Observation of Chaotic and Regular Dynamics in Atom-Optics Billiards

Nir Friedman, Ariel Kaplan, Dina Carasso, and Nir Davidson
Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 22 September 2000)

We report on experimental observations of chaotic and regular motion of ultracold atoms confined by a billiard-shaped optical dipole potential induced by a rapidly scanning laser beam. To investigate the dynamics of the atoms confined by such an "atom-optics" billiard we measure the decay of the number of trapped atoms through a hole on the boundary. A fast and purely exponential decay, the clear signature of chaotic motion, is found for a stadium billiard, but not for a circular or an elliptical billiard, in agreement with theory. We also investigated the effects of decoherence, velocity spread, and gravity on regular and chaotic motion.

DOI: 10.1103/PhysRevLett.86.1518 PACS numbers: 05.45.Mt, 32.80.Pj, 42.50.Vk

The dynamics of particles moving in a closed region of coordinate space and scattering elastically from the surrounding boundary (a "billiard") is governed by the shape of the boundary [1]. For example, the motion in a plane circular or elliptical billiard is integrable, whereas it has been shown that the motion in the Bunimovich stadium is fully chaotic [2]. Much theoretical work has been done on the features of ergodic motion in billiards, including the decay of correlations [3–8], quantum manifestations of chaos [9], and also on the origin of thermodynamical laws [10]. Several experimental realizations of billiards include the measurement of conductance fluctuations in semiconductor microstructures [11] and level statistics of the eigenmodes of microwave cavities [12].

In this Letter we present experimental observations of integrable and chaotic motion of ultracold rubidium atoms confined by a billiard-shaped optical dipole potential induced by a rapidly scanning laser beam. Such atom-optics billiards offer unique flexibility and control over the parameters of both the billiard and the confined particles. These include the ability to form arbitrary billiard shapes and to change them in real time, to control the spatial and velocity distributions of the atoms and the interactions between them, to add a controlled amount of decoherence and noise to the system, and to combine dynamic measurements with high-precision spectroscopy. To investigate the dynamics of the atoms confined by such an "atom-optics" billiard we measure the decay of the number of confined atoms through a small hole on the boundary. A fast and purely exponential decay, the clear signature of chaotic motion, is found for the stadium-shape billiard, but not for the circular or the elliptical billiard, as indeed predicted by theory [3,4,8]. Moreover, we present the effects of velocity spread, decoherence, and gravity on the regular and chaotic motion, effects that could not be approached with existing billiard systems.

The three potential shapes that we investigate are shown in Fig. 1, each with a typical (classical) atomic trajectory of identical length, using our experimental parameters. The dynamics reveals two types of phase space effects: macroscopic separation and microscopic separation. The first are illustrated for the elliptical billiard in Fig. 1a. Here, phase space is macroscopically divided into two separate regions [13]: "external" trajectories that are confined outside elliptical caustics (smaller than the billiard itself but with the same focal points), and "internal" trajectories confined by a hyperbolic caustics, again with the same focal points, as shown in Fig. 1a. Hence, if a hole exists at the short side of the ellipse (upper inset in Fig. 2), atoms in those trajectories remain confined and never reach the hole. Alternatively, all trajectories, excluding a zero-measure amount, reach the vicinity of a hole on the long side of the ellipse (lower inset in Fig. 2) and hence the number of confined atoms decays indefinitely.

Comparing the atomic trajectories for the circular billiard (Fig. 1b) and the tilted-stadium billiard (Fig. 1c) illustrates the microscopic effects in phase space. Neither of these shapes has a macroscopic stable region in phase space for a hole at any point on the boundary. However, for the circular billiard nearly periodic trajectories exist (see Fig. 1b) that require an increasingly long time to sample all regions on the boundary (the exactly periodic trajectories that are completely stable have only a zero measure and hence can be neglected). This yields many time scales for the decay through a small hole on the boundary and results in an algebraic decay [3]. Finally, for the tilted-stadium billiard, phase space is chaotic, and hence each trajectory covers the entire phase space with a comparable time scale (for monoenergetic atoms). This results in a pure exponential decay [3,4], with a $(1/e)$ decay time scale of $\tau_c = \pi A/vL$, with A the billiard area, v the atomic velocity, and L the length of the hole [3]. We used a "tilted" stadium (and not the regular Bunimovich stadium) in order to reduce the number of nearly stable trajectories [6].

In our experiments we measure the decay in the number of ultracold rubidium atoms confined by a billiard-shaped optical dipole potential through a hole in the potential boundary. Our techniques to form, load, and detect such scanning-beam optical traps are described elsewhere [14]. Briefly, the billiard-shape repulsive optical potential is formed with a rapidly scanning (100 kHz) and tightly focused (e^{-2} radius $W_0 = 16\ \mu$m) blue-detuned

 0031-9007/01/86(8)/1518(4)$15.00

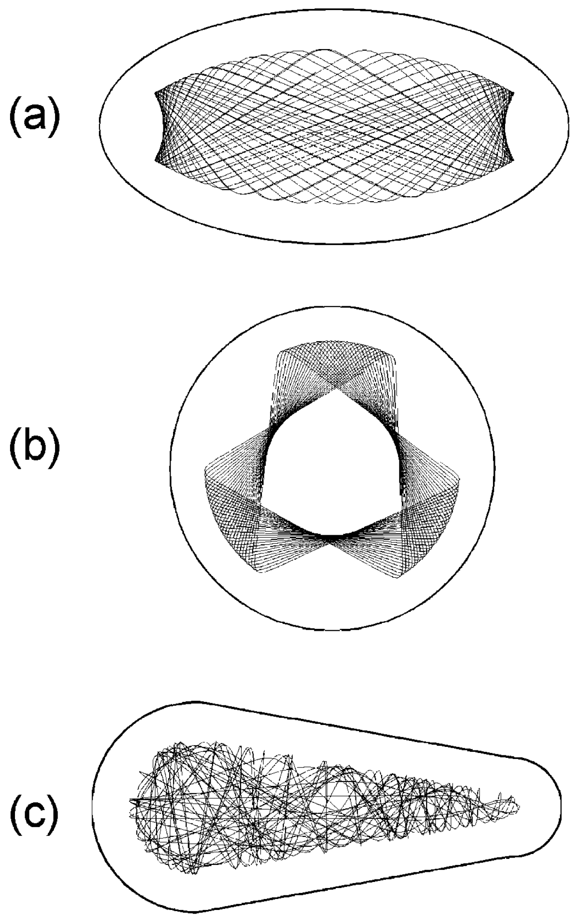

FIG. 1. Numerical simulation of two-dimensional trajectories of rubidium atoms in different atom-optics billiards. A 16 μm Gaussian beam is scanned along the boundary (shown in the figure), at 100 kHz. It yields a time-averaged potential of $\sim 1300 E_{\mathrm{recoil}}$. (a) Elliptical billiard (270 × 135 μm): only one of the two existing types of trajectories is shown, for which atoms are confined by a hyperbolic caustic and thus excluded from a certain part of the boundary. (b) Circular billiard (195 μm diameter): nearly periodic trajectories demand an increasingly long time to sample all regions on the boundary. (c) "Tilted" stadium (two half circles with 115 and 50 μm diameter, connected by two 170 μm straight lines): every atomic trajectory would reach a certain region in the boundary with a comparable time scale.

laser beam using two perpendicular acousto-optic scanners (AOS's). For such high scanning frequency, the optical potential is well approximated as a time-averaged quasistatic potential [14]. By controlling the deflection angles of both AOS's using arbitrary function generators, we create various billiard shapes, such as an ellipse, a circle, and a tilted Bunimovich stadium that confined the atoms in the transverse direction. The in-focus light distributions measured with a CCD camera, are shown in the insets in Figs. 2 and 4.

In the beam propagation direction, the atoms are confined by the divergence of the focused laser beam [14]. Alternatively, for tighter longitudinal confinement and to better approximate a true two-dimensional system we confine the atoms by a stationary blue-detuned standing wave along the optical axis. Here, the atoms are tightly confined near the node planes of the standing wave but move freely in these planes, forming "pancake" shaped traps separated by ~400 nm (half the wavelength of the standing wave laser) all with a nearly identical billiard potential in the other two dimensions [15]. Our experiments are conducted both with and without the longitudinal standing wave and the results are fairly similar. To minimize photon scattering, and thus ensure ballistic motion of the atoms and elastic reflections from the billiard potential walls, we use far-detuned laser beams to form the billiard and standing wave potentials −1.5 and 4 nm, respectively, above the atomic transition. The scattering rate is measured using a spontaneous Raman scattering technique [16] and found to be $<10\ \mathrm{s}^{-1}$, far too small to affect our results.

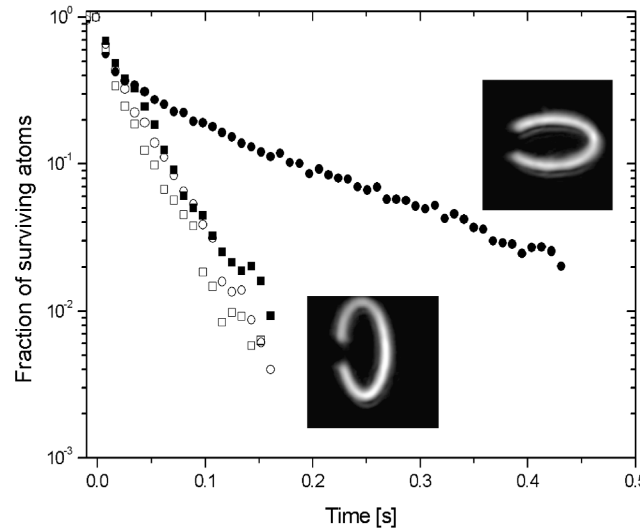

FIG. 2. Decay of the number of atoms from an elliptical billiard: the full symbols denote the unperturbed case, in which the surviving fraction for the ellipse with the hole on the long side (■) decays much faster than for the hole on the short side (●). The insets show charge-coupled device (CCD) camera images of the billiards' cross sections at the beam focus. The empty symbols show the case in which a 10 μs velocity randomizing molasses pulse is applied every 3 ms (see text).

The billiards are loaded from a compressed ^{85}Rb magneto-optical trap (MOT) [14]. The MOT beams are shut off after 700 ms of loading, 47 ms of compression, and 3 ms of polarization gradient cooling, leaving $\sim 10^6$ confined atoms with a temperature of 9 μK and a peak density of $\sim 5 \times 10^{10}\ \mathrm{cm}^{-3}$. 100 ms after the MOT turnoff, a hole is opened in the billiard potential, by switching off one of the AOS's for ~1 μs, synchronously with the scan. The number of atoms remaining in the trap is measured using fluorescence detection [14]. The ratio of the number of trapped atoms with and without the hole, as a function of time, is the main data of our experiments.

Figure 2 shows the measured survival probability for the elliptical billiard with a 60 μm hole on the long side and on the short side. To minimize the effect of gravity, the ellipse was rotated such that the hole's direction is perpendicular to gravity for both cases (see inset). The results reveal that the initial decay rate (for the first few points) is identical for both cases, as expected, confirming the experimental accuracy of our shapes and hole sizes [17]. However, at longer times, the survival probability for the hole on the short side became much higher than for the hole on the long side, as expected from the discussion above. In general, the stability due to such a macroscopic phase-space separation was very robust and remained nearly unchanged with or without the standing wave, for a large variety of hole sizes, for different orientations of the ellipse relative to gravity, and with small distortions of the ellipse's potential shape.

Next, we introduce a controlled amount of scattering to the atomic motion. We expose the confined atoms to a series of 10 μs pulses of optical molasses (using the six MOT beams) every 3 ms. During each molasses pulse, each atom scatters $\sim$30 photons, and hence its direction of motion is completely randomized, whereas the total velocity distribution remains statistically unchanged. Moreover, the atoms barely move during each short molasses pulse and maintain ballistic motion between the pulses. Thus, the effects of the molasses pulses resemble the effect of random scattering from fixed points (impurities) or of short range binary atomic collisions. The measured decay curves for this case are also shown in Fig. 2, for the two hole positions of the ellipse. As seen, for the hole on the long side the randomizing molasses pulses causes little change. However, for the hole on the short side, a complete destruction of the stability occurs, and the decay curves for the two hole positions approximately coincide.

This effect is further illustrated in Fig. 3 that shows the measured survival probability, 150 ms after the hole was open, for the two hole positions in the ellipse as a function

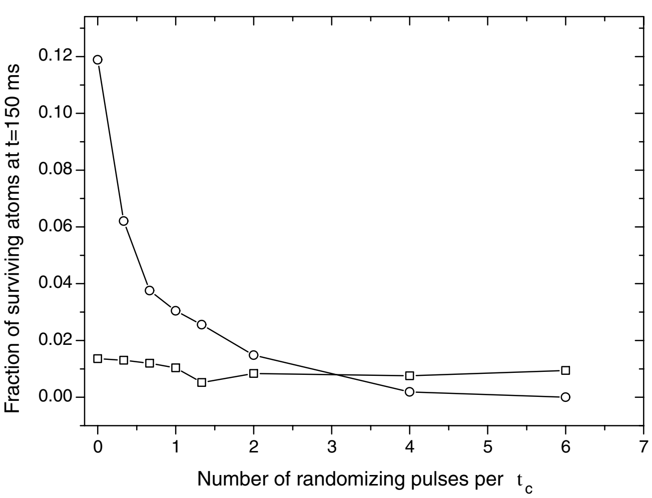

FIG. 3. The surviving fraction of atoms 150 ms after the hole was opened on the long (□) and the short (○) side of the elliptical billiard of Fig. 2, as a function of the repetition rate of the velocity randomizing molasses pulses. At a rate comparable to the escape rate, τ_c^{-1}, the stability of the trajectories was significantly reduced.

of the repetition rate of the velocity-randomizing pulses. As seen, the pulse rate required to significantly reduce the stability is approximately one pulse per τ_c, the $1/e$ decay time (calculated as 12 ms here, when averaging over the thermal velocity distribution of the atoms).

Next, we compare the decay curves through holes in the circular and the tilted-stadium billiards, where integrable and chaotic dynamics, respectively, are expected. Since the atoms are loaded into the billiard from a cloud in thermal equilibrium, their velocity is distributed around zero. Their rms transverse velocity distribution width was measured to be $11v_{\text{recoil}}$ [14]. To better approximate the monoenergetic case, where all atoms have identical velocity, and to reduce the relative contribution of gravitational energy ($\leq 50E_{\text{recoil}}$ for our billiards), we illuminated the atoms with a 1.5 μs pulse of a strong, on-resonance pushing beam perpendicular to the billiard beams and at 45° to the symmetry axis of the billiard. Following this pushing beam, the center of the velocity distribution was shifted to $20v_{\text{recoil}}$, while the rms width grew to $12v_{\text{recoil}}$. After an additional 50 ms of collisions with the billiard's boundaries, the direction of the transverse velocity distribution was completely randomized by the time the hole was opened.

The decay curves for the circular billiard and the tilted stadium with an identical potential height, identical area [18] and hole length [17], and the hole located at the bottom are shown in Fig. 4. Again, the initial decay is nearly identical for the two shapes, as expected. However, for longer times the decay curve for the circular billiard flattens, indicating the existence of nearly stable trajectories, whereas that of the stadium remains a nearly pure exponential. Also shown in the figure are the results of full numerical simulations that contain no fitting parameters. The simulations include the measured three-dimensional atomic and laser-beam distributions, atomic velocity spread, laser beam scanning, and gravity. As seen, the simulations fit the data quite well. For long times the circular billiard simulations predict greater stability than the experimental data. A possible explanation for this difference is the imperfections of the billiard beam shape (such imperfections were visible in particular away from the beam focus) [19]. The deviation of the stadium decay curve from the simulation results can be explained by the existence of a small amount of atoms trapped in areas far from the focus, where the real intensity distribution deviates from the simulated one to a greater extent. Finally, Fig. 4 also shows a pure exponential decay curve with a $(1/e)$ time constant $\tau_c = 4.9$ ms, calculated with the measured billiard parameters. As seen, it closely resembles the full simulation results, indicating that for this configuration, the stadium behaves nearly like an ideal chaotic billiard (two dimensions, no gravity, monoenergetic, zero wall thickness). Note that a strong deviation from a pure exponential decay, representing correlations due to the finite hole size, is expected at long times for chaotic billiards [4,6] but occurs below the noise level of our experiment.

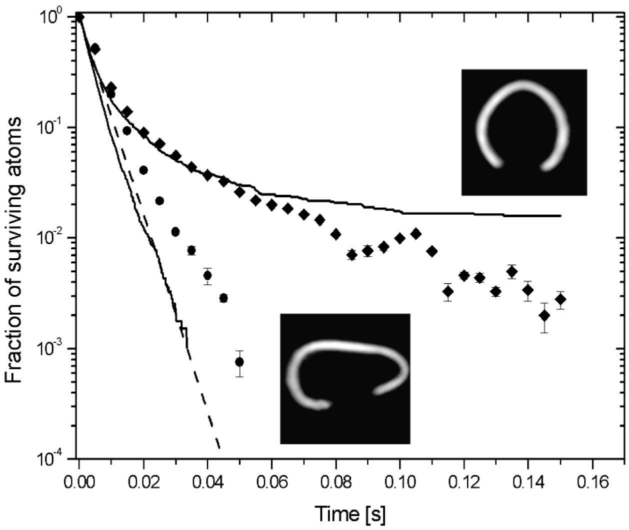

FIG. 4. Decay of atoms from circular and stadium billiards: the decay from the stadium billiard (●) shows a nearly pure exponential decay. For the circle (◆) the decay curve flattens, indicating the existence of nearly stable trajectories. Error bars were calculated from the spread in the experimental data for each point. The full lines represent numerical simulations, including all the experimental parameters, and no fitting parameters. The dashed line represents $\exp(-t/\tau_c)$, where τ_c is the escape time calculated for the experimental parameters.

Differences in the decay curves are also observed when the billiards are rotated by 90°, for smaller holes and without the standing wave. Without the pushing beam, the long-time decay for the stadium strongly deviates from an exponential shape and nearly coincides with that of the circular billiard. This is expected since many time scales exist even for the chaotic shape, due to the presence of many velocity classes.

Finally, both theory [20] and our simulations predict that as the walls of the chaotic billiard become softer, islands of stability appear and eventually the motion becomes completely integrable. We have seen evidence for this effect by observing a reduced difference between the decay of a circular billiard and stadium, when the relative wall thickness was increased.

In conclusion, an experimental observation of integrable and chaotic motion in atom-optics billiards was presented, and effects of scattering, velocity spread, gravity, and wall softness were investigated. The ability to form arbitrarily shaped atom-optics billiards and the precise control of parameters offered by ultracold atoms provide a powerful tool for the study of both classical and quantum chaos and their dependence on atomic collisions, quantum statistics, external fields, real-time temporal changes, and noise. Chaotic dark optical traps may also be useful for spectroscopy, since uniform phase-space mixing can suppress inhomogeneous broadening due to the ac Stark shift.

We gratefully acknowledge very helpful discussions with U. Smilansky. This work was supported in part by the Israel Science Foundation, the Minerva Foundation, and the United States–Israel Binational Science Foundation. N. F. acknowledges support from the Israel Ministry of Science.

[1] M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York, 1990).
[2] L. A. Bunimovich, Funct. Anal. Appl. **8**, 254 (1974).
[3] W. Bauer and G. F. Bertsch, Phys. Rev. Lett. **65**, 2213 (1990).
[4] H. Alt *et al.,* Phys. Rev. E **53**, 2217 (1996).
[5] R. Artuso, G. Casati, and I. Guarneri, J. Stat. Phys. **83**, 145 (1996).
[6] F. Vivaldi, G. Casati, and I. Guarneri, Phys. Rev. Lett. **51**, 727 (1983).
[7] B. Friedman and R. F. Martin, Jr., Phys. Lett. **105A**, 23 (1984).
[8] V. B. Kokshenev and M. C. Nemes, Physica (Amsterdam) **275A**, 70 (2000).
[9] S. W. McDonald and A. N. Kaufman, Phys. Rev. Lett. **42**, 1189 (1979).
[10] G. M. Zaslavsky, Phys. Today **52**, No. 8, 39 (1999).
[11] R. A. Jalabert, H. U. Baranger, and A. D. Stone, Phys. Rev. Lett. **65**, 2442 (1990); C. M. Marcus *et al.,* Phys. Rev. Lett. **69**, 506 (1992).
[12] H.-J. Stöckmann and J. Stein, Phys. Rev. Lett. **64**, 2215 (1990); E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990); S. Sridhar and E. J. Heller, Phys. Rev. A **46**, R1728 (1992).
[13] J. Koiller, R. Markarian, S. Oliffson Kamphorst, and S. Pinto de Carvalho, J. Stat. Phys. **83**, 127 (1996).
[14] N. Friedman, L. Khaykovich, R. Ozeri, and N. Davidson, Phys. Rev. A **61**, 031403(R) (2000).
[15] The diameter of the standing wave ($\sim$600 μm) is much larger than the size of any of the billiards to suppress its effects on the transverse motion.
[16] R. Ozeri, L. Khaykovich, and N. Davidson, Phys. Rev. A **59**, R1750 (1990).
[17] We define the size of the hole as its projection on a straight line at half the maximal potential.
[18] The billiard's effective area is the area enclosed by the potential contour line equal to the atomic average kinetic energy. The area of the circular billiard and the stadium were found to be equal, with a precision of $\sim$10%.
[19] Including imperfections in the beam shape in the simulations yielded similar effects of reducing the long-time stability of the circular billiard.
[20] V. Rom-Kedar and D. Turaev, Physica (Amsterdam) **130D**, 187 (1999).