Discrete Solitons

FALK LEDERER and
YARON SILBERBERG

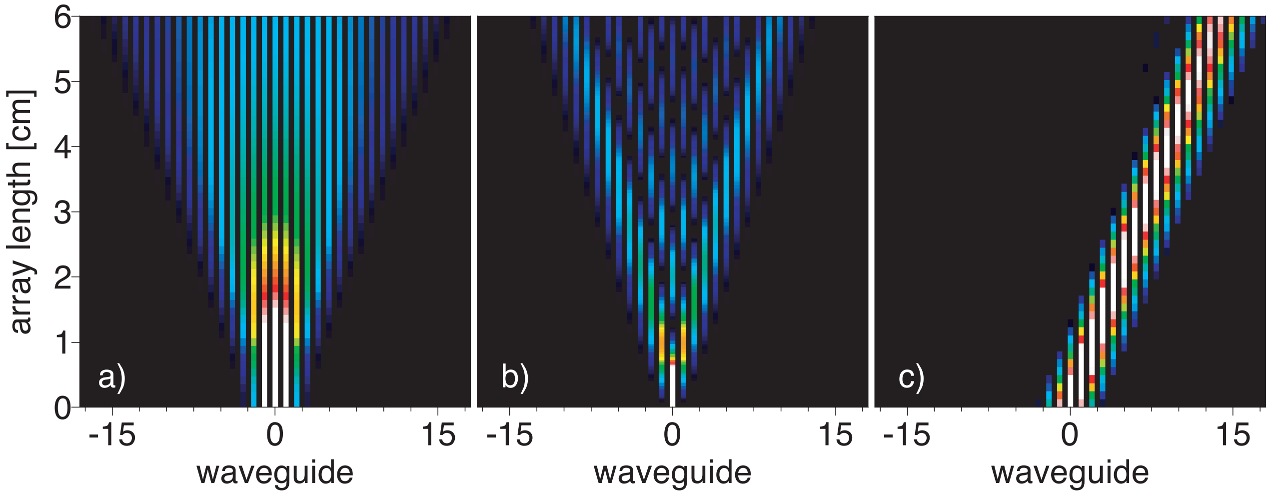

Figure 1. Discrete diffraction: numerical simulation of the field evolution in an array consisting of 75 evanescently coupled waveguides. a) Discrete diffraction of a finite beam (about eight waveguides are excited). b) Single waveguide excitation—visualization of Green's function (note that the energy is spread mainly into two lobes). c) Diffractionless propagation (about eight waveguides are excited).

Optics deals with continuous objects: The electromagnetic fields are continuous functions of space and time. There are situations, however, in which the evolution of an optical field can be represented as a discrete problem. This happens when the field can be described as a sum of discrete modes. One such simple and important case is that of a coupled one-dimensional (1D) waveguide array.[1,2] In a waveguide array, a large number (infinite in principle) of single-mode channel waveguides are laid one near the other such that their individual modes overlap. The evolution of the transversal field distribution is described by an infinite sum of coupled complex amplitudes of the individual modes. The problem of light propagation in a linear array was first treated theoretically by Jones,[1] and later studied by Yariv and co-workers,[2] who fabricated and tested such arrays in gallium arsenide.

The equivalent nonlinear problem was studied fifteen years later by Christodoulides and Joseph.[3] They showed that the field evolution in an array exhibiting a Kerr nonlinearity can be described by a discrete version of the nonlinear Schrödinger equation (NLSE). They found solitary solutions, which are now frequently termed discrete solitons. The study of these nonlinear objects became quite popular in the 1990s and it was shown that they also exist as dichromatic discrete solitons in a quadratic nonlinear environment. In particular, it was found that if the transverse excitation involves only a few waveguides, the discreteness of the respective nonlinear systems matters and any continuous approximation fails to correctly predict the nonlinear dynamics of the excitation. Moreover, it has been proven that in this case many novel exotic soliton solutions exist unknown from continuous systems. For a systematic overview of discrete soliton theory and a discussion of various novel soliton types, such as twisted, flat-top or dark solitons with internal phase dynamics, the reader is referred to the review *Spatial Solitons* and to the references therein.[4] The study of discrete optical solitons has remained primarily an area of theoretical research until recently, when several experimental studies were reported. Experiments in indium gallium arsenide (InGaAs) waveguide arrays demonstrated the formation of discrete solitons[5] and later involved the study of a few of their distinct properties.[6]

Discrete solitons are a particular variety of spatial solitons...spatial soliton formation requires interplay between nonlinear phase modulation and a linear correlation effect (diffraction).

Discrete solitons are a particular variety of spatial solitons. As we are aware from our knowledge of continuous media (bulk, or planar waveguides), spatial soliton formation requires interplay between nonlinear phase modulation and a linear correlation effect (diffraction). In discrete systems, this correlation is achieved by evanescent coupling between modes in adjacent waveguides. This coupling resembles diffraction in that it spreads an excitation of finite width across the array and can thus be termed discrete diffraction. Diffraction has its origin in a varying phase shift in propagation direction for different transverse wave-vector components. This is a consequence of the dependence of the longitudinal wave vector on the transverse vector (diffraction relation). The particular form of this relation controls the nature of diffraction, which is very peculiar in a discrete system. For this reason, a prerequisite to understanding discrete soliton formation is a complete understanding of discrete diffraction.

Discrete diffraction

In an optical directional coupler consisting of only two waveguides, the overlap between nearest-neighbor modes has a dominant effect on light propagation. It results in a phase-sensitive linear coupling between the waveguides. Upon propagation, the optical power is periodically exchanged between them. The period, or "beat length," is proportional to the inverse coupling strength.

The same type of coupling takes place

1047-6938/02/02/0048/6-$0015.00

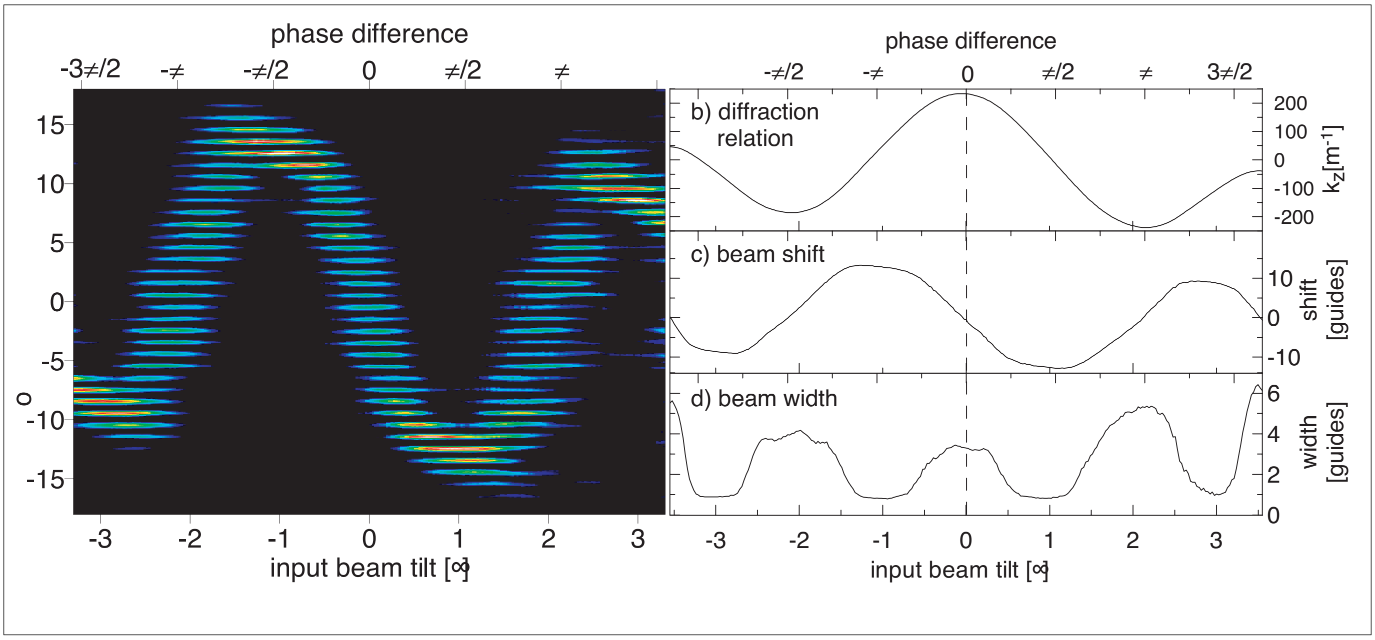

Figure 2. Experimental visualization of the diffraction relation in a polymer array (75 waveguides, waveguide separation d = 8.5 μm). *Left*: output distribution as a function of the beam tilt; *right*: a) diffraction relation k_z (k_x); b) beam shift of the output beam $\Delta x(k_x)$; c) beam width of the output beam as a measure for the diffraction coefficient. (From Ref. 9)

in an infinite array of identical waveguides. But because of the discrete translation symmetry of the infinite array, an initially strongly localized excitation (a few waveguides are excited) tends to spread over the whole array [Fig.1 (a)]. Mathematically, this spreading can be described by an infinite set of coupled-mode equations with nearest-neighbor interaction. The solution of this set has been known for many years.[1] The linear evolution when only one guide is excited is depicted in Fig.1 (b) and represents the Green's function of the discrete diffraction problem. Just as in canonical diffraction, the light distribution tends to broaden as it propagates. Notice how most of the light is concentrated in two distinct outermost lobes. This is in clear contrast to canonical diffraction in continuous systems.

But discrete diffraction has much more to offer. The magnitude of diffraction, normally fixed at a given wavelength, can now be engineered through the design of the array, i.e., the coupling strength. Moreover, this magnitude depends harmonically on the propagation direction. This has an exciting consequence: Diffraction can reverse its sign (anomalous diffraction) and can thus be eliminated completely [see Fig.1 (c)] in close analogy to dispersion in the temporal case.

To understand discrete diffraction, it is worthwhile to first examine the continuous case. Because we are assuming that beams are not too narrow, we can restrict ourselves to the paraxial approximation. As already mentioned, the nature of diffraction derives from the respective diffraction relation, i.e., the relation between longitudinal (k_z) and transverse wave-vector component. Each spectral component (k_x) of a finite beam is accumulating optical phase differently while propagating. The amount of phase gained by each component after propagating distance z is $\Phi(k_x)=k_z(k_x)z$. A group of transverse components centered at this component is transversely shifted, i.e., refracted, by an amount of

$$\Delta x = -\frac{\partial \varphi}{\partial k_x} = -\frac{\partial k_z}{\partial k_x} z .$$

The beam broadens because of the divergence between the different displacements $\Delta x(kx)$. The divergence

$$D = -\frac{1}{z}\frac{\partial^2 \varphi}{\partial k_x^2} = -\frac{\partial^2 k_z}{\partial k_x^2}$$

is the magnitude of diffraction or the diffraction coefficient, in analogy to the definition of dispersion.

The diffraction relation for a specific system can be derived from the evolution equation by assuming a plane wave solution. For a scalar propagation in a 2D continuous system, i.e., a film waveguide, the diffraction relation in paraxial approximation is parabolic, as

$$k_z(k_x) = \beta - \frac{k_x^2}{2\beta},$$

where β is the propagation constant of the film mode. Consequently, the transverse shift is proportional to the transverse wave-vector component (normal refraction) and the diffraction coefficient is constant ($\sim 1/\beta$) and positive (normal diffraction).

The evolution equation for the mode amplitudes E_n reads as

$$i\frac{dE_n}{dz} = \beta E_n + C\left(E_{n-1} + E_{n+1}\right).$$

In close analogy to a linear chain of weakly coupled atoms, the diffraction relation for the waveguide array is $k_z(k_x)=\beta+2C\cos(k_x d)$, where β is now the propagation constant of the channel waveguide modes, C the coupling constant and d the waveguide spacing. Because it is periodic, there are an infinite number of components k_x propagating for each k_z, spaced equally by $2\pi/d$. It suffices to restrict them to the first Brillouin zone ($|k_x|<\pi/d$). Now the beam shift upon propagation is also periodic and attains a maximum value $|\Delta x|=2Cd_z$ at $|k_x|<\pi/2d$. For increasing k_x the shift decreases for $\pi/2d < |k_x| < \pi/d$ (anomalous refraction). The diffraction coefficient is now $D=2Cd^2\cos(k_x d)$. Thus it can be engineered in sign and size by varying the waveguide spacing and/or the transverse wave vector (initial phase tilt). For $|k_x| = \pi/2d$ diffraction is arrested [see Fig.1 (b)] and for $\pi/2d < |k_x| < \pi/d$ it is anomalous. These peculiarities led to a renewed interest in the linear properties of discrete optical structures. Recently, photonic Bloch oscillations in waveguide arrays were predicted and experimentally verified.[7-9] Engineering of diffraction[10] and the experimental visualization of the periodic diffraction relation were also reported.[11] In Fig. 2, the experimentally obtained diffraction relation, the anomalous beam shift (refraction), and the output beam width as a measure of the diffraction coefficient are displayed for a polymer waveguide array.[11]

Discrete diffraction and nonlinearity

With regard to the formation of resting or slowly traveling discrete solitons, the immediate environment of the top ($k_x d=0$) and the bottom of the dispersion curve $|k_x d| = \pi$ are relevant. In both cases the diffraction coefficient has the same modulus but a different sign. Thus, in the linear array for both cases (phase difference between adjacent guides is zero for unstaggered solutions or π for staggered solutions), beam spreading is identical because it is only governed by the square of the diffraction coefficient [see Fig. 1 (c)]. But phase evolution will be reversed. Thus, when nonlinearity comes into play, we expect a richer spectrum of nonlinear effects.

A prerequisite to understanding discrete soliton formation is a complete understanding of discrete diffraction.

For example, it is known that a nonlinear increase of refractive index leads to self-focusing. However, this occurs only in a medium with normal diffraction. If discrete diffraction is present, a nonlinearly induced increase of the refraction coefficient leads to defocusing and to a decrease in focusing provided that in both cases the initial excitation is staggered (phase difference π between adjacent guides, bottom of the dispersion curve). This defocusing and decrease in focusing will crucially affect soliton formation.

In Fig. 3 we show an experimental demonstration of self-focusing and defocusing in an AlGaAs waveguide array achieved by working at different points on the dispersion curve.[10] By applying tilts to the input beam, different groups of k_x modes are launched. Figure 3 (a) shows an image of the input beam. Figures 3 (b-c) are images of the output facet when $k_x=0$ for low and high input powers, respectively. As can be seen, in this condition of normal diffraction, self-focusing, which leads to the formation of bright solitons, occurs. Figures 3 (d-e) are again for low and high power, but when $k_x d=\pi$ and the diffraction is anomalous. The dark notches that are seen between the waveguides in this case are an indication of the π phase jumps between adjacent guides. When low optical power is injected, the beam is slightly broadened, as in the normal case.

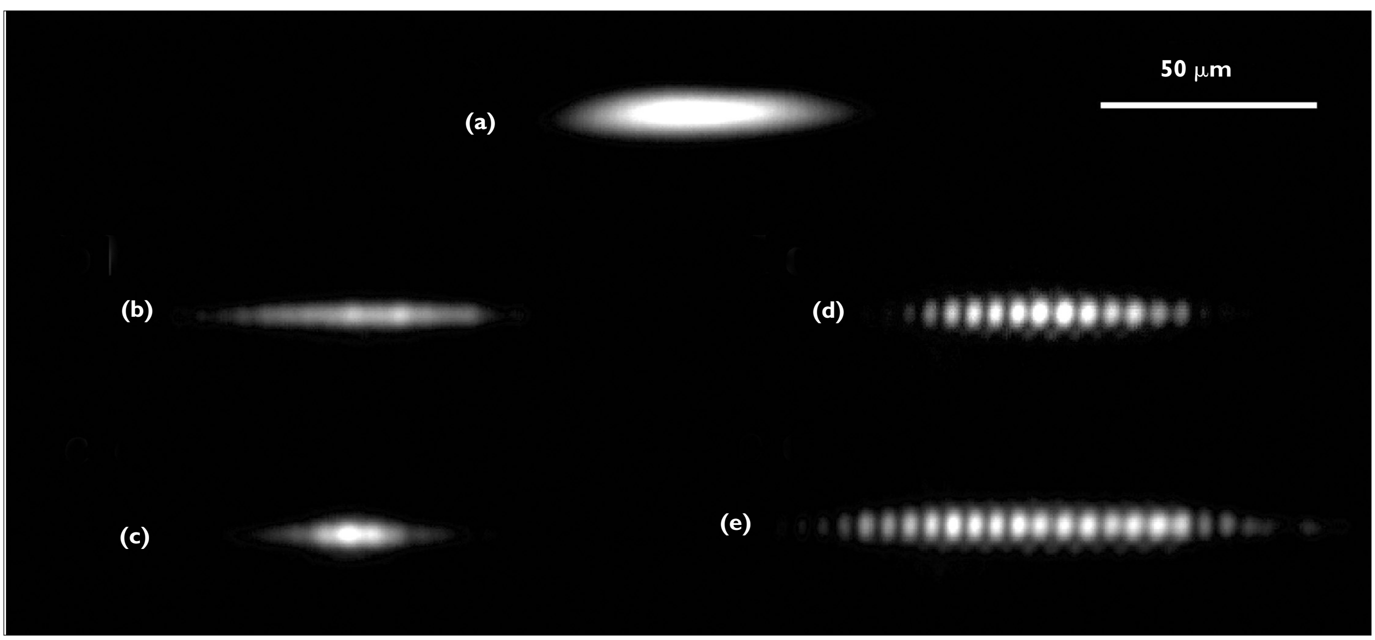

Figure 3. Experimental results showing both nonlinear self-focusing and self-defocusing in an array of GaAlAs waveguides, for slightly different initial conditions. (a) The input beam, ~35 μm wide at FWHM. (b) Light distribution at the output facet for normal dispersion in the linear regime. The beam slightly broadens through discrete diffraction. (c) At high power (I_{peak}~150W), the field shrinks and evolves into a discrete bright soliton. (d) For an anomalous dispersion condition, when the beam is injected at an angle of 2.6±0.4° inside the array, it broadens slightly as in (b). Note the dark lines between the optical modes resulting from the π-phase flips between adjacent waveguides. (e) When the power is increased (I_{peak}~100W), the distribution broadens significantly due to self-defocusing. (From Ref. 10)

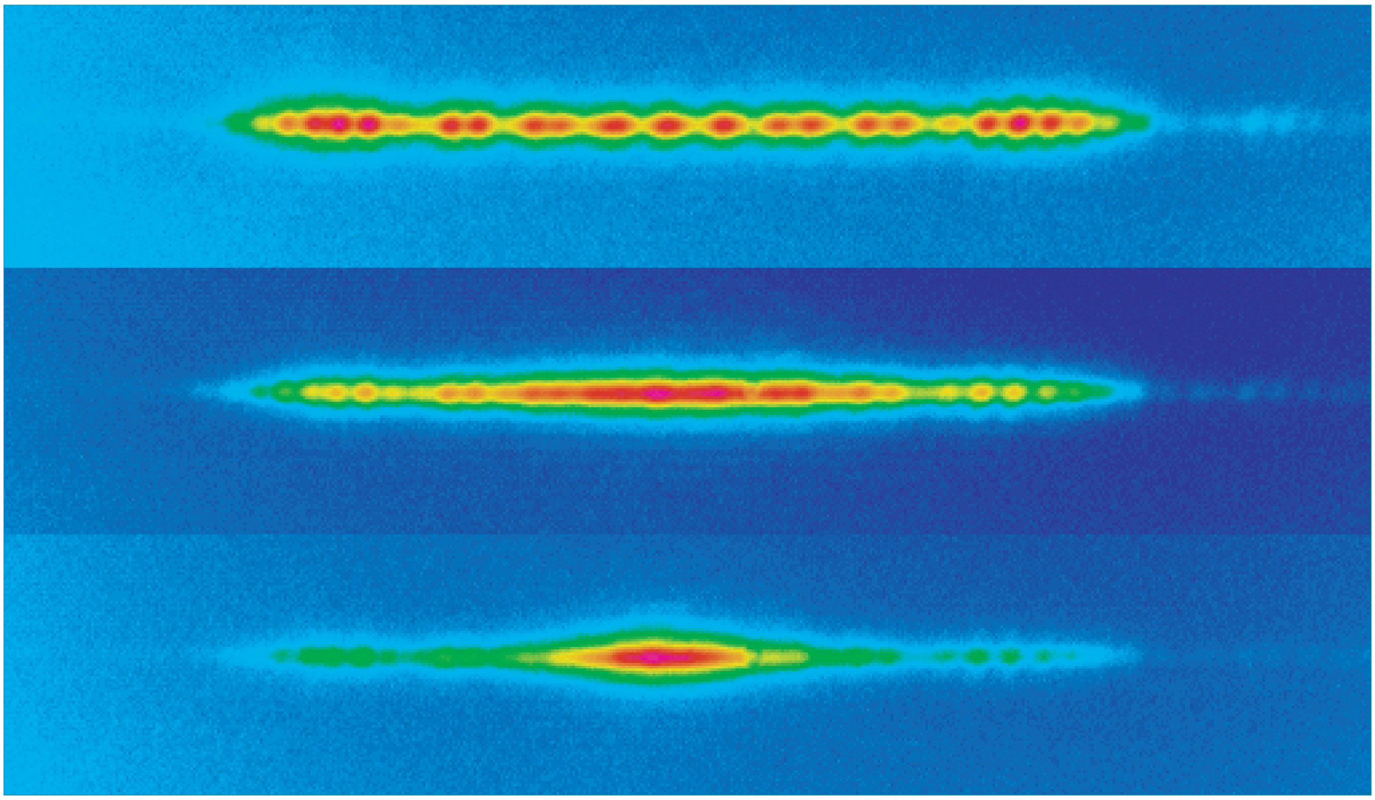

Figure 4. Images of the output facet of a waveguide array at different input powers. *Top:* Peak power 70 W. Linear features are demonstrated: Two main lobes and a few secondary peaks in between. *Center:* Peak power 320 W. Intermediate power, the distribution is narrowing. *Bottom:* Peak power 500 W. A discrete soliton is formed. (From Ref. 5)

The difference is obvious when the power is increased. Instead of focusing, which would be expected in such positive n_2 material, the light distribution expands considerably.

Discrete solitons

Discrete solitons are stationary solutions of discrete Schrödinger-type equations with a nonlinearity. Unlike the cubic NLSE in a 2D continuous medium, these discrete nonlinear equations are not integrable, and therefore they do not possess genuine soliton solutions. However, numerical and analytic studies show that stable solitary-wave solutions propagating without diffraction exist. Although formally speaking, all of these solutions are solitary waves, the term discrete soliton is commonly used to describe all such non-diffracting waves. Until now these solitons could be identified in instantaneous cubic (Kerr) and quadratic nonlinearities. Regions for the existence of various types of discrete solitons can be identified by systematically studying modulational instability of nonlinear plane wave solutions (see Ref. 4 for details). Strongly localized bright and dark soliton solutions of different topologies, symmetry, and shape can even be found analytically, and they can be probed against their stability by a conventional linear stability analysis.[4] Numerical propagation studies can be used for confirming the analytical findings. It results that the family of dichromatic discrete solitons in quadratic media is even richer than that of cubic media because of the vectorial nature of interaction. But because there is no experimental evidence of these solitons to date, we focus here on discrete solitons in Kerr media.

Theory and experiments show that discrete solitons are sensitive to their position and their transverse momentum.

Figure 4 reproduces the first observation of discrete solitons. Light was injected into a single central waveguide in a wide array of InGaAs waveguides. The figure shows images of the output facet for various input powers. At low power, a wide distribution is obtained, covering about 35 waveguides. This distribution matches what is expected from a sample a few coupling lengths long [see Fig. 1 (c)]. When the power is increased, we first see the light distribution converging to form a bell shape. Launching even more power leads to the formation of a confined distribution around the input waveguide, which propagates as a discrete soliton.[5]

Since the diffraction properties can be modified, waveguide arrays can be used to form both bright solitons (as shown in Fig. 4) and dark solitons in the same material system. Figure 5 shows that by modifying the input condition (staggered excitation around the bottom of the diffraction curve, i.e. anomalous diffraction), it is possible in principle to propagate in the excitation field a dark notch, which preserves its dimension and does not broaden.[12]

When the discrete soliton is broad and covers many waveguides, it is very similar in many of its properties to a spatial soliton in a planar continuous medium. In particular, it can move across the array. However, narrow discrete solitons, which contain just a few waveguides, behave in a significantly different manner. Continuous solitons possess two fundamental geometrical symmetries of space. Under rotations of the axes and translation of their origin, exactly the same mathematical solution is reproduced. On the other hand, for discrete optical systems both symmetries are broken due to the direction and position of the array waveguides. Rotational symmetry is gone altogether while translation symmetry is reduced. These differences in symmetry affect in particular the dynamic properties of discrete solitons: The way they propagate when launched at an angle with respect to the waveguide direction.

Theory and experiments show that discrete solitons are sensitive to their position and their transverse momentum. Briefly, the most stable situation is when a discrete soliton is symmetrically excited around a central waveguide. Such a soliton tends to lock in the waveguide direction and to resist any perturbations that tend to drive it sideways. Solitons can also be excited symmetrically around a point between two waveguides, but they are unstable and tend to acquire transverse momentum.[6] These dynamic properties could in principle be used to steer solitons into a desired location by applying small perturbations on the input conditions.

There is a well-known way to form a linearly localized state in a discrete system. When one waveguide in an otherwise uniform array is different, a confined mode around this site is created. Today, a topic of study is the interaction of discrete solitons with defects and boundaries in the arrays, and the interactions of two or more discrete solitons. It would also be interesting to identify some of the exotic discrete solitons without the continuous analog predicted in Ref. 4. Another research goal that will be addressed in the near future is soliton excitation in arrays with a quadratic nonlinearity, e.g., in periodically poled lithium niobate waveguide arrays. Other topics that will be a subject of future research are the physics of discrete solitons in higher dimensions—in 2D arrays, for example—or in more complex 1D arrays embedded in two dimensions.

Acknowledgments

We acknowledge numerous contributions of our colleagues S.J. Aitchison, A. Bräuer, S. Darmanyan, H.S. Eisenberg, A. Kobyakov, R. Morandoti, T. Pertsch, and U. Peschel.

References

1. A.L. Jones, J. Opt. Soc. Am. **55**, 261 (1965).
2. S. Somekh, E. Garmire, A. Yariv, H.L. Garvin, and R.G. Hunsperger, Appl. Phys. Lett. **22**, 46 (1973).
3. D.N. Christodoulides and R.I. Joseph, Opt. Lett. 13, 794 (1988).
4. F. Lederer, S. Darmanyan, and A, Kobyakov, 'Discrete solitons', in: *Spatial Solitons*, eds.: S. Trillo and W. Torruellas, Springer Series on Optical Sciences, Vol. 82, Springer, Berlin, New York, 2001.
5. H.S. Eisenberg, Y. Silberberg, R. Morandotti, A.R. Boyd, and J.S. Aitchison, Phys. Rev. Lett. **81**, 3383 (1998).
6. R. Morandotti, U. Peschel, J.S. Aitchison, H.S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83**, 2726 (1999).
7. U. Peschel, T. Pertsch, and F. Lederer, Opt. Lett. **23**, 1701 (1998)
8. T. Pertsch, P. Dannberg, W. Elflein, A. Bräuer and F. Lederer, Phys. Rev. Lett. **83**, 4752 (1999)
9. R. Morandotti, U. Peschel, J.S. Aitchison, H.S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83**, 4756 (1999).
10. H.S. Eisenberg, Y. Silberberg, R. Morandotti, and J.S. Aitchison, Phys. Rev. Lett. **85**, 1863 (2000).
11. T. Pertsch, T. Zentgraf, U. Peschel, A. Bräuer, and F. Lederer, "Anomalous Refraction and Diffraction in Discrete Optical Systems," Phys. Rev. Lett. (submit-ted)
12. R. Morandotti, H. S. Eisenberg, Y. Silberberg, M. Sorel, and J. S. Aitchison, Phys. Rev. Lett. **86**, 3296 (2001).

Falk Lederer is with the Friedrich-Schiller-Universitaet Jena, in Jena, Germany. His e-mail address is pfl@uni-jena.de. Yaron Silberberg is with the Weizmann Institute of Science, Rehovot, Israel. His e-mail address is yaron.silberberg@weizmann.ac.il.

Figure 5. Generation of a dark discrete solitary wave in the case of anomalous diffraction. (a) The input profile, ~40 µm wide at FWHM. (b) For normal diffraction at low power, a notch is visible in the output profile. The beam evolves into two repulsive bright solitons when the intensity is increased [(c) I_{peak}~250 W]. For anomalous diffraction (beam tilt=2.0±0.4° in this array), the dark notch initially present in the output profile [(d) linear case] slightly narrows and becomes more marked when the power is increased [(e) I_{peak}~250 W)]. As a result of anomalous diffraction, the dark localization is self-sustained in a defocusing bright background and does not disappear when the beam broadens nonlinearly. (From Ref. 10).