An improved method for estimation of Jupiter’s gravity field using the Juno expected measurements, a trajectory estimation model, and an adiabat based thermal wind model

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Abstract

The upcoming high precision measurements of the Juno flybys around Jupiter, have the potential of improving the estimation of Jupiter’s gravity field. The analysis of the Juno Doppler data will provide a very accurate reconstruction of spatial gravity variations, but these measurements will be over a limited latitudal and longitudinal range. In order to deduce the full gravity field of Jupiter, additional information needs to be incorporated into the analysis, especially with regards to Juno’s trajectory. In this work, we propose a new iterative method for the estimation of the gravity field, using the Juno expected measurements, a trajectory estimation model, and an adiabat based inverse thermal wind model.

The results show that using this method the gravitational field is improved better to the ‘observed’ one, mainly due to the fact that the thermal wind model is taking into consideration the wind structure and depth. Thus, it is suggested that the method presented here has the potential of improving the accuracy of the gravity field calculated from the expected Juno flyby observations.

Introduction

Measurable perturbations to the gravity fields of Jupiter can result from mass anomalies due to two sources—theg rapid rotation of these planets, which distorts the planets into a non-spherical (oblate) shape; and density perturbations, which result from fast atmospheric winds organized on both planets into a broad zone of westward flow near the equator and easterly flow at high latitudes. The gravity field can be decomposed into spherical gravity harmonics, , which are defined as a weighted integral over the planet's density distribution:

where , the number of the Legendre polynomial, is the mean planetary radius, is the local density and is the local radius. On planets with internal dynamics (winds), the density is perturbed by the flow so that the total density is given by:

where the density is the hydrostatic density, and are the density fluctuations arising from internal dynamics. The gravity harmonics, can be then similarly decomposed into parts , , , where is the first part due to the gravitational potential of Jupiter, and is the second part due to dynamical perturbations arising from winds. In this work we aim to use data from the wind dynamics to estimate the gravity field.

Method

The forward model

The forward model calculates the gravity harmonics based on a given velocity structure. Given the planet rapidly we assume the fluid motion can be only a long cylinder parallel to the axis of rotation. Nonetheless, the wind speed can still decay forward the high pressure. We therefore assume that the zonal wind field has the general form

where , are the observed cloud level zonal winds extended along the direction of the axis of rotation, and is an unfolding decay depth of the cloud level winds representing the possible shear of the winds. The -folding depth of the winds is a free parameter, and varying it systematically allows us to explore the dependence of the gravity harmonics on the vertical extent of the winds. Since the flow to leading order is in geostrophic balance the wind structure must hold so that

\[ \bar{\Omega}(x) V(x) \rho_0 = \nabla \times \bar{\Omega} \times \nabla \rho_0, \]

where is the planetary rotation rate, is the 3D velocity and is the mean gravity vector. The adjoint model

An efficient way to tackle the problem of determining the internal structure of the wind field, given the observations of the gravity harmonics , and the forward model given above, is the adjoint method. This method allows for an effective optimization of the model solution with respect to a cost function and control variables. The cost function in our case is a function of the difference between the model calculated harmonics and those observed, and the control variable is the decay parameter . That cost function can have the form

\[ J = \int \left( \mathcal{L}_\omega (\Delta \omega) + \mathcal{L}_\rho (\Delta \rho) \right) d^3r, \]

where the model solution, is the observed one and is a weight given to each harmonics term according to the observational uncertainty. Our goal is to minimize the cost function, i.e. bring the model solution closer to the observed one, therefore we need to calculate its sensitivity to changes in the decay parameter . The adjoint model was therefore computed for the Juno wind field, and was shown to be able to reach a solution within to 20 iterations (see example in Fig. 1).

Combining the adjoint optimization with a trajectory estimation model

The trajectory estimation model is the primary tool for deriving the gravity variations from the Juno spacecraft trajectories. The Juno mission includes 25 passes in which the spacecraft would observe the planet at high latitudes. The deviation of those trajectories, from those expected from a spherical solid body, will be used to estimate the gravitational moments of Jupiter, both the static and the dynamic. The main limitation in that estimation is the limited spatial coverage of the 25 passages, resulting in large uncertainties in most regions of the planet (Fig. 2). In order to overcome this limitation, we propose a new iterative method for the estimation of the Jupiter gravity field, using the Juno expected measurements, the trajectory estimation model, and the adiabat based inverse thermal wind model. Beginning with an artificial gravitational field (taken from the thermal wind solution), the trajectory estimation model together with an optimization procedure is used to obtain an initial solution of the gravitational moments. As upper limit constraints, the model applies the gravity harmonics obtained from the thermal wind model with barotropic assumption. The solution from the trajectory model is then used as ‘observations’ for the thermal wind model, and together with an adjoint optimization method, the optimal perturbation depth of the wind is computed. As a final step, the gravity harmonics solution from the thermal wind model is given back to the trajectory model, along with uncertainties estimates, to be used as constraints for a new calculation of the gravity field.

Next, the estimated gravity field is used as ‘observations’ for the thermal wind model, and using the adjoint optimization, a solution is found with a corresponding wind depth (which we know should be 3000km). A solution is found for a depth of H=3164km. Next, this solution (not shown here) is used as initial guess for the estimation model, and its associated uncertainties are used as bounds for the solution search by the estimation model. In physical terms, the thermal wind model uses the partial solution obtained by the estimation model to calculate the solution in the entire spatial domain, taking advantage of the knowledge of the winds all around the planet.

As expected, the gravity field is poorly estimated by the estimation model (Fig. 4). Using the results of this run, the adjoint based thermal wind model lead to a solution of H=3069km. Repeating the same methodology as before, the estimation model is now to come with an estimation that is much closer to the observations (Fig. 8), therefore there is improvement in the case of shallow winds as well.

Conclusion

A new approach is proposed for the estimation of Jupiter gravity field from the Juno mission

An adiabat based thermal wind model is used to find the depth of the winds, given the observed gravity field

An iterative process leads, at least within the context of the thermal wind solution, to a much improved estimation of Jupiter gravity field for both deep and shallow winds

The method could be implemented in the Juno mission, as well as the Cassini gas Orbiter.

References