Mathematical models explaining regulating processes and stability of open marine systems

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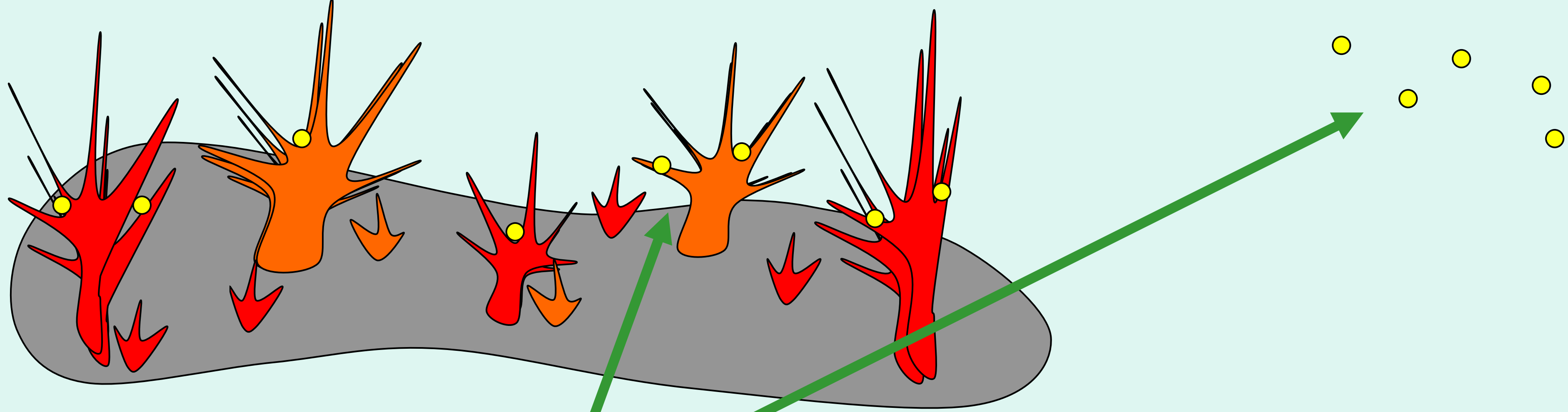


A bit about open systems...

[illegible] a **two-phase life cycle**:

a small **planktonic** larval phase

a larger **sedimentary** adult phase

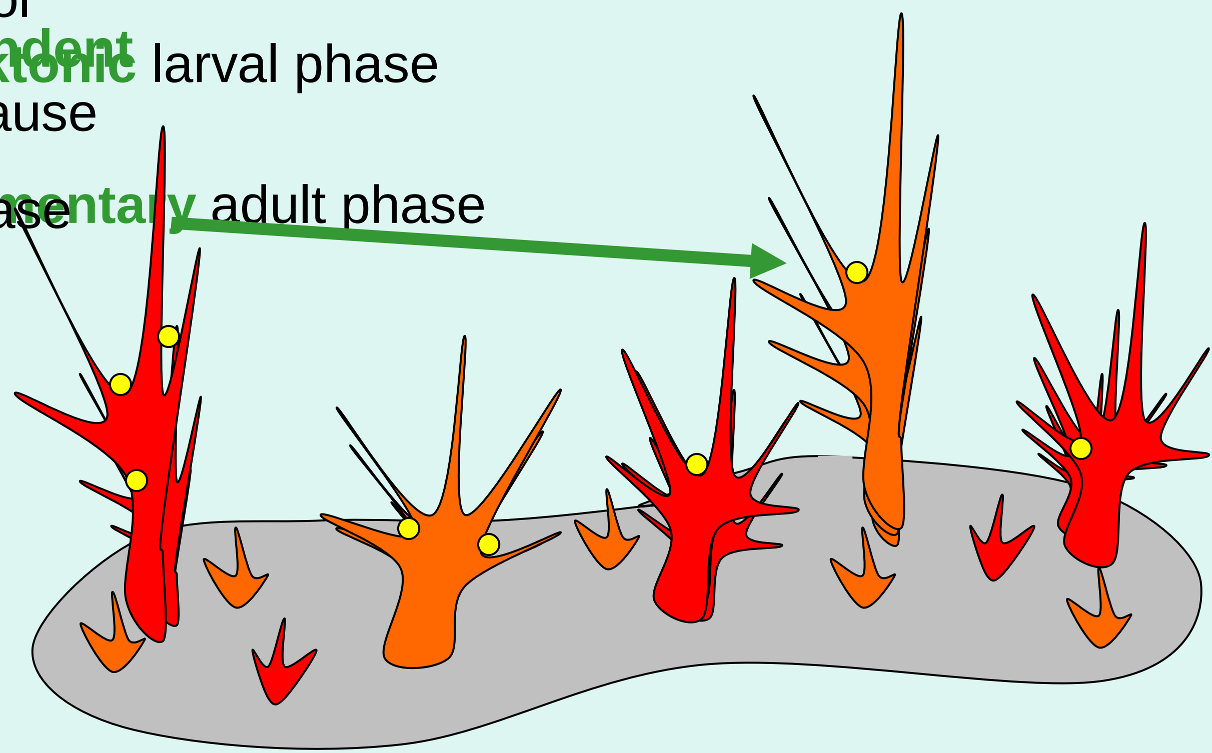
Larvae is dispersed by adults by **sexual reproduction**

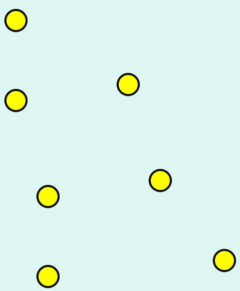
Sedimentary colonies increase their size by **asexual reproduction**.

Larvae produced on different habitats mix in a pelagic pool making the system **demographically open**

New recruits settle onto local habitats

Recruitment is **independent** of local habitat density because the system is open

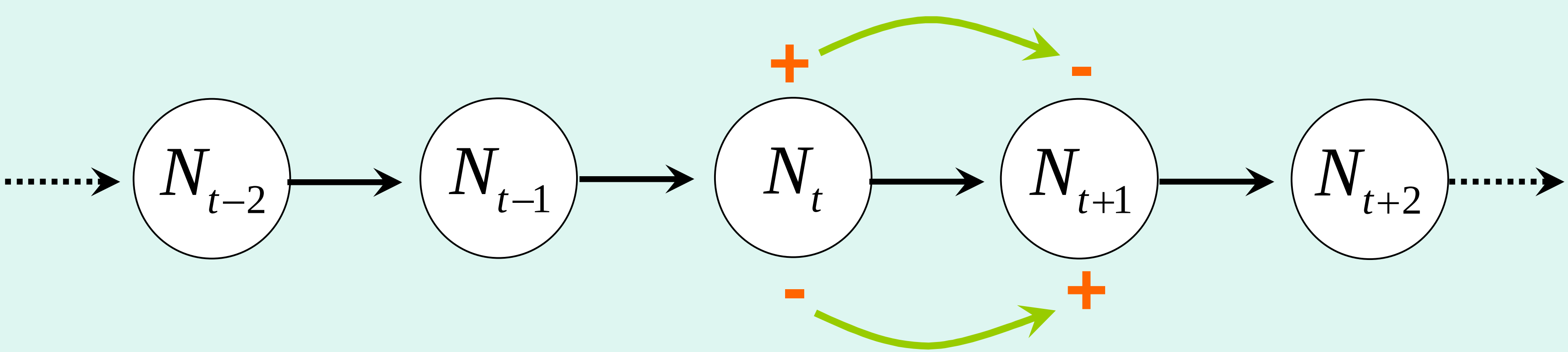




Regulation

A population is **regulated** if it **persists** for many generations with fluctuations bounded above zero and below infinity.

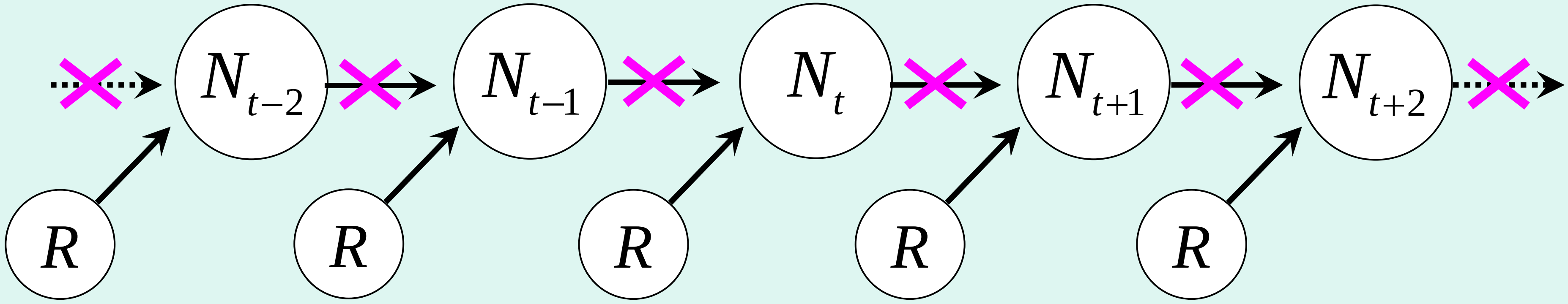
Regulation requires **negative feedback** such that the population will have the tendency to increase when small and decrease when large.



The paradox of open systems

- Stability of coral reefs

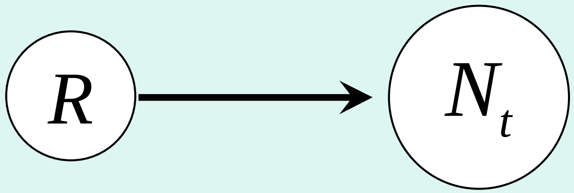
- Open systems don't seem to have any intrinsic regulation.



Controversy regarding persistence of open marine systems

- Recruitment Limitation hypothesis
- An increase in recruitment rate will cause an increase in local adult population size
- Populations that are locally open may persist indefinitely without density dependence

"recruitment regulation"



$$N_{t+1} = AN_t + R$$

The equilibrium solution is (when $t \to \infty$):

$$\hat{N} = R(I - A)^{-1}e = R \cdot \left(1, p_1, p_2, \ldots, \prod p_i\right)^T$$

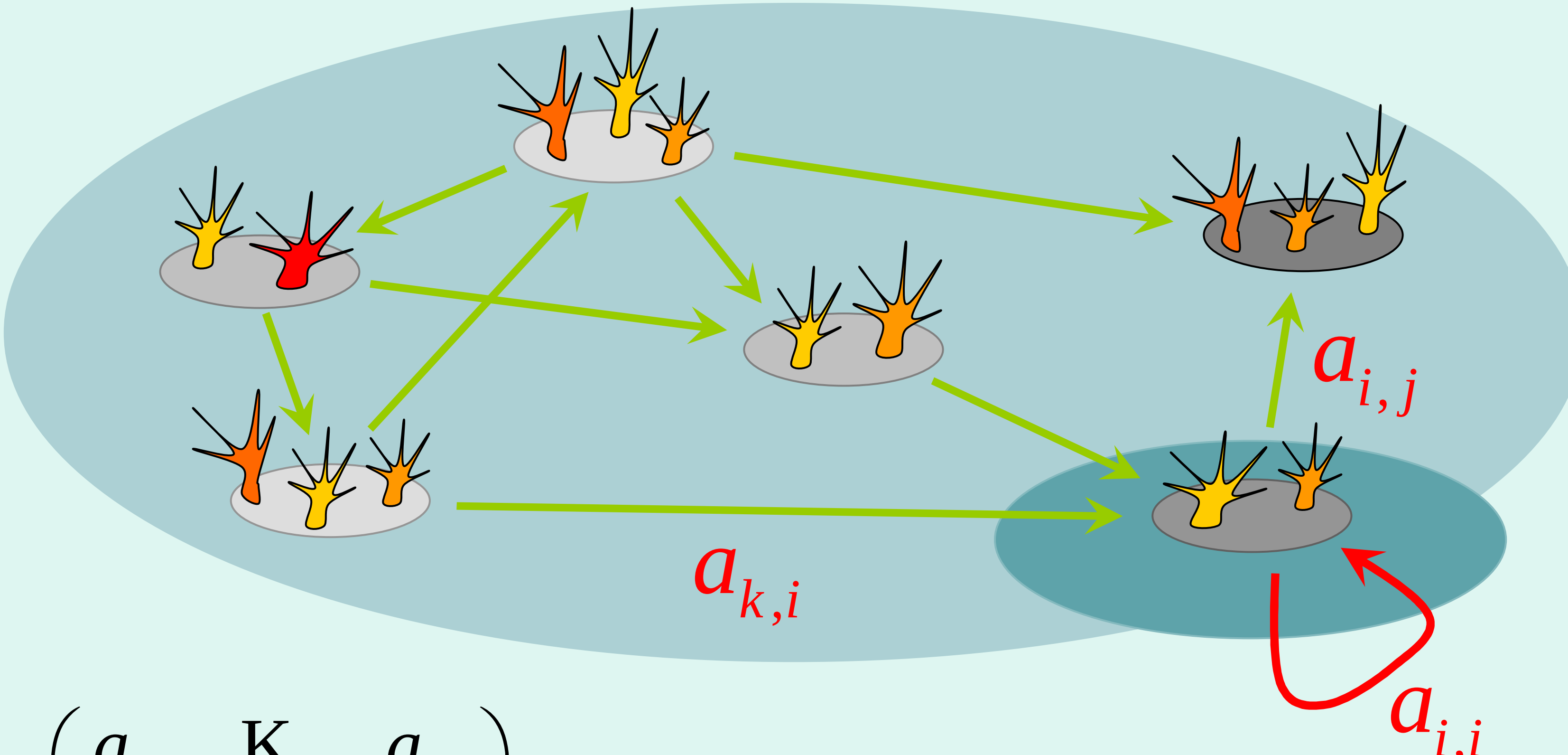
This solution is globally stable

- Density dependence is unnecessary for persistence at a local population.
- Supporting the Recruitment Limitation hypothesis.
- Even though **recruitment remains constant,** the **number** of recruits entering the system is high when local density is low, and low when density is high.

Metapopulations

- Looking only at the local population might be **incomplete**.
- On a wider scale, interconnected groups of local populations that are **linked by dispersal** form what is known as a metapopulation.
- A local population going extinct would be **re-colonized** by dispersed recruits.

On the small scale populations are open or partially open, but on the large scale they are closed:

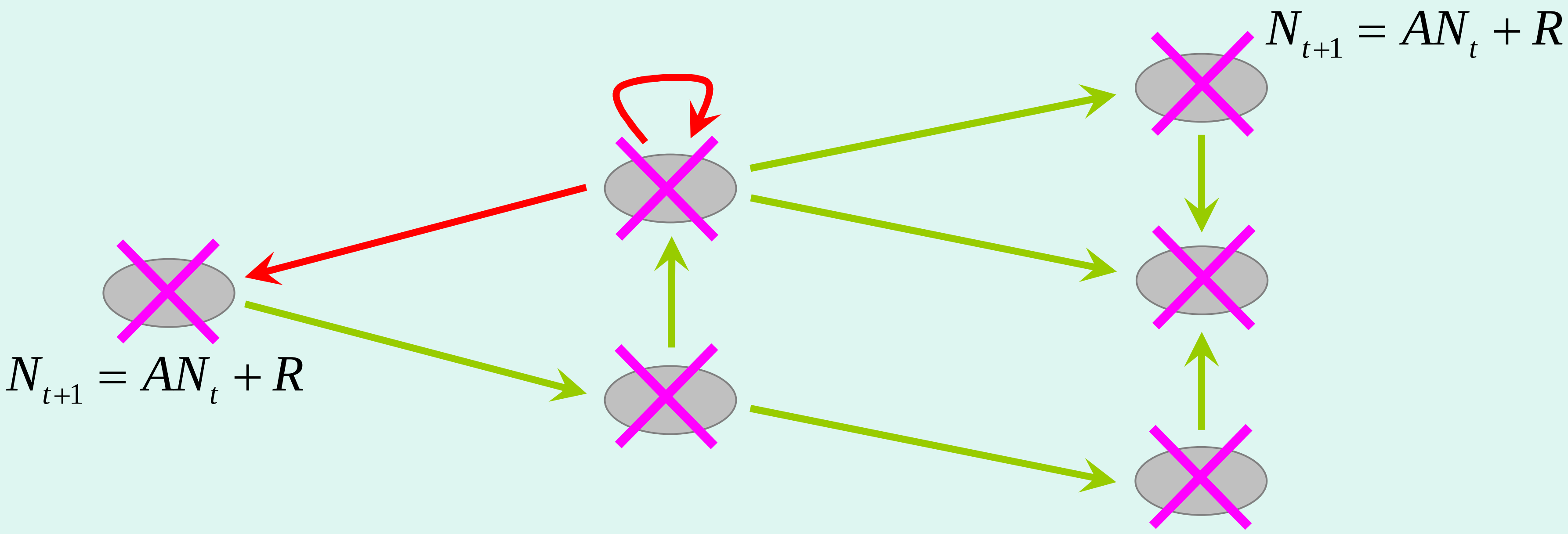


$$A = \begin{pmatrix} a_{11} & \mathrm{K} & a_{1n} \\ \mathrm{M} & \mathrm{O} & \mathrm{M} \\ a_{m1} & \mathrm{L} & a_{mn} \end{pmatrix}$$

The metapopulation can be described as a **network of connections**

A closer look at the metapopulation…

If there are no closed path of larval dispersal:



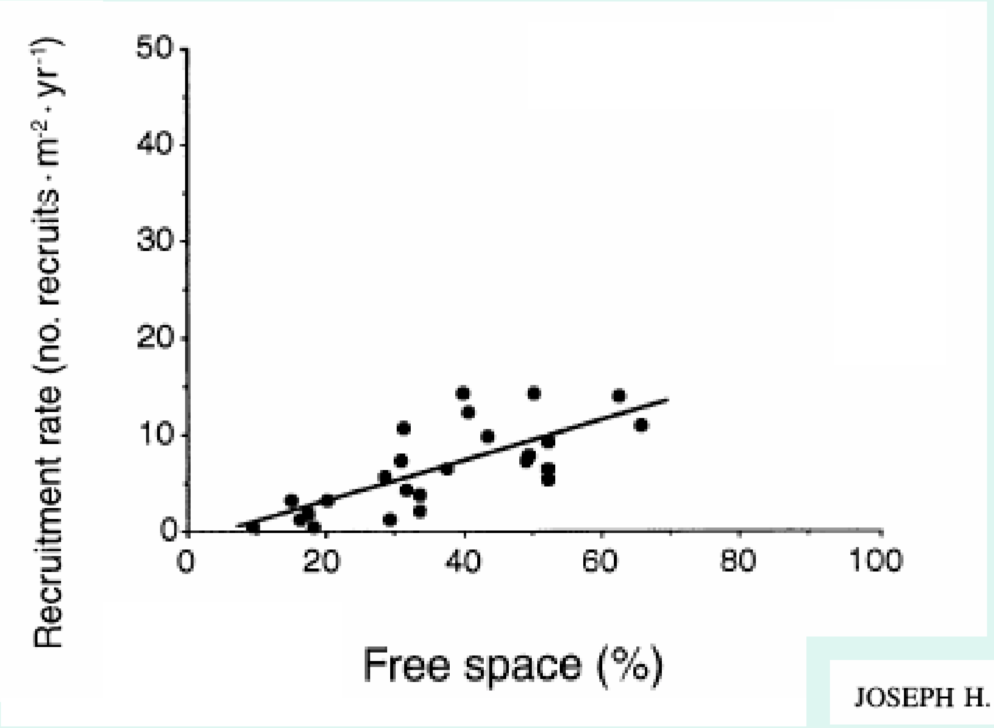
On the other hand, the existence of self cycles leads to positive feed back affecting the entire metapopulation.

- We have just seen that for a metapopulation to persist there must be some form of density dependence in the system.

- Local population models that assume a constant larval supply rate, **implicitly assume** that the wider meta-population is regulated, so the assumptions used in the local model are problematic.

- Incomplete models ignore critical feed backs at the scale of the metapopulation necessary to address questions of regulation

Free-space limitation



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Can the **limitation** of **free-space** provide **regulation** in both local and metapopulation cases?

Local size structured model

$$n_t(x,t) + (\alpha(x)n(x,t))_x = -u(x)n(x,t)$$

- growth rate
- number of individuals of size x at time t
- death rate
- size
- time

$$n(x_0,t)\alpha(x_0) = sF(t)$$

- minimum size
- recruitment rate
- free space at time t

$$F(t) = A - \int_{x_0}^{x_{\max}} xn(x,t)\,dx$$

- total area

Free space at equilibrium:

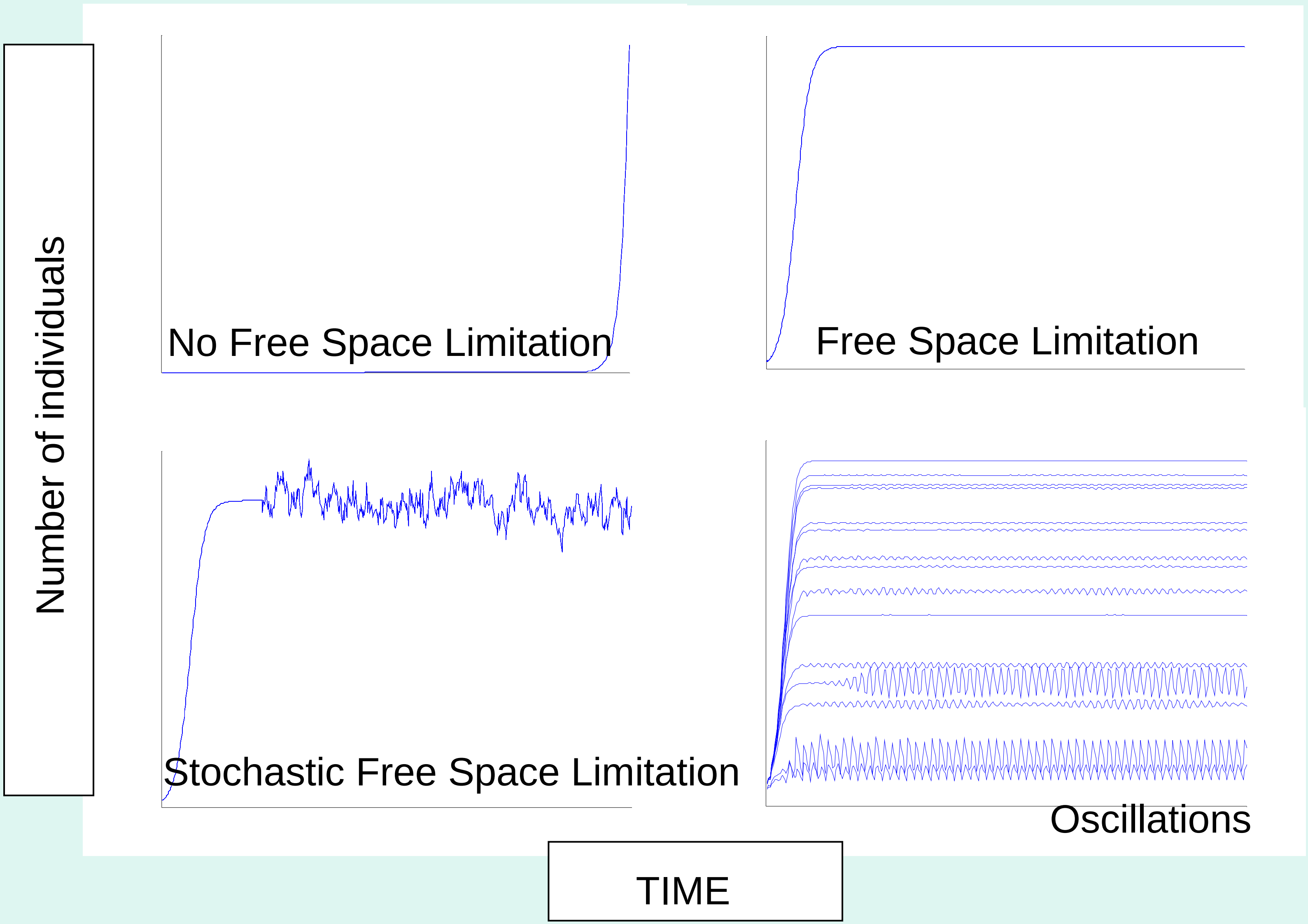
$$\frac{\hat{F}}{A} = \frac{1}{1 + s\int_{x_0}^{x_{max}} \frac{x\exp\left(a_1(x)\right)}{\alpha(x)}dx}$$

Stability analysis shows the addition of free space dependency leads to stable regimes, and thus acts as a **regulating** mechanism.

This is always true when growth is slow relative to mortality.

Otherwise, settlement rate higher than a critical value leads to **limit cycles**.

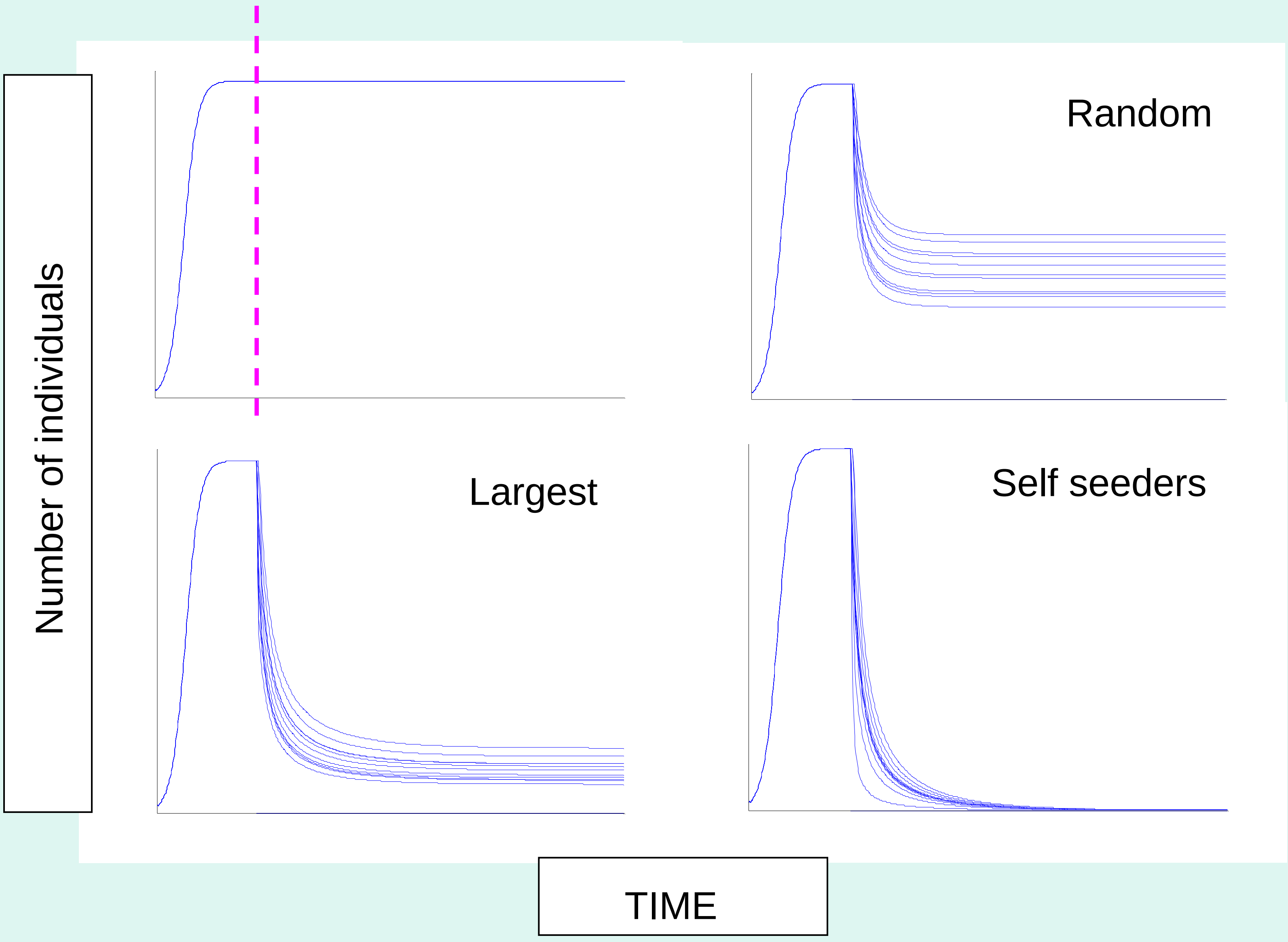
Adding free-space limitation to the meta-population case



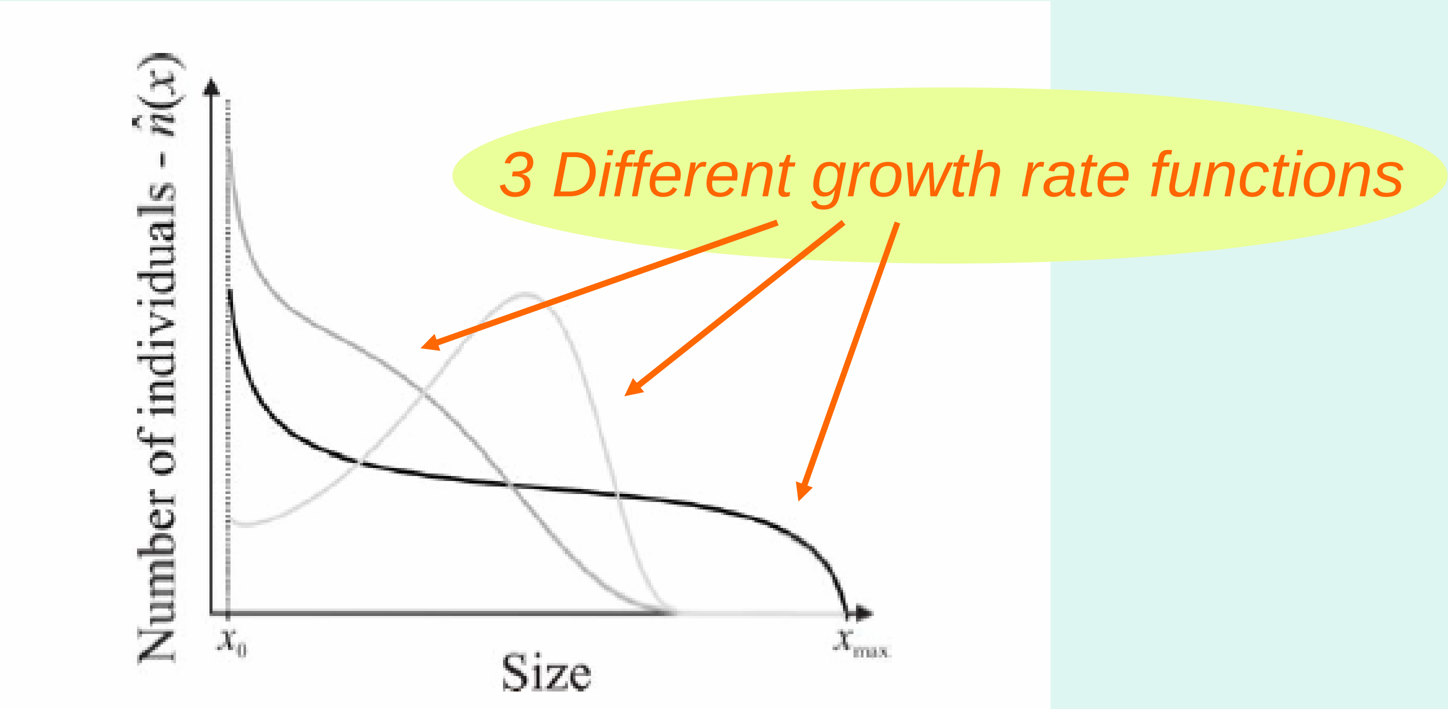
Importance of understanding population regulation

- Understanding the role of physical environment on regulation
- Protecting mechanisms responsible for the persistence of marine populations.
- Identify potential sensitivities

1) Persistence with ‘knock outs’



2) Size distribution at equilibrium:

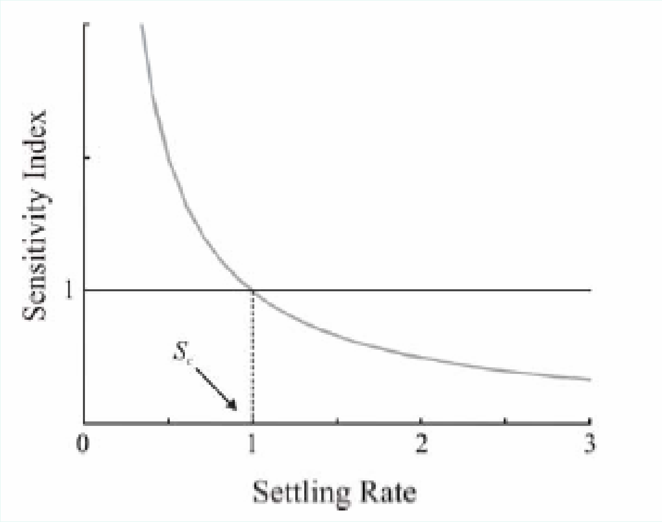


$$\hat{n}(x) = \frac{s\hat{F}\exp\left(a_1(x)\right)}{\alpha(x)}$$

Where:

$$a_1(x) = -\int_{x_0}^{x} \frac{u(z)}{\alpha(z)} dz$$

3) Sensitivity index: $I(s)=\left|\frac{\partial\hat{F}}{\partial s}\Big/\frac{\partial\hat{F}}{\partial u}\right|$



We find that systems characterized by low recruitment rate will be more sensitive to changes in recruitment rate relative to death rate.