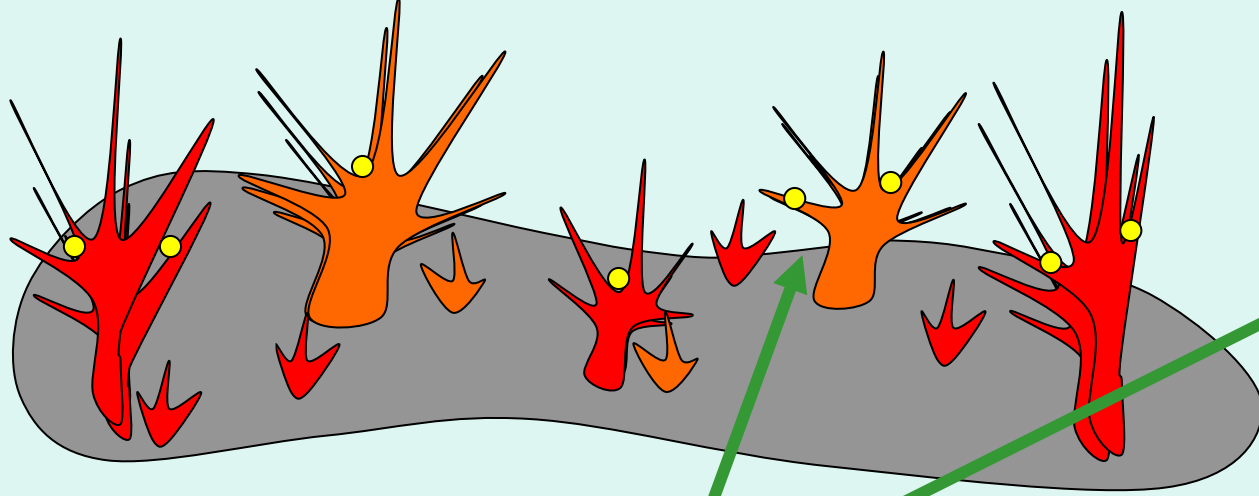


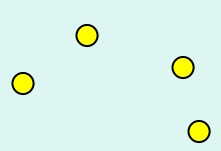
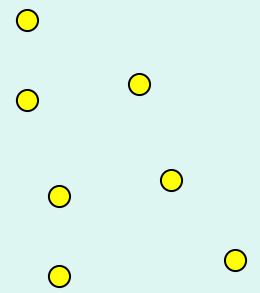
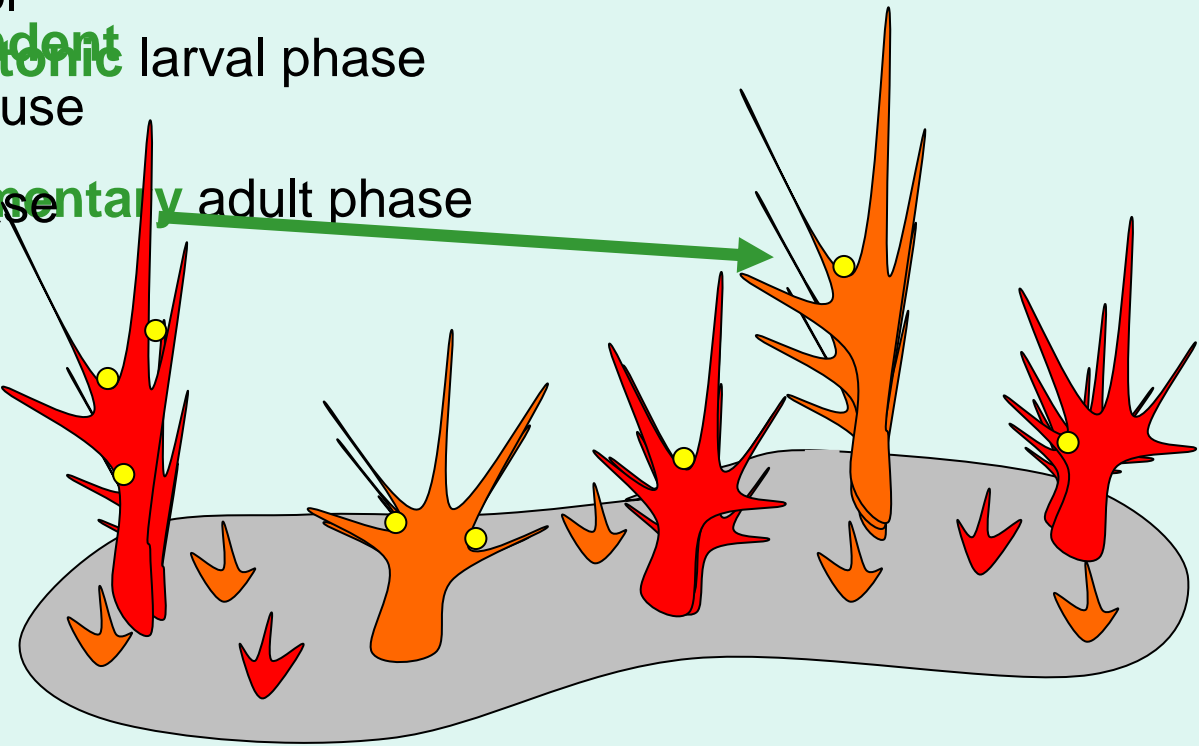
**Mathematical models explaining
regulating processes
and stability
of open marine systems**

Yael Artzy-Randrup, Lewi Stone
Tel-Aviv University



Life cycle: **open systems...**

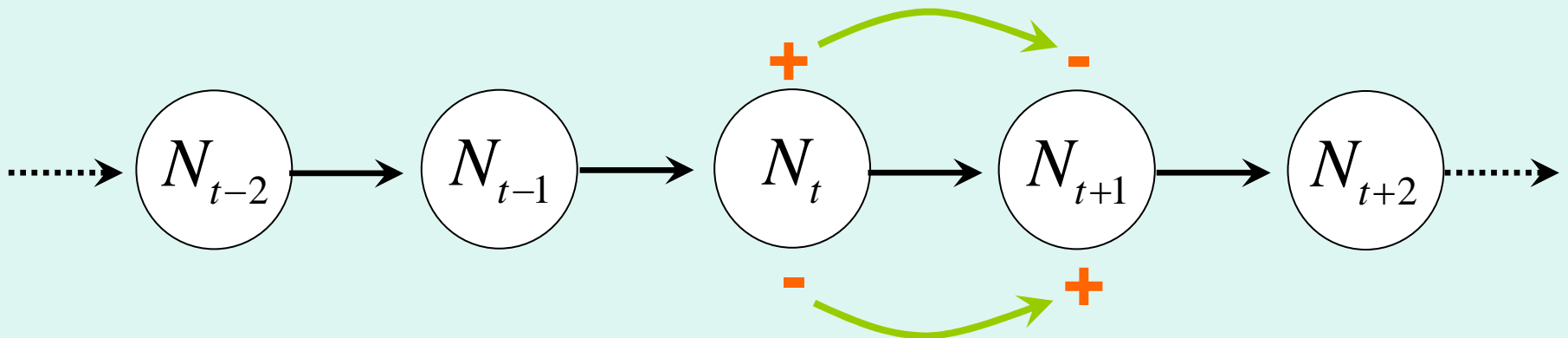
New recruits enter the habitat
 Larvae is dispersed by adults
 habitats mix in a pelagic pool
 by **sexual reproduction** independent
 making the system **planktonic** larval phase
demographically open of local habitat density because
 the system is open
 Sedimentary colonies **increase** their size by **asexual reproduction**.



Regulation

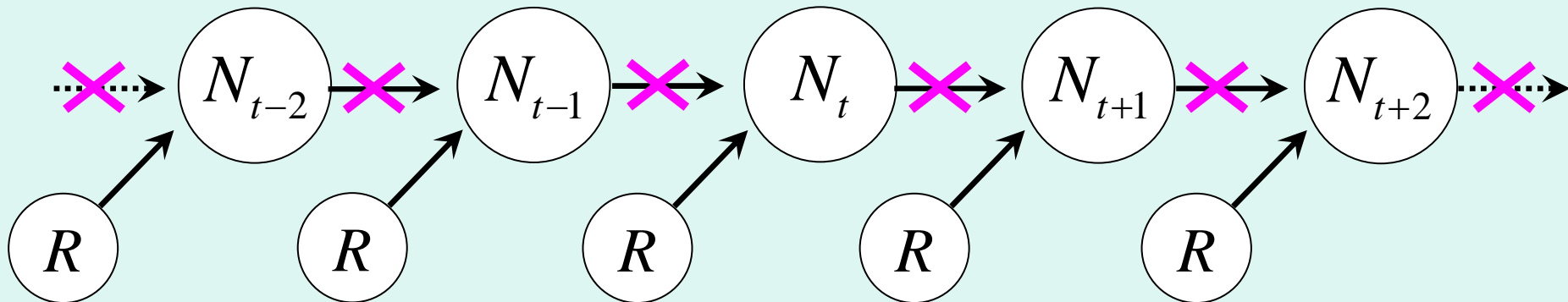
A population is **regulated** if it **persists** for many generations with fluctuations bounded above zero and below infinity.

Regulation requires **negative feedback** such that the population will have the tendency to increase when small and decrease when large.



The paradox of open systems

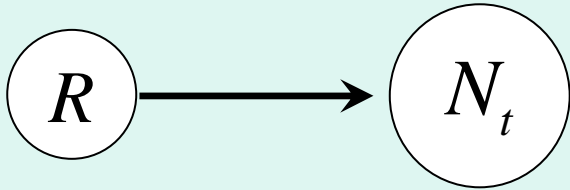
- Stability of coral reefs
- Open systems don't seem to have any intrinsic regulation.



Controversy regarding persistence of open marine systems

- Recruitment Limitation hypothesis
- An increase in recruitment rate will cause an increase in local adult population size
- Populations that are locally open may persist indefinitely without density dependence

“recruitment regulation”



$$N_{t+1} = AN_t + R$$

The equilibrium solution is (when $t \rightarrow \infty$):

$$\hat{N} = R(I - A)^{-1}e = R \cdot \left(1, p_1, p_2, \dots, \prod p_i\right)^T$$

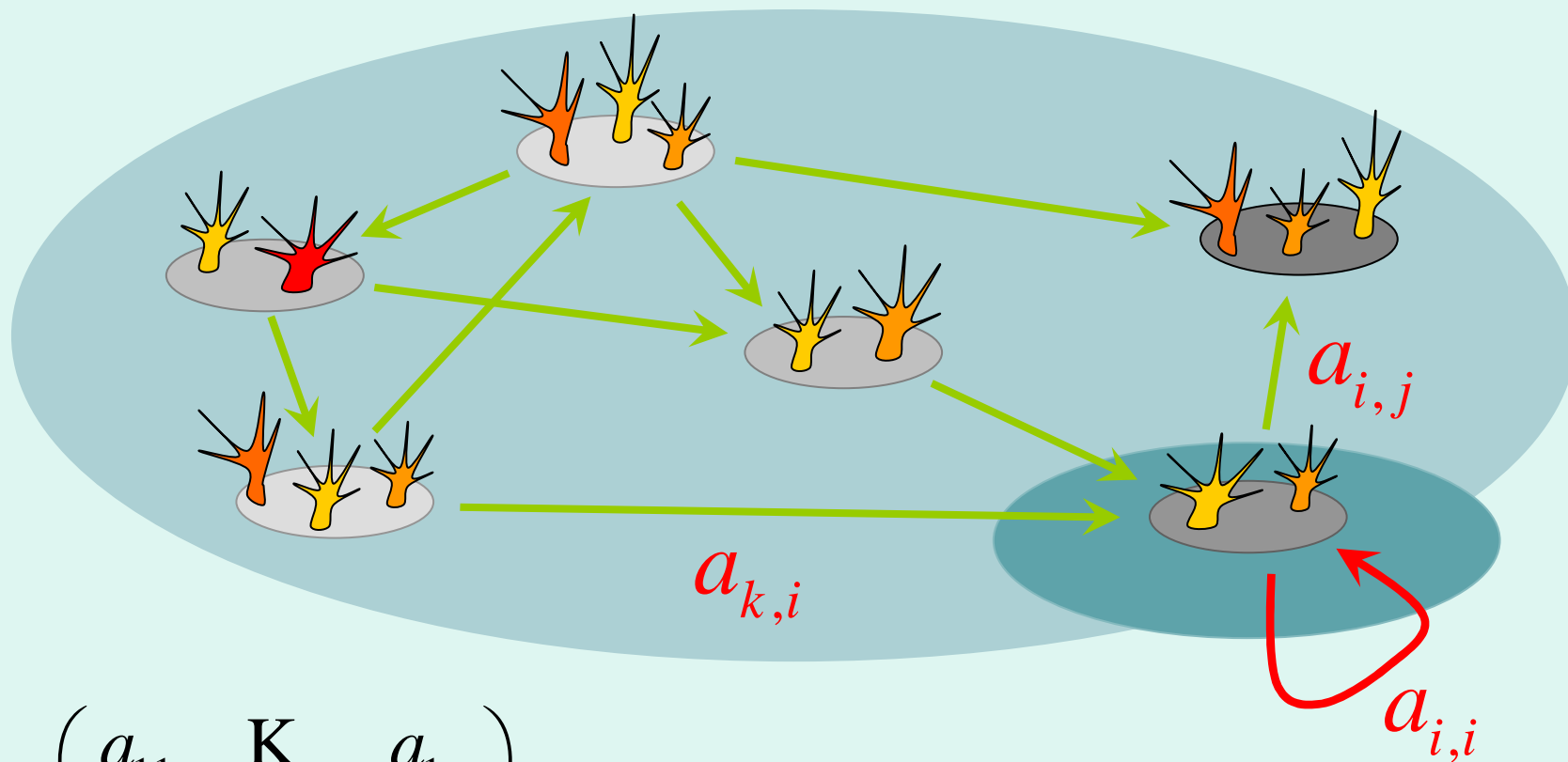
This solution is globally stable

- Density dependence is unnecessary for persistence at a local population.
- Supporting the Recruitment Limitation hypothesis.
- Even though **recruitment remains constant**, the **number** of recruits entering the system is high when local density is low, and low when density is high.

Metapopulations

- Looking only at the local population might be **incomplete**.
- On a wider scale, interconnected groups of local populations that are **linked by dispersal** form what is known as a metapopulation.
- A local population going extinct would be **re-colonized** by dispersed recruits.

On the small scale populations are open or partially open, but on the large scale they are closed:

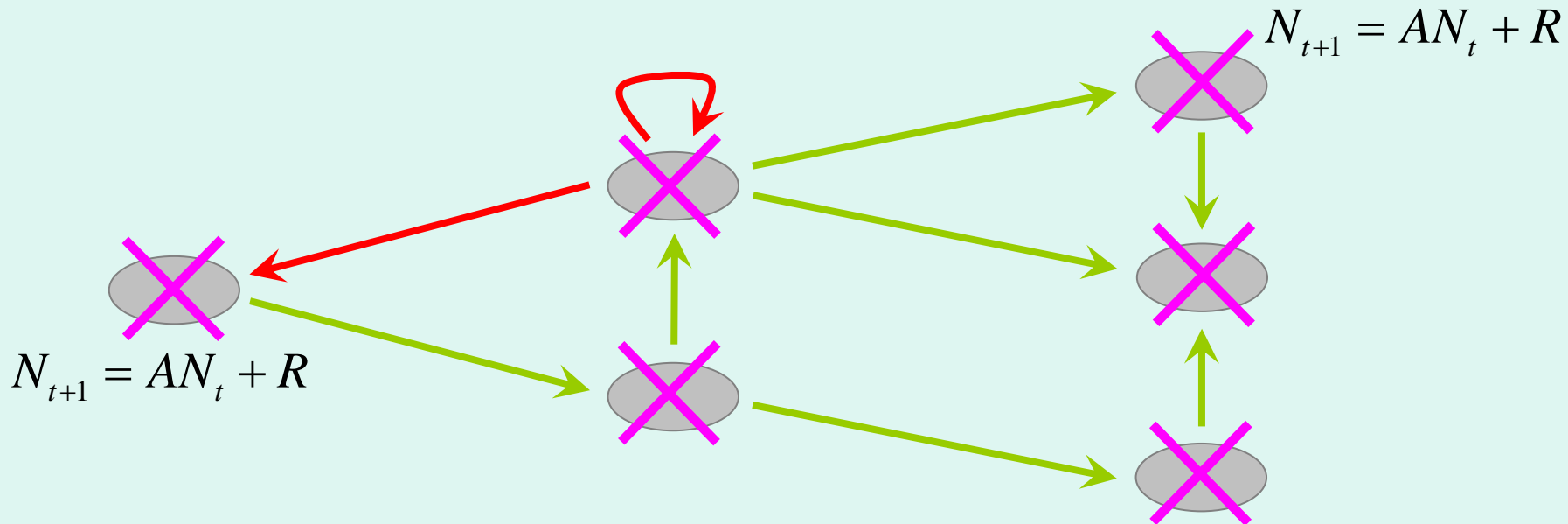


$$A = \begin{pmatrix} a_{11} & \mathbf{K} & a_{1n} \\ \mathbf{M} & \mathbf{O} & \mathbf{M} \\ a_{m1} & \mathbf{L} & a_{mn} \end{pmatrix}$$

The metapopulation can be described as a **network of connections**

A closer look at the metapopulation...

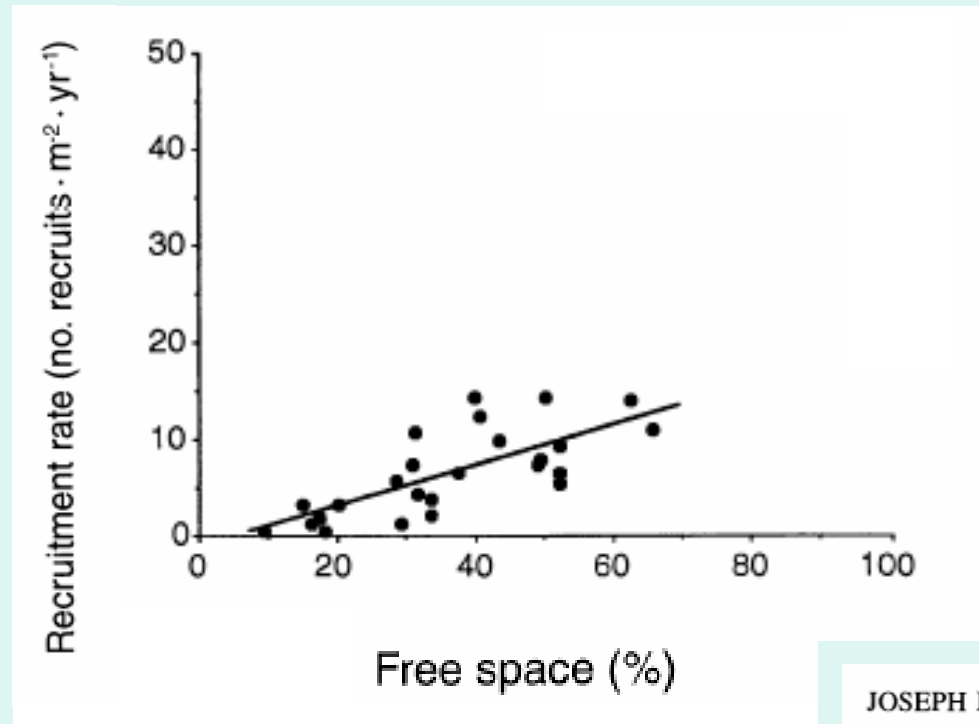
If there are no closed path of larval dispersal:



On the other hand, the existence of self cycles leads to positive feed back affecting the entire metapopulation.

- We have just seen that for a metapopulation to persist there must be some form of density dependence in the system.
- Local population models that assume a constant larval supply rate, **implicitly assume** that the wider meta-population is regulated, so the assumptions used in the local model are problematic.
- Incomplete models ignore critical feed backs at the scale of the metapopulation necessary to address questions of regulation

Free-space limitation



Can the **limitation** of **free-space** provide **regulation** in both local and metapopulation cases?

Local size structured model

$$n_t(x, t) + (\alpha(x)n(x, t))_x = -u(x)n(x, t)$$

growth
rate

number of
individuals of
size x at time t

death rate

size

time

$$n(x_0, t)\alpha(x_0) = sF(t)$$

minimum size

recruitment
rate

free space at time t

$$F(t) = A - \int_{x_0}^{x_{\max}} xn(x, t)dx$$

total area

Free space at equilibrium:

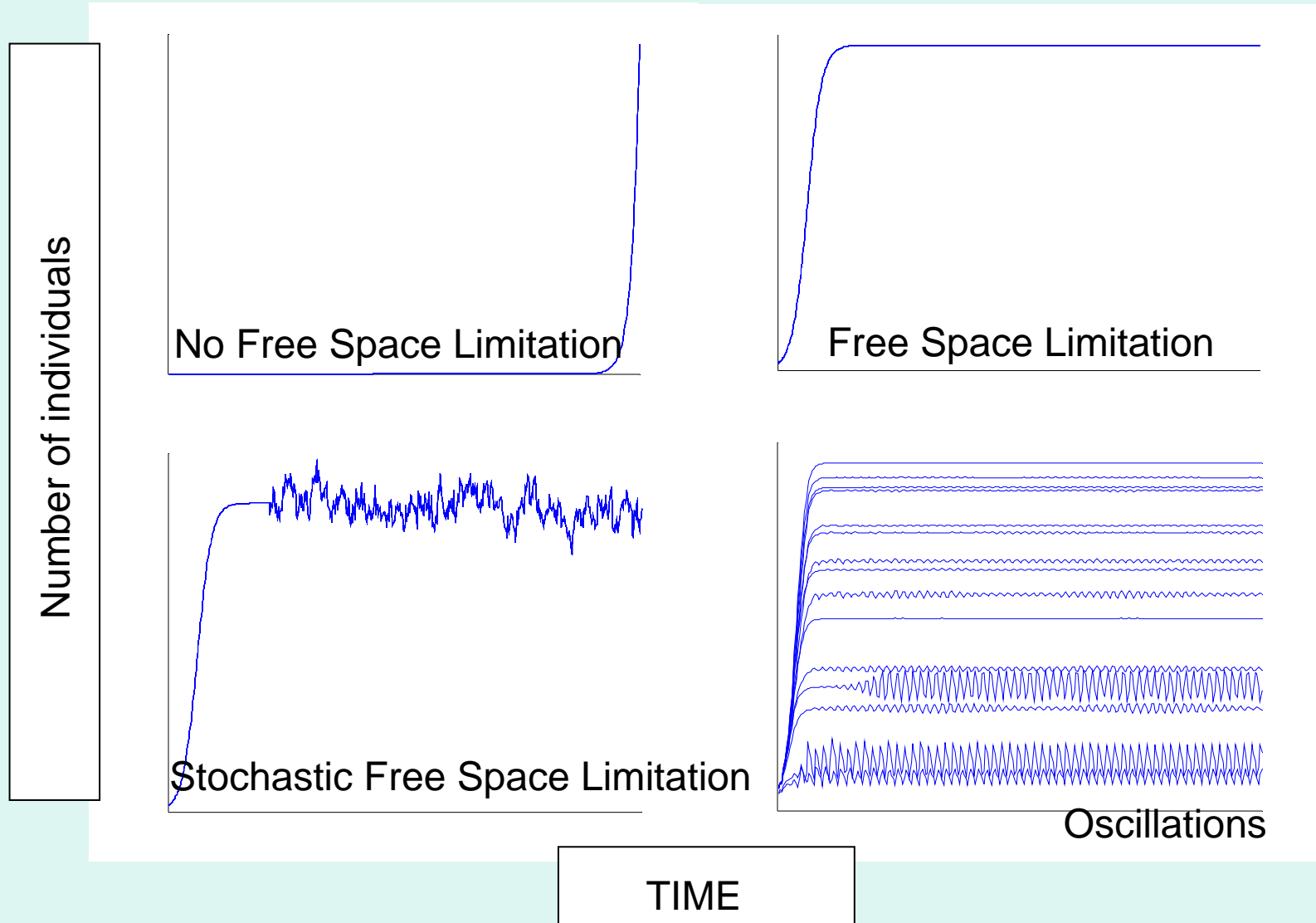
$$\frac{\hat{F}}{A} = \frac{1}{1 + s \int_{x_0}^{x_{\max}} \frac{x \exp(a_1(x))}{\alpha(x)} dx}$$

Stability analysis shows the addition of free space dependency leads to stable regimes, and thus acts as a **regulating** mechanism.

This is always true when growth is slow relative to mortality.

Otherwise, settlement rate higher than a critical value leads to **limit cycles**.

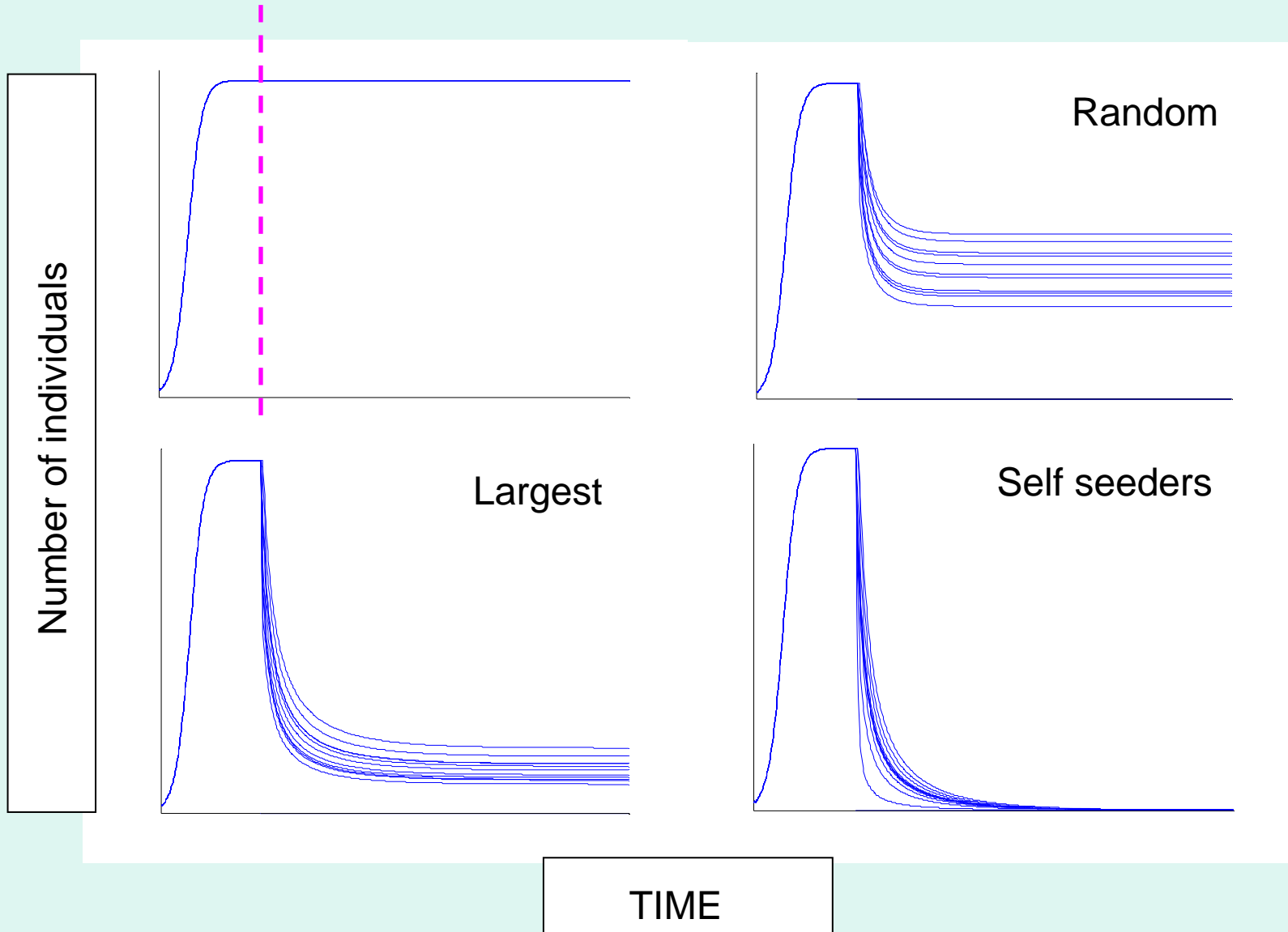
Adding free-space limitation to the meta-population case



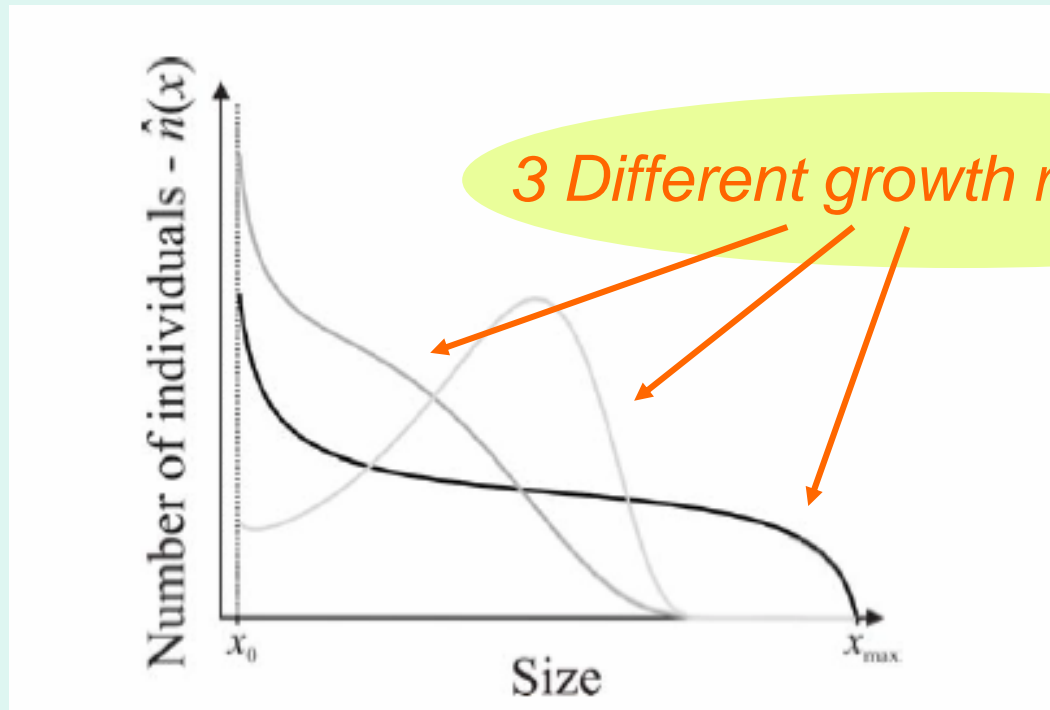
Importance of understanding population regulation

- Understanding the role of physical environment on regulation
- Protecting mechanisms responsible for the persistence of marine populations.
- Identify potential sensitivities

1) Persistence with 'knock outs'



2) Size distribution at equilibrium:



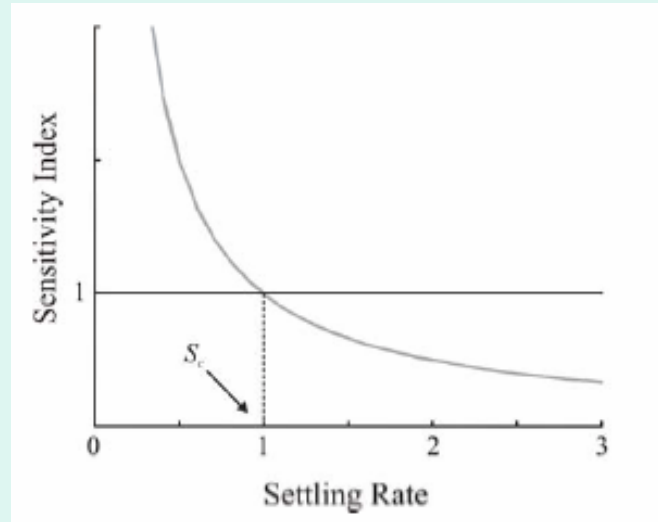
$$\hat{n}(x) = \frac{s\hat{F} \exp(a_1(x))}{\alpha(x)}$$

Where:

$$a_1(x) = -\int_{x_0}^x \frac{u(z)}{\alpha(z)} dz$$

3) Sensitivity index:

$$I(s) = \left| \frac{\partial \hat{F}}{\partial s} / \frac{\partial \hat{F}}{\partial u} \right|$$



We find that systems characterized by low recruitment rate will be more sensitive to changes in recruitment rate relative to death rate.