

with

$$\chi_s^{(n)} \big|_{\mu=0} = d h^{(2-\alpha-n/2) / \beta \delta} f_{\pm}^{(n)}(z)$$

$$\chi_s^{(n)} \big|_{\mu \neq 0} = d h^{(2-\alpha-n) / \beta \delta} f_{\pm}^{(n)}(z)$$

At $\mu = 0$ only $\chi_B^{(n)}$ with $n \geq 6$ receive contribution from $\chi_S^{(n)}$

At $\mu \neq 0$ only $\chi_B^{(n)}$ with $n \geq 3$ receive contribution from $\chi_S^{(n)}$

$\chi_B^{n=2}$ generalized susceptibilities of the net baryon number is non critical with respect to $O(4)$
Consider fluctuations and correlations of conserved charges to be compared with LQCD

- They are quantified by susceptibilities:
  
  \[
  \frac{\chi_N}{T^2} = \frac{\partial^2 (P)}{\partial (\mu_N)^2} \quad \frac{\chi_{NM}}{T^2} = \frac{\partial^2 (P)}{\partial \mu_N \partial \mu_M} 
  \]

  \[
  N = N_q - N_{-q}, \quad N, M = (B, S, Q), \quad \mu = \mu / T, \quad P = P / T^4
  \]

- Susceptibility is connected with variance
  
  \[
  \frac{\chi_N}{T^2} = \frac{1}{VT^3} (<N^2> - <N>^2)
  \]

- If \( P(N) \) probability distribution of \( N \) then
  
  \[
  <N^n> = \sum_N N^n P(N)
  \]

Excellent probe of:

- QCD criticality
  - A. Asakawa at. al.
  - S. Ejiri et al.,…
  - M. Stephanov et al.,
  - K. Rajagopal et al.
  - B. Frimann et al.

- Freezeout conditions in HIC
  - F. Karsch &
  - S. Mukherjee et al.,
  - P. Braun-Munzinger et al.,
Consider special case:

- Charge and anti-charge uncorrelated and Poisson distributed, then
- \( P(N) \) the Skellam distribution

\[
P(N) = \left( \frac{\sqrt{N_q}}{\sqrt{N_{-q}}} \right)^{N/2} I_N \left( 2 \sqrt{N_{-q} N_q} \right) \exp\left[ -(N_{-q} + N_q) \right]
\]

- Then the susceptibility

\[
\frac{\chi_N}{T^2} = \frac{1}{VT^3} \left( \langle N_q \rangle + \langle N_{-q} \rangle \right)
\]

\( <N_q> \equiv \bar{N}_q \) =>

Charge carrying by particles  \( q = \pm 1 \)
Consider special case: particles carrying \( q = \pm 1, \pm 2, \pm 3 \)

**The probability distribution**

\[
P(S) = \left( \frac{\bar{S}_1}{S_1} \right)^{\frac{3}{2}} \exp \left[ \sum_{n=1}^{3} (\bar{S}_n + \bar{S}_n) \right] \\
\sum_{i=\infty}^{\infty} \sum_{k=\infty}^{\infty} \frac{(\bar{S}_3)^{k/2}}{\bar{S}_3} I_k(2\sqrt{\bar{S}_1 \bar{S}_1}) \\
\frac{(\bar{S}_2)^{i/2}}{\bar{S}_2} I_i(2\sqrt{\bar{S}_1 \bar{S}_1}) \\
\frac{(\bar{S}_1)^{-i-3k/2}}{S_1} I_{2i+3k-3}(2\sqrt{\bar{S}_1 \bar{S}_1})
\]

\[
\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{q} n^2 (\langle S_n \rangle + \langle S_{-n} \rangle)
\]

\[
\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{m=-q_M}^{q_M} \sum_{n=-q_N}^{q_N} nm \langle S_{n,m} \rangle
\]

\( \langle S_{n,m} \rangle \): is the mean number of particles carrying charge \( N = n \) and \( M = m \)

P. Braun-Munzinger, B. Friman, F. Karsch, V. Skokov & K.R.
Variance at 200 GeV AA central coll. at RHIC

STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

\[
\frac{\langle p \rangle + \langle \bar{p} \rangle}{\sigma^2} = 1.022 \pm 0.016 \\
\chi_1 = 1.076 \pm 0.035
\]

- ALICE data consistent with Skellam \( \Delta \eta < 1 \)

- Skellam distribution is a good approximation to calculate 2\textsuperscript{nd} order charge fluctuations in HIC

See also talk of Lijun Ruan
Variance at 200 GeV AA central coll. at RHIC


STAR Collaboration data in central coll. 200 GeV Consistent with Skellam distribution

\[
\frac{\langle p \rangle + \langle \bar{p} \rangle}{\sigma^2} = 1.022 \pm 0.016 \quad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035
\]

ALICE data consistent with Skellam \( \Delta \eta < 1 \)

- Shrinking of Skellam dis. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

A. Rustamov, QM2017

\[
\Delta \eta
\]
Can the thermal nature and composition of the collision fireball in HIC be verified?
Constructing net charge fluctuations and correlation from ALICE data with Skellam distribution

- Net baryon number susceptibility

\[
\frac{\chi_B}{T^2} \approx \frac{1}{VT^3} \left( \langle p \rangle + \langle N \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + \langle \Xi^- \rangle + \langle \Xi^0 \rangle + \langle \Omega^- \rangle + \overline{par} \right)
\]

- Net strangeness

\[
\frac{\chi_S}{T^2} \approx \frac{1}{VT^3} \left( \langle K^+ \rangle + \langle K_S^0 \rangle + \langle \Lambda + \Sigma_0 \rangle + \langle \Sigma^+ \rangle + \langle \Sigma^- \rangle + 4 \langle \Xi^- \rangle + 4 \langle \Xi^0 \rangle + 9 \langle \Omega^- \rangle + \overline{par} \right)
\]

\[-(\Gamma_{\phi \rightarrow K^+} + \Gamma_{\phi \rightarrow K^-} + \Gamma_{\phi \rightarrow K_S^0} + \Gamma_{\phi \rightarrow K_L^0}) \langle \phi \rangle \]

- Charge-strangeness correlation

\[
\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3} \left( \langle K^+ \rangle + 2 \langle \Xi^- \rangle + 3 \langle \Omega^- \rangle + \overline{par} \right)
\]

\[-(\Gamma_{\phi \rightarrow K^+} + \Gamma_{\phi \rightarrow K^-}) \langle \phi \rangle - (\Gamma_{K_0^* \rightarrow K^+} + \Gamma_{K_0^* \rightarrow K^-}) \langle K_0^* \rangle \]
Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee


- Is there a temperature where calculated ratios from ALICE data agree with LQCD?
Direct comparisons of Heavy ion data at LHC with LQCD

- STAR results => the 2nd order cumulants $\kappa_2$ are consistent with Skellam distribution, thus $\kappa_N$ and $\kappa_{NM}$ with $N,M = \{B,Q,S\}$ are expressed by particle yields. Consider LHC data

$$\frac{\kappa_B}{T^2} = \frac{1}{VT^3} (203.7 \pm 11.4)$$

$$\frac{\kappa_S}{T^2} = \frac{1}{VT^3} (504.2 \pm 16.8)$$

$$\frac{\kappa_{QS}}{T^2} = \frac{1}{VT^3} (191.1 \pm 12)$$

- The Volume at $T_c$

$$V_{T_c} = 3800 \pm 500 \text{ fm}^3$$

Compare ratios with LQCD at chiral crossover
P. Braun-Munzinger, A. Kalweit, J. Stachel, & K.R.

The cumulant ratios extracted from ALICE data are consistent with LQCD at the chiral crossover: Evidence for thermalization at the phase boundary
Constraining chemical freezeout temperature at the LHC

At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

\[ C_{BS} = -\frac{\langle \delta B \rangle \langle \delta S \rangle}{\langle (\delta S)^2 \rangle} = -\frac{\chi_{BS}}{\chi_S} \]

- Excellent observable to fix the temperature

\[ -\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3} [2 < \Lambda + \Sigma^0 > + 4 < \Sigma^+ > + 8 < \Xi > + 6 < \Omega^- >] = (97.4 \pm 5.8)/VT^3 \]

However, this is the lower limit since e.g. \( \Sigma^* (\geq 1660) \rightarrow N\bar{K} \)
\( \Lambda^* (\geq 1520) \rightarrow N\bar{K} \) are not included.

- Data on \( \chi_B / \chi_S \) and \( \chi_B / \chi_{QS} \) consistent with LQCD results for

\[ 0.148 \leq T_f < 160 \text{ MeV} \]
Fluctuations of net baryon number sensitive to deconfinement in QCD

S. Ejiri, F. Karsch & K.R. (06)
A. Bazavov et al. arXiv. 1701.04325

\[ \chi^B_n = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n} \]

- HRG factorization of pressure:
  \[ P^B(T, \mu_q) = F(T) \cosh(B\mu_B / T) \]

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

\[ \kappa \sigma^2 = \frac{\chi^B_4}{\chi^B_2} \approx B^2 = \begin{cases} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{cases} \]
Modelling fluctuations in the $O(4)/Z(2)$ universality class

$$\mathcal{L}_{QM} = \bar{q} [i \gamma_\mu \partial^\mu - g(\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi})] q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi)^2 - U(\sigma, \pi)$$

Effective potential is obtained by solving \textit{the exact flow equation} (Wetterich eq.) with the approximations resulting in the $O(4)/Z(2)$ critical exponents

$$\partial_k \Omega_k(\sigma) = \frac{V k^4}{12 \pi^2} \left[ \sum_{i=\pi,\sigma} \frac{d_i}{E_{i,k}} [1 + 2n_B(E_{i,k})] - \frac{2v_q}{E_{q,k}} [1 - n_F(E_{q,k}^+) - n_F(E_{q,k}^-)] \right]$$

$$E_{\pi,k} = \sqrt{k^2 + \Omega_k^0}$$
$$E_{\sigma,k} = \sqrt{k^2 + \Omega_k^0 + 2g^2 \Omega_k'}$$
$$E_{q,k} = \sqrt{k^2 + 2g^2 \rho}$$
$$\Omega_k' = \frac{\partial \Omega_k}{\partial (\sigma^2/2)}$$

Full propagators with $k < q < \Lambda$

$$\Gamma_{\Lambda} = \mathcal{S}_{\text{classical}}$$

Integrating from $k=\Lambda$ to $k=0$ gives full quantum effective potential
Deviations of cumulant ratios from Skellam distribution are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC.
STAR data on the cumulants of the net baryon number

Deviations from the HRG

\[ S \sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}}, \quad \kappa \sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}} \]

\[ S \sigma|_{HRG} = \frac{N_p - N_{-p}}{N_p + N_{-p}}, \quad \kappa \sigma|_{HRG} = 1 \]

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy
STAR “BES” and recent results on net-proton fluctuations

With increasing acceptance of the transverse momentum, large increase of net-proton fluctuations at $\sqrt{s} < 20$ GeV beyond that of a non-critical reference of a HRG

Is the above an Indication of the CEP?

At $\sqrt{s} > 20$ GeV data consistent with LQCD results near the chiral crossover

See also talk of Lijun Ruan
It is possible to find the freeze-out line such that kurtosis exhibits the energy dependence as seen in data.

However, are other cumulants consistent?
Self-consistent freeze-out and STAR data

- Freeze-out line in \((T, \mu)\) – plain determined by fitting \(\chi_B^3 / \chi_B^1\) to data
- Ratio \(\chi_B^1 / \chi_B^2 \approx \tanh(\mu / T)\) => further evidence of equilibrium and thermalisation at \(7 \text{ GeV} \leq \sqrt{s} < 5 \text{ TeV}\)
- Ratio \(\chi_B^1 / \chi_B^2 \neq \chi_B^3 / \chi_B^2\) expected due to critical chiral dynamics
- Enhancement of \(\chi_B^4 / \chi_B^2\) at \(\sqrt{s} < 20 \text{ GeV}\) not reproduced
Higher order cumulants - energy dependence

- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC

At freeze-out fixed by $\chi_B^3 / \chi_B^1$, the ratio $\chi_B^2 / \chi_B^2 \approx 0$ which agrees with preliminary STAR data, albeit within still very large error.
Conclusions:

- From LQCD: chiral crossover in QCD is the remnant of the 2nd order phase transition belonging to the O(4) universality class.

  Very good prospects for exploring the phase diagram of QCD in nuclear collisions with fluctuations.

- The medium created in HIC is of thermal origin and follows the properties expected in LQCD near the phase boundary.

- Systematics of the net-proton number fluctuations at $\sqrt{s} \geq 20$ GeV measured by STAR Coll. in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics.
Hagedorn’s continuum mass spectrum contribution to strangeness fluctuations

- Missing strange baryon and meson resonances in the PDG
- Satisfactory description of LGT with asymptotic states from Hagedorn’s exponential mass spectrum $\rho^H(m) = m^{a} e^{m/T_H}$ fitted to PDG
Charge - Strangeness correlations

- The ratio
  \[ 1.014 \leq \frac{\chi_2^B}{\chi_2^{QS}} \leq 1.267 \]
  extracted from ALICE data is consistent with LQCD for
  \[ 148 < T_f \leq 170 \text{ MeV} \]
  when combined with \( T_f \) obtained from \( \chi_2^B / \chi_2^{S} \) one concludes that, data consistent with LGT for
  \[ 148 < T_f \leq 160 \]
The ratio of cumulants in LGT and ALICE data

- The ratio

\[ 0.376 \leq \frac{\chi_2^B}{\chi_2^S} \leq 0.432 \]

extracted from ALICE data is consistent with LQCD for

\[ 142 < T_f \leq 160 \text{ MeV} \]

thus excellently overlaps with chiral crossover

\[ 145 < T_c \leq 163 \text{ MeV} \]

F. Karsch QM 21017
Modelling QCD Statistical Operator in hadronic phase

Interacting hadron gas $\Rightarrow$ S matrix approach (Dashen, Ma & Bernstein, Phys. Rev (1969): “uncorrelated” gas of hadrons and resonances (HRG)

$$P_H^{\text{int.}} \approx \sum_{i=\text{stabel hadrons}} P_i^{id} + \sum_{k=\text{all resonances}} P_k^{id}$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

A. Bazavov et al. HotQCD Coll. 2014, 2017

HRG provides very good description of yields data and LQCD equation of state