Optimized single-beam dark optical trap

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We propose a new scheme for constructing a single-beam dark optical trap that minimizes light-induced perturbations of the trapped atoms. The proposed scheme optimizes the trap depth for given trapping laser power and detuning by creating a light envelope with (a) an almost minimal surface area for a given volume and (b) the minimal wall thickness that is allowed by diffraction. The stiffness of the trap's walls, combined with the large detuning allowed by the efficient distribution of light intensity, yields a low spontaneous photon scattering rate for the trapped atoms. Our trap also optimizes the loading efficiency by maximizing the geometrical overlap between a magneto-optical trap and the dipole trap. We demonstrate this new scheme by generating the proposed light distribution of a single-beam dark trap with a trap depth that is ~33 times larger than that of existing blue-detuned traps and ~13 times larger than that of a red-detuned trap with the same diameter, detuning, and laser power. Trapped atoms are predicted to have a decoherence rate that is >200 times smaller than in existing single-beam dark traps and ~1800 times smaller than in a red-detuned trap with the same diameter, depth, and laser power.

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1. INTRODUCTION

Blue-detuned optical traps confine neutral atoms mostly in the dark, therefore reducing the perturbations, such as spontaneous scattering, light-assisted losses, and ac Stark shift of the atomic levels, induced by the trapping light on the atoms. Such traps facilitate long lifetimes and long atomic coherence times,[1] which are important for precision spectroscopy. The basic idea of blue-detuned traps is to surround a dark region with a repulsive dipole potential by use of laser light that is detuned above an atomic resonance.[2] Experimentally, blue-detuned traps are harder to achieve than red-detuned ones, for which a single focused beam constitutes a trap.[3] Several configurations for far-detuned dark optical traps were demonstrated in which gravity provided the confinement in one direction[2]: Light sheet traps were generated by elliptical focusing of two laser beams and overlap of the two propagating light sheets to form a V-shaped cross-section that resists the effects of gravity, whereas confinement in the laser propagation direction is provided by the beam's divergence.[1] Later, and to achieve larger trapping volume, a different trap was constructed with four light sheets that produced an inverted pyramid.[4] A single beam trap was demonstrated by use of two axicons and a spherical lens to generate a conical hollow beam propagating upward.[5] All these schemes rely on gravity (they are actually gravito-optical traps) and are limited to weak confinement.

Traps in which light provided the confinement in all directions were developed with hollow beams. Laguerre–Gaussian modes (doughnut beams) were used, together with additional plug-in beams, to form such a trap.[6]

Recently several dark traps based on a single laser beam were demonstrated[7–9] that provide greater experimental simplicity and permit dynamic changes in the trap geometry and strength that were used to increase the atomic density,[9] perform sensitive spectroscopic measurements,[10] and investigate atomic dynamics in atom-optics billiards.[11,12] In the first scheme the trapping beam was produced by passage of a Gaussian beam through a phase plate of appropriate size, which shifted the optical phase at the center of the beam by π rad. Interference led to a dark volume at the focus of the lens, surrounded by light in all directions.[7] An additional method, which produced a trap with a much larger volume and a more symmetric shape, was realized by simultaneous focusing of two diffraction orders of a properly designed binary phase element, consisting of concentric phase rings with a π-phase difference between subsequent rings.[8] Finally, a tightly focused rapidly rotating laser beam was used to create a time-averaged potential that confined the atoms.[9]

Existing single-beam dark optical traps have two main disadvantages: First, there is a large difference between optical potential barriers in the radial and in the axial directions, which results in inefficient use of the available laser power. Second, the traps show poor loading efficiency from the magneto-optical trap (MOT) that usually serves as the source of cold atoms. As opposed to the loading of attractive (red-detuned) traps, which is a continuous process in which the atoms cooled by the MOT beams are trapped by diffusion into the confining range of the trap[13] and other mechanisms,[14] loading of a blue-detuned trap is limited by the trap's geometrical overlap with the atomic cloud. The different volumes and shapes of the nearly spherical MOT and the highly elongated dipole trap are then detrimental to loading efficiency.

In all these single-beam configurations[7–9] the confinement in the beam propagation direction (z axis) is provided by the beam's divergence. As a result, the optical potential far from the beam's focus is much weaker than the potential in focus, and the trap depth (defined as the minimal potential height on the trap's surrounding wall) is greatly reduced. To demonstrate this quantitatively, we consider a blue-detuned trap with radius r (ideally, similar to the size of the atomic cloud from which the trap is loaded) and, for comparison, a red-detuned trap formed by two focused Gaussian beams intersecting at a right angle[15] and generating a trap with comparable dimensions ($w_0 = r$, where w_0 is the beam waist[16]). We chose a crossed trap, not the simpler focused Gaussian beam trap, because with a single focused beam a trap radius of 0.5 mm will result in an extremely large axial size (~1 m for the parameters that we use). However, our analysis also applies to a single focused beam. The red-detuned trap depth $\hat{U}$ is proportional to the laser intensity and for large detunings from resonance is given by

$$\hat{U}_{\text{red}} = k\frac{1}{2}\frac{2P}{\pi {w_0}^2} = k\frac{P}{\pi r^2}, \qquad (1)$$

where $k = \hbar\gamma^2/(8\delta I_S)$, γ is the natural linewidth, I_S is the transition saturation intensity, δ is the detuning, and P is the total laser power. [See Fig. 2(a) below for the potential of this trap.] For a blue-detuned trap, $\hat{U} = kI_{\min}$, where $I_{\min}$ is the minimal light intensity on the trap's surface. This trap depth is approximately the same for all the existing single-beam configurations described above[7–9] and for others.[17] For example, in our rotating-beam trap[9] the minimum of the potential barrier is on the z axis, and in this direction the maximum of the potential is achieved for z that satisfies $w(z) = \sqrt{2}r$, where $w(z)$ is the beam waist at z. [See Fig. 2(b) below for the potential of this trap]. Hence the trap depth is given by

$$\hat{U}_{\text{rotating}} = k\frac{2P}{\pi(\sqrt{2}r)^2}\exp\left[-\frac{2r^2}{(\sqrt{2}r)^2}\right] = \frac{U_{\text{red}}}{e}. \qquad (2)$$

Equation (2) shows that the trap depth of existing blue-detuned traps is ~$1/e$ of the trap depth of a crossed red-detuned trap with the same trap diameter, laser power, and detuning. As a result, to achieve the same trap depth with a given laser power requires smaller detuning for the dark trap, and therefore the reduction in light-induced perturbations is not fully exploited.

In this paper we propose, analyze, and experimentally demonstrate a novel optical scheme to generate a so-called optimal trap, which maximizes the trap depth and greatly reduces the light-induced perturbations to the atoms. We implement the scheme by surrounding a large dark volume by a light envelope with (a) an almost minimal surface area for a given volume,[18] (b) the minimal wall thickness that is allowed by diffraction, and (c) an almost constant wall height on every point on the envelope. The stiffness of the trap's walls, combined with the large detuning allowed by the efficient distribution of light intensity, yields a low spontaneous photon scattering rate for the trapped atoms.

2. METHOD AND ANALYSIS

Our optical configuration is illustrated in Fig. 1. With two refractive axicons, a Gaussian beam is converted into a collimated hollow beam with a ring cross section.[19] Surprisingly, the cross section of the ring is a near-ideal Gaussian, as we confirmed experimentally and also by solving the Fresnel diffraction integrals. Hence diffraction from the discontinuous apex of the axicon is found to be negligible. This beam is incident onto a binary phase element (BPE) composed of concentric phase rings with a π-phase difference between subsequent rings, thus creating a radial grating with uniform spacing (and, hence, a diffractive axicon) that has a diffraction efficiency of 40.5% into the +1 and −1 orders of diffraction.[20] The +1 order forms an outgoing cone of light with its base on the BPE and its apex to the left. The −1 order can be treated as a part of a virtual cone with its apex to the right of the BPE. When these two orders are imaged by an imaging system, they generate a spatial light distribution consisting of two hollow cones attached at their bases and completely surrounding a dark region.

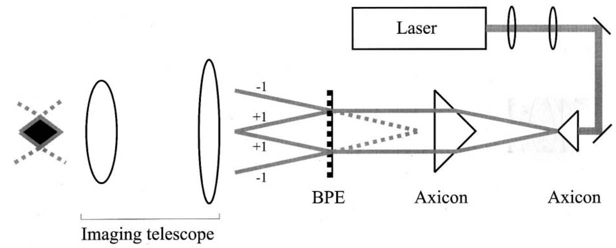

Fig. 1. Optimal dark optical trap generated by use of a telescope, two refractive axicons, and a BPE. The +1 and −1 orders of diffraction are imaged to generate the trap (printed in black). Note that all the elements in this figure, and hence the trap itself, have circular symmetry.

Two factors affect the potential height as a function of the distance, z, from the trap center: First, the radius of the trap's cross sections decreases (and thus the peak intensity increases) and, second, the Gaussian beam divergence causes the peak intensity to drop. The confining potential can then be approximated by

$$U(z) = \frac{U_{(z=0)}}{2}\frac{L}{[1 + (z/z_r)^2]^{1/2}(L - z)}, \qquad (3)$$

where L is the trap's half-length, $z_r = (\pi {w_0}^2/\lambda)$ is the beam's Rayleigh range, and λ is the laser's wavelength. Beam waist w_0 is chosen to partially balance these two effects and maintain a nearly constant potential height along the beam propagation axis. As a first approximation we require that the potential at $z = L/2$ be equal to the potential at $z = 0$ and then obtain $L = 3.5z_r$. An accurate numerical calculation that requires a nearly constant potential for the interval $z \in [0,(2/3)L]$ yields $L \approx 3.6z_r$.

Because the potential is nearly constant for any z,[21] the trap depth is equal to $U_{(z=0)}$, and for $r \gg w_0$ it is

$$\hat{U}_{\text{optimal}} = k\frac{1}{2}\left(\frac{2}{\pi}\right)^{1/2}\frac{P}{2\pi r w_0} \approx \frac{1}{5}\frac{r}{w_0}U_{\text{red}}. \qquad (4)$$

We define $R = r/w_0$ as the trap resolution (the ratio of the trap radius to the wall thickness); relation (4) shows that $\hat{U}$ is expected to be ~$R/5$ times larger than in a crossed red detuned trap and ~$R/2$ times larger than in existing single-beam blue-detuned traps.

This configuration also has a geometrical advantage over existing schemes: The trap aspect ratio is determined completely by the characteristics of the BPE, the axicons, and the imaging telescope and is independent of the beam width. It is possible then to construct an optimal trap, in the sense described above, that will also have a small aspect ratio, improving the match between the dipole trap and the nearly spherical MOT and thus gaining loading efficiency.

As a specific example of such an optimal trap we assume a trapping laser with $P = 1$ W and a sample of ^{85}Rb atoms, that are laser cooled in a MOT to a temperature of $\sim 5\ \mu$K and form a nearly spherical cloud with radius $r \sim 0.5$ mm. Our aim is to trap most of the MOT atoms in a dipole trap and minimize light-induced perturbations. For the crossed red-detuned trap we assume that $w_0 = r = 0.5$ mm, and, for both blue-detuned traps (rotating beam[9] and optimal), $r = 0.5$ mm and $w_0 = 10\ \mu$m (and therefore $R = 50$). Finally, for the optimal trap we choose $L = 1.5$ mm, yielding a cone half-angle of $\sim 18°$, which represents a trade-off between the true optimum (achieved for $L \approx r$ and corresponding to a cone half-angle of 45°) and a numerical aperture that ensures a moderate amount of aberrations from the imaging optics.

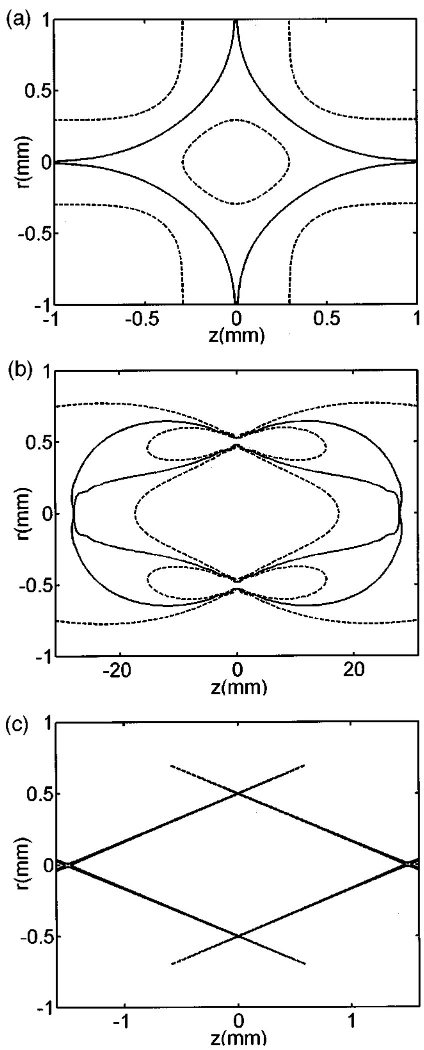

Fig. 2. Contour maps of the calculated potential for three different optical traps. Each solid curve is the contour corresponding to the trap depth. The dashed curves are contours at 0.5 and 1.5 times the trap depth. All the traps have the same radial dimension. (a) Crossed red-detuned trap, (b) rotating beam trap, (c) the proposed optimal trap (the contour lines practically coincide in this case).

Table 1. Maximum Possible Detuning, Atomic Darkness Factor, and Mean Spontaneous Photon Scattering Rate for the Three Configurations Analyzed in Text[a]

Type of Trap	Detuning (nm)	$\langle U\rangle/\langle E_k\rangle$	$\langle \gamma_s\rangle$ (s^{-1})
Red-detuned	1.35	5.670	31.057
Rotating-beam	0.5	0.258	3.847
Optimal	12.5	0.028	0.017

[a] A radius of 0.5 mm, a laser power of 1 W, a trap depth of $\sim 15\ \mu$K, and an atomic temperature of 5 μK are assumed for all three cases.

Note that $w_0 \approx 10\ \mu$m is the optimal value for this trap, as shown below. Figure 2 shows the calculated optical potential distribution of these three traps in the $x-z$ plane. Each solid curve in Fig. 2 is the contour corresponding to the trap depth, and the dashed curves are contours at 0.5 and 1.5 times the trap depth.

For the rotating beam trap [Fig. 2(b)] and a detuning $\delta = 0.5$ nm from the ^{85}Rb D_2 line, the trap depth is equal to approximately three times the mean kinetic energy of the atoms. According to Eq. (1) and relation (4), to obtain the same trap depth with the red-detuned trap [Fig. 2(a)] requires a detuning of $\delta = 1.4$ nm, and with the optimal trap [Fig. 2(c)] the required detuning is $\delta = 12.5$ nm.

We define the atomic darkness ratio of each trap as the ratio of ensemble-averaged potential to kinetic energies of the trapped atoms, $\langle U\rangle/\langle E_k\rangle$. Assuming trapped atomic gas in thermal equilibrium[2] and neglecting gravity, we find the ensemble-averaged potential energy:

$$\langle U\rangle = \int \mathrm{d}\mathbf{r} U(\mathbf{r}) \exp\left[-\frac{U(\mathbf{r}) - U_0}{k_b T}\right] \Big/ \int \mathrm{d}\mathbf{r} \exp\left[-\frac{U(\mathbf{r}) - U_0}{k_b T}\right], \tag{5}$$

where $U(\mathbf{r})$ is the three-dimensional potential function, U_0 is the potential at the trap center, and the integration is over the entire trap volume. The ensemble-averaged kinetic energy is $\langle E_k\rangle = (3/2)k_b T$. We numerically calculated $\langle U\rangle/\langle E_k\rangle$ and the mean spontaneous photon scattering rate $\langle \gamma_s\rangle = (\gamma/\hbar\delta)\langle U\rangle$, which determine the heating and decoherence rates of the trapped atoms.[2] Table 1 lists shows the results for $\langle U\rangle/\langle E_k\rangle$ and γ_s for each of the three traps (all of which have the same radius, laser power, and trap depth but different detunings). The darkness ratio for the optimal trap is improved by 1 order of magnitude compared with the rotating beam trap and is ~ 200 times better than for a red-detuned trap. The advantage of the optimal trap is even larger when the mean spontaneous photon scattering rates are compared: The improved darkness ratio combined with the efficient distribution of optical power that permits increased detuning results in a scattering rate that is ~ 226 times smaller than the rate in the rotating-beam trap and ~ 1800 times smaller than in the red-detuned trap. In this example we find that the mean time to scatter a photon is ~ 1 min, compared with the calculated ~ 250 ms for the rotating-beam trap.

For a given trap size the power efficiency, and therefore the minimal light intensity on the trap surface, is a function of w_0, as shown in Fig. 3(a). As can be seen, for our parameters the optimal power efficiency is achieved for $w_0 \approx 10.5\ \mu\text{m}$. Next, the detuning for each w_0 is adjusted (as $1/I_{\min}$) to maintain a constant trap depth. The darkness ratio and the mean spontaneous photon scattering rate are now calculated as functions of w_0, and the results are shown in Fig. 3(b). The optimal w_0 in terms of darkness ratio ($w_0 \approx 12.5\ \mu\text{m}$) is slightly different from the one that optimizes the power efficiency. The spontaneous scattering rate is minimized for $w_0 \approx 10.7\ \mu\text{m}$. Similar calculations for existing single-beam dark optical traps show that darkness ratio and scattering rate are almost independent of w_0.

3. EXPERIMENT

We demonstrated the proposed optical configuration of Fig. 1 with a He–Ne laser ($\lambda = 633$ nm), two refractive axicons with an apex angle of 175°, a BPE with 15-μm-wide rings, and a 1:5 imaging telescope. The laser beam waist was located on the BPE (and hence on its image plane at $z = 0$) and was set to the value $w_0 = 22.6\ \mu\text{m}$, which was calculated to be optimal for these parameters, by use of an additional telescope located before the two axicons. Figure 4 shows cross sections of the trap light's intensity at several planes along the beam propagation axis, measured with a CCD camera and a digital frame grabber. The light distribution generates the expected double-conical trap with an apex angle of 25° (aspect ratio 1:4.4, compared to 1:90 for our rotating-beam trap[9]). The trap's resolution was found to be $R = 65$, compared to $R = 6.5$ given in Ref. 9. At the $z = 0$ plane (the image plane of the BPE) the $+1$ and -1 diffraction orders exactly overlap and form a single ring with $r = 1.47$ mm. At $z < 0$ ($z > 0$) the $+1$ (-1) order ring linearly decreases in size (up to $|z| = L$). We introduced a stop between the telescope's lenses (not shown in Fig. 1) to suppress the BPE's unwanted diffraction orders and scattered light, resulting in a light intensity at the trap's center of <1/1000 of the wall's intensity.

Figure 5 shows the measured potential height as a function of the distance from the trap center, z. Also shown is a calculation from Eq. (3), and the measured val-

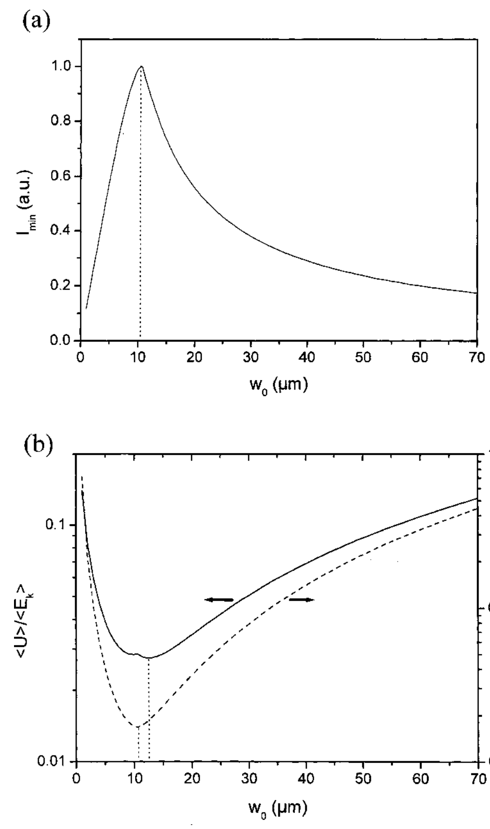

Fig. 3. (a) Calculated minimal light intensity $I_{\min}$ on the surface of the trap of Fig. 1 as a function of beam waist w_0 at $z = 0$, with the parameters specified in the text. For $w_0 = 10.5\ \mu\text{m}$ the light intensity on the trap's surface is almost constant, and therefore the weakest point is maximized. (b) The trap's darkness ratio, $\langle U\rangle/\langle E_k\rangle$ (solid curve), and the average spontaneous photon scattering rate $\langle \gamma_s\rangle$ (dashed line) as functions of w_0. $\langle U\rangle/\langle E_k\rangle$ and $\langle \gamma_s\rangle$ are minimized for $w_0 = 12.5\ \mu\text{m}$ and $w_0 = 10.7\ \mu\text{m}$, respectively. Note that for each value of w_0 the laser detuning varies as $(1/I_{\min})$ to maintain a constant trapping depth.

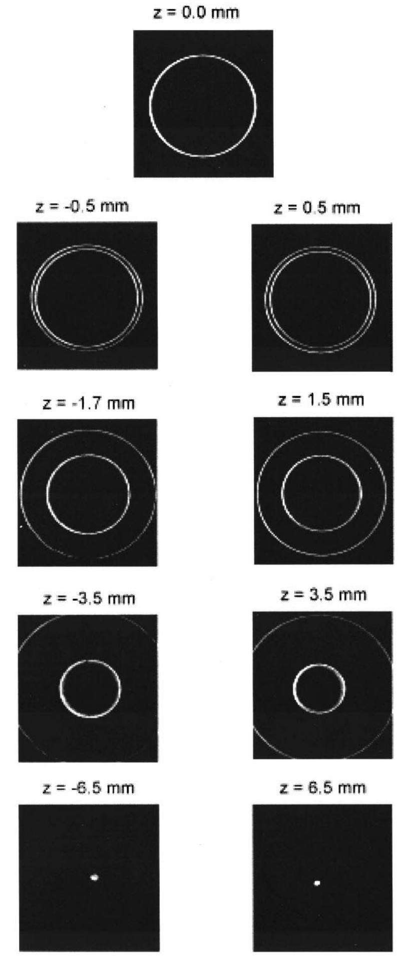

Fig. 4. Measured light-intensity cross sections at several planes along the optical axis for the optical configuration of Fig. 1, with the parameters described in the text. The internal circle for $z < 0$ ($z > 0$) is formed by the $+1$ (-1) diffraction order of the BPE and provides the radial confinement. The external circle is formed by the other diffraction order. For $z = 0$ the two orders exactly overlap and form a single ring with $r = 1.47$ mm.

ues of w_0, r, and L. A nearly constant potential height is indeed obtained along z, conclusive evidence of the optimal use of the laser power.[21]

4. CONCLUSIONS

We have proposed a novel scheme for constructing a single-beam dark optical trap that optimizes the trap depth for a given laser power and detuning. We achieved this trap by first minimizing the surface-to-volume ratio of the trap (to almost that of a sphere) and then choosing the minimal wall thickness that is allowed by diffraction. Because the interaction volume (i.e., the volume at which the trapped atoms feel a considerable dipole potential) is approximately the surface area multiplied by the wall thickness, the considerations described above inherently improve the darkness ratio of the trap and thus increase the atomic coherence times.[7] Moreover, for a given trap size, atomic temperature, and laser power, our configuration permits much larger detunings from resonance, which ensure even longer atomic coherence times. Our trap also optimizes the geometrical loading efficiency by permitting a small aspect ratio, thus maximizing the mode matching between the MOT and the dipole trap.

We experimentally demonstrated this new scheme and generated light distribution with a resolution (ratio of the trap radius to the wall thickness) of $R = 65$. Such a distribution corresponds to a single-beam dark trap with trap depth $R/2 \approx 33$ times larger than that of existing blue-detuned traps and $R/5 \approx 13$ times larger than that of a red-detuned trap with the same diameter, detuning, and laser power. This trap is expected to have a photon scattering time (and hence an atomic coherence time) that is ~200 times longer (for a given laser power) than existing single-beam dark traps and ~1800 times longer than red-detuned traps. Note that these exact ratios also apply to the relative ac-Stark shift of the hyperfine ground-state transition (known as an atomic-clock transition). These shifts are responsible for inhomogeneous broadening in microwave spectroscopy of the atomic clock transition and are proportional to $\langle \gamma_s \rangle$ for far-detuned dipole traps.[1]

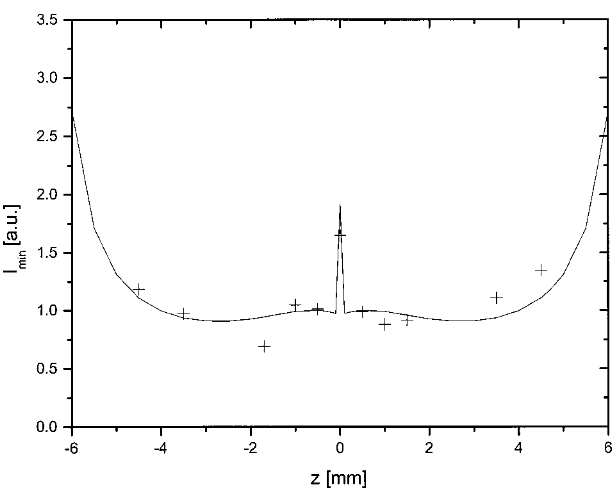

Fig. 5. Measured (+) and calculated (solid curve) minimal potential height as a function of the distance along beam propagation direction z for the trap of Fig. 1, with the parameters described in the text. The nearly constant potential height over most of the trap's length is evidence of optimal use of the laser power.

This trap can be used for precision measurements for which the long coherence times and the improved signal-to-noise ratio that can be obtained with a high number of atoms confined at a relatively low density suppress density-related effects such as collision-induced shifts of the atomic levels.[22] Finally, this optical configuration may also be used as dark optical tweezers[23] with which the combination of large trapping volume and thin light walls will enable either very large (hundreds of micrometers) particles or many small ones to be trapped in the same trap.

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