A General Circulation Model for Deep Circulation on Gas Giants:
Internal Convection, Solar Heating and Zonal Flows

Part I: Axisymmetric Calculations

Yohai Kaspi and Glenn R. Flierl

Department of Earth, Atmospheric, and Planetary Sciences, MIT

Email: yohai@mit.edu

I. Introduction

The circulations on the gas giant planets are quite different from those in Earth’s atmosphere: the flows are dominantly zonal, having occasional vortices or waves rather than strong symmetric meridional flows as large as the zonal ones. The gas giants have both internal (gravitational collapse and cooling of cores) and external (solar) heat sources, with internal/external ratios of 0.9, 1.7, 2.3 for Uranus, Jupiter, and Saturn, respectively. The more recent Galileo and Cassini missions to the outer planets have yielded tremendous data bases, yet many questions as to the energy source, the depth and the stability of the jet remain unresolved.

We develop a new general circulation model (based on the MTGcm) in order to understand more fully the role of rotation, a large vertical density gradient, internal heat sources, and heating from the star. The interior stratification and zonal flows are certainly influenced strongly by convection while the large solar heating in the weather layer can create large-scale, baroclinically unstable shears. The evolution of such discontinuities, and the subsequent generation of the zonal jets will depend on the densities of the interior structure, therefore, we model the interior convection (with an equation of state suitable for hydrogen and helium mixtures) and the exterior baroclinic dynamics.

II. The Model

We modified the MTG general circulation model (MTGcm), designed for the dynamics of oceans and atmospheres, augmenting the non-hydrostatic version so that the grid can reach deep into the planet’s interior (including the strong variations in gravity). This extension basically allows the GCM to treat the complete dynamics of a whole sphere of gas instead of just a spherical shell. To account for the vertical change in mean density we use the anelastic approximation. We use an equation of state for hydrogen and helium mixtures (Saumon et al. 1995) which accounts for some of the complex interior thermodynamic processes. In summary the main features of the model are:

- Non-hydrostatic
- Deep spherical geometry
- Non-Boussinesq - Anelastic
- Varying gravitational acceleration
- Hydrogen-Helium equation of state
- Focusing by interior and solar heat fluxes

Model Thermodynamics

The gas is primarily composed of hydrogen and helium with small amounts of heavier elements. At low temperatures and pressures in the outer regions of the planet, hydrogen and helium is a molecular gas and the EOS may be approximated as an ideal gas. Deep into the interior, however, due to the high densities and relatively low temperatures (compared to stars), the giant planets lie in an extremely complex thermodynamic region. The many factors that separate the gas under these conditions from ideal gas behavior are pressure ionization, electron degeneracy, and Coulomb interactions. We use the SCVH EOS (Saumon et al, 1995), calculated specifically for high pressure hydrogen and helium mixtures, including these thermodynamic complexities. The pressure-temperature-density relationship is shown in Fig. 1, where it can be seen that up to about 10^15bars (~0.9 the radius of the planet) the SCVH EOS is close to an ideal gas, but it differs substantially for the deep interior.

For the dynamical equations to remain energetically consistent when incorporating the anelastic approximation the system must have an adiabatic reference state. We set the reference state to follow the adiabat that matches the Galileo probe measurements (Atkinson et al., 1996). The variation from this reference entropy is computed dynamically.

III. Effect of Rotation

The model is defined by four Control Parameters: the Prandtl, Rayleigh and Taylor numbers and a fourth parameter A, varying logarithmically between a constant mean density Boussinesq model, and a fully anelastic model. The Rayleigh number sets the internal horizontal heat flux driving the system.

For small Rayleigh numbers the anelastic vorticity equation takes the form:

\[ \mathbf{r} \cdot \nabla \mathbf{u} + \frac{\Omega}{\rho} \mathbf{r} \times \mathbf{u} + \frac{\rho}{\rho_0} g \mathbf{r} = \frac{1}{\rho} \frac{\partial p}{\partial r} \mathbf{r} + \frac{1}{\rho} \frac{\partial \theta}{\partial r} \mathbf{r} - \frac{1}{\rho_0} \frac{\partial p}{\partial r} \mathbf{r} - \frac{1}{\rho_0} \frac{\partial \theta}{\partial r} \mathbf{r} \]

The ratio of the baroclinic term to the right hand side and term on the left hand side, scales as the ratio of the magnitude of the buoyancy force to the rotation force. Thus keeping the Rayleigh and Prandtl numbers constant, changing the Taylor number would alter this balance. Below we show two cases, where only the Rayleigh number has been varied from a slowly rotating planet to one with a 10 hour period and a Rossby number similar to Jupiter’s. Both runs are well into the turbulent regime (Rayleigh number is >100 times critical) and, after the initial spin-up have reached a statistical steady state with sporadic emergence of plumes. The high Taylor number experiment (right side of Fig. 3) is dominated by Taylor columns, which extend to low latitudes as depicted by Boisse (1976). In the low Taylor number run the plumes are more sporadic, with no alignment along the rotation axis.

IV. The Effect of the Mean Density Gradient

To study the effect of the mean density gradient on the depth of the zonal flows, we vary the anelastic parameter A, ranging from a Boussinesq model to one with mean density variations from 0.1 Kg/m^3 at 1bar to 2000 Kg/m^3 at 10Mbar. The Taylor-Proudman theorem does not depend on the fluid being Boussinesq, and if the fluid were purely stratooid and had a very small Rossby number, the zonal velocities would extend deep into the planet interior. However the contribution from the baroclinic term may not be negligible. The 2D calculations indicate that \( \mathbf{r} \cdot \mathbf{u} = \frac{\partial \mathbf{u}}{\partial r} \) may be more nearly constant along the rotation axis, and as seen in Fig. 4, convection columns become more confined to the interior.

V. The Effect of Solar Forcing

We are interested to study the interaction between flow driven by internal convection and the flow resulting of exterior solar forcing. In Fig. 5 we add a solar heat flux absorbed in the top few layers. The plot on the right has only solar forcing, creating a meridional Hady cell extending all the way to the pole, while the left panel has both interior and solar forcings. Convection columns develop (constrained to high latitudes-elastic effects) and, at the higher levels, interact with the Hady cell, breaking it up into several cells.

VI. The Critical Rayleigh Number

Focusing on the stage of the initial convection before the system reaches a statistical steady state, we solve the linear stability problem for a perturbation in velocity, pressure and buoyancy. During this stage while perturbation is small the system is close to a state of linear growth (Fig. 6), and the velocities though weak extend to lower latitudes. We can solve the eigenvalue problem to find the critical Rayleigh number which depends on the Taylor number the latitude and the largest growing wave number, giving:

\[ Re_c = \left( \frac{1}{\lambda} \right) \left( \frac{\partial \mathbf{u}}{\partial r} \right) \left( \frac{1}{\lambda} \right) \frac{1}{\sin \lambda + \cos \lambda \theta} \]

For low Taylor numbers the first term is dominant giving uniform convection latitudinally (Fig. 3), but for fast rotating systems the second term is dominant giving a higher critical Rayleigh number at low latitudes and therefore preference for convection at higher latitudes.

VII. Summary

- in a rapidly rotating system the character of convective turbulence is strongly affected by the ratio of the magnitude of the buoyancy force to the rotation frequency (see part III)
- in a convectively driven anelastic system the property conserved in Taylor columns is \( \mathbf{r} \cdot \mathbf{u} \) rather than the velocity itself, (see part IV)
- interaction of convection columns with solar forced Hady cells may be stronger at high latitudes and can drive high latitude surface zonal jets (see part V)
- for large Taylor numbers the critical Rayleigh number has a latitudinal dependence that creates stronger convection at higher latitudes, and limits the extent of convective column in low latitudes. (see part VI)

VIII. Looking Forward: 3D Calculations

The next stage in this study is extending the model to 3-D, to further illuminate the role of rotation, internal heat sources, and heating from the star on the dynamics. We expect to see baroclinic instability in the solar-forced cases (even in the presence of convection) and will explore the interactions of the eddies with the deep flow and surface jets. Some initial 3D calculations are shown in Fig. 8.