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Analysis of a three-core adiabatic directional coupler

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ABSTRACT

We present an analytical description of a new class of three-core adiabatic following directional couplers. Using a multiple-scale WKB method we obtain closed-form expressions describing the optical field dynamics in such structures. The adiabatic evolution occurring in this particular three-core configuration can lead to a spatial switch over of local supermodes and to an irreversible power transfer.

1. Introduction

Waveguide couplers are important elements in integrated optics and microwave circuits [1–3]. In its simplest realization, a directional coupler involves two evanescently coupled parallel waveguides in close proximity. In this arrangement, power is periodically exchanged among waveguides during propagation [2]. In general, the field evolution in such structures can be effectively described within the framework of coupled-mode theory and for basic configurations analytical solutions have been reported in the literature [4]. Evidently, for more involved coupled geometries, the field dynamics tend to become richer in behavior and typically not easily amenable to analytical approaches. One such class is that of adiabatic directional couplers where the local eigenstates remain instantaneously invariant under slowly acting perturbations [4].

In this letter, we present a theoretical study of a new class of three-core adiabatic couplers. In this arrangement the intermediate waveguide is slanted with respect to an uncoupled pair of parallel channels (Fig. 1). The power exchange and oscillations in this system is analytically described in terms of the adiabatic and coupling parameters. It is worth noting that this same configuration was recently studied [5–7] as an optical analogue of the stimulated Raman adiabatic passage (STIRAP) process [8]. The adiabatic evolution occurring in this particular type of three-core couplers can lead to a spatial switch over of local supermodes and to a non-periodic quasi-monotonic power transfer.

2. Analysis and results

We consider a system of three single-mode weakly-guiding, evanescently coupled waveguides. Such a system may be realized for instance in a ridged waveguide geometry similar to the one shown in Fig. 1a. In this case the guidance is provided by a local increase of the effective index due the thickness modulation of the top layer. The problem, under these conditions, reduces to the study of the modal amplitudes in the three channels depicted in Fig. 1b and this irrespective of the actual field profiles in these identical guides.

To analyze this three-core coupled waveguide system, we employ a coupled-mode formulation. All waveguide channels are assumed here to be identical with propagation constants β. The center of the coordinate system $z = 0$ is placed at the middle of this device, where the intermediate waveguide is equidistant from the other two. At this point the intermediate waveguide has the same coupling constant κ_0 with respect to both the left and right channels. Given that the coupling coefficient varies exponentially with waveguide separation [9], then in this linearly slanted arrangement $\kappa(z) = \kappa_0 \exp(\pm\hat{\gamma}z)$ where $\hat{\gamma}$ is proportional to the slant angle and is associated with the rate of increase/decrease of the coupling process. The total length of this structure is L, with its input at $z = -L/2$ and its output at $L/2$. By introducing a normalized propagation coordinate $\xi = \kappa_0 z$ and a small dimensionless perturbation parameter $\gamma = \hat{\gamma}/\kappa_0$, we then find that the modal field amplitudes $a, ,b, c$ obey the following set of coupled differential equations:

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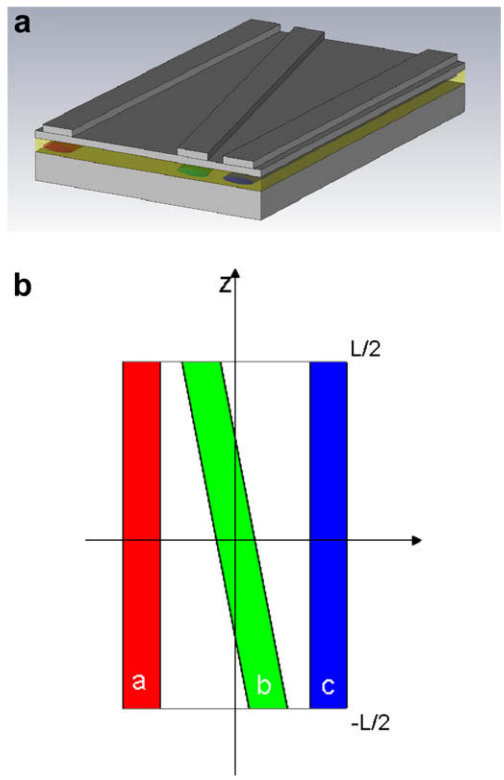

Fig. 1. (a) Ridged waveguide structure geometry. The modes are guided beneath the raised sections of the top layer. (b) Schematic representation of the three channels in the reference system adopted for the analysis.

$$i\frac{da}{d\xi} + \exp(\gamma\xi)b = 0 \tag{1a}$$
$$i\frac{db}{d\xi} + \exp(\gamma\xi)a + \exp(-\gamma\xi)c = 0 \tag{1b}$$
$$i\frac{dc}{d\xi} + \exp(-\gamma\xi)b = 0 \tag{1c}$$

This three-core coupler will be studied under two different excitation conditions at $z = -L/2$: $(a_{in} = 1,\ b_{in} = c_{in} = 0)$ and $(a_{in} = b_{in} = 0,\ c_{in} = 1)$. In all cases the following conservation law (total power) applies during propagation, $|a|^2 + |b|^2 + |c|^2 = 1$. Eqs. (1) can be directly decoupled in the form:

$$\gamma^2\left(\frac{d^3a}{dt^3} - \frac{d^2a}{dt^2}\right) + 2\cosh(2t)\frac{da}{dt} + 2a\exp(2t) = 0 \tag{2a}$$
$$\gamma^2\left(\frac{d^3c}{dt^3} + \frac{d^2c}{dt^2}\right) + 2\cosh(2t)\frac{dc}{dt} - 2c\exp(-2t) = 0 \tag{2b}$$

where the new coordinate variable t is here defined as $t = \gamma\xi = \hat{\gamma}z$. Due to the symmetry of these two equations, once a general solution is obtained for any one of them, the other follows through t reversal. To solve Eq. (2a) we use the following WKB ansatz [10]:

$$a(t) = \exp\left[\frac{S_0(t) + \gamma S_1(t)}{\gamma}\right] \tag{3}$$

By substituting (3) into Eq. (2a) and by collecting terms γ^{-1}, γ^0 of similar order in the adiabatic parameter γ, we find that:

$$\dot{S}_0\lfloor\dot{S}_0^2 + 2\cosh(2t)\rfloor = 0 \tag{4a}$$
$$\dot{S}_0^2(3\dot{S}_1 - 1) + 2[\exp(2t) + \cosh(2t)\dot{S}_1] + 3\dot{S}_0\ddot{S}_0 = 0 \tag{4b}$$

where $\dot{S}_0 = dS_0/dt$. Eq. (4a) has three independent solutions, namely:

$$S_0 = A = const \tag{5a}$$
$$S_0(t) = \pm i\sqrt{2}\int_{-t_0}^{t} dx\sqrt{\cosh(2x)} = \pm i\gamma Q(t) \tag{5b}$$

where $t_0 = \gamma\kappa_0 L/2$. Note that the dimensionless distance t belongs to the range $-t_0 \leqslant t \leqslant t_0$. From (4b) one can obtain after integration the first order correction term $S_1(t) = \tilde{c} - (1/2)\ln(1 + \exp(\pm 4t))$ corresponding to three branch solutions given in Eq. (5). Therefore, from Eq. (3), the general solution of Eq. (2a) is given by:

$$a(t) = A_1\frac{\sqrt{1+\exp(-4t_0)}}{\sqrt{1+\exp(4t)}} + A_2\frac{\sqrt{1+\exp(4t_0)}}{\sqrt{1+\exp(-4t)}}\cos[Q(t) + \varphi_a] \tag{6}$$

where the constants $A_{1,2}, \varphi_a$ are to be determined from boundary conditions. A solution of a similar functional form can be obtained for $c(t)$ in Eq. (2b) through space-reversal, i.e.,

$$c(t) = C_1\frac{\sqrt{1+\exp(4t_0)}}{\sqrt{1+\exp(-4t)}} + C_2\frac{\sqrt{1+\exp(-4t_0)}}{\sqrt{1+\exp(4t)}}\cos[Q(t) + \varphi_c] \tag{7}$$

The phase function $Q(t)$ can be expressed in terms of elliptic integrals $F(u,k)$ and $E(u,k)$ of the first and second kind respectively [11], that is

$$\begin{aligned} Q(t) = {} & \gamma^{-1}[F(\alpha(t_0), 2^{-1/2}) - 2E(\alpha(t_0), 2^{-1/2})] + 2^{1/2}\gamma^{-1} \\ & \times \sinh(2t_0)[\cosh(2t_0)]^{-1/2} + (t/|t|)\{\gamma^{-1}[F(\alpha(t), 2^{-1/2}) \\ & - 2E(\alpha(t), 2^{-1/2})] + 2^{1/2}\gamma^{-1}\sinh(2|t|)[\cosh(2t)]^{-1/2}\} \end{aligned} \tag{8}$$

where $\alpha(t) = \arcsin\{[(\cosh(2t) - 1)/\cosh(2t)]^{1/2}\}$.

Under the first excitation condition, e.g. $a_{in} = 1$, $b_{in} = c_{in} = 0$, the following initial conditions hold true: $c(-t_0) = 0$, $\dot{c}(-t_0) = 0$, $\ddot{c}(-t_0) = -1/\gamma^2$. From here the constants involved in Eq. (7) can be directly evaluated:

$$\begin{aligned} C_1 &= f_1(t_0,\gamma) = -\frac{1}{2[\gamma^2\tanh(2t_0) + 2\gamma^2 + \cosh(2t_0)]} \\ C_2 &= f_2(t_0,\gamma) = \frac{\sqrt{1 + 2\gamma^2\,\mathrm{sec}\,h(2t_0)}}{2[\gamma^2\tanh(2t_0) + 2\gamma^2 + \cosh(2t_0)]} \\ \varphi_c &= f_3(t_0,\gamma) = -\arctan\left[\frac{\gamma\sqrt{2}}{\sqrt{\cosh(2t_0)}}\right]. \end{aligned} \tag{9}$$

In this case the evolution of the optical field $c(t)$ in the waveguide ***c*** can be completely determined from Eqs. (7)–(9) and in turn $b(t)$ from Eq. (1c). The power in the first channel $|a(t)|^2$ can then be obtained from the power conservation law. As an example let us consider a $L = 2.5$ cm long three-core adiabatic AlGaAs structure similar to that examined in the experimental study of Ref. [5]. Let all three waveguides be 3 μm wide and the edge to edge distance between the two parallel channels be 12 μm. The intermediate waveguide is slanted at 0.2 mrad with an edge to edge distance of 2 μm from the third waveguide at the input $z = -L/2$. The coupling constant at the middle of the device is taken here to be $\kappa_0 = 7.9\ \mathrm{cm}^{-1}$ with a growth rate of $\hat{\gamma} = 0.9\ \mathrm{cm}^{-1}$. In this case the resulting adiabatic parameter is $\gamma \cong 0.1$ and $t_0 = 1.15$. Fig. 2a depicts the evolution of the power levels in this particular structure as a function of distance. The error resulting from the WKB approximation in this case is very small and for this reason is not shown here. In fact the error remains below 3% even for adiabatic parameters as high as $\gamma \approx 0.2$. As Fig. 2a indicates, the power is adiabatically transferred from the initially excited waveguide ***a*** to ***c***. Note that at all stages of propagation the intermediate channel ***b*** remains almost devoid of energy. In other words, the local supermode [1, 0, 0] is switched over to [0, 0, 1] in a non-periodic quasi-monotonic fashion-the energy never returns back to channel ***a***. We note that this almost 100% exchange is rather insensitive to any dependence the coupling may have on the wavelength or the

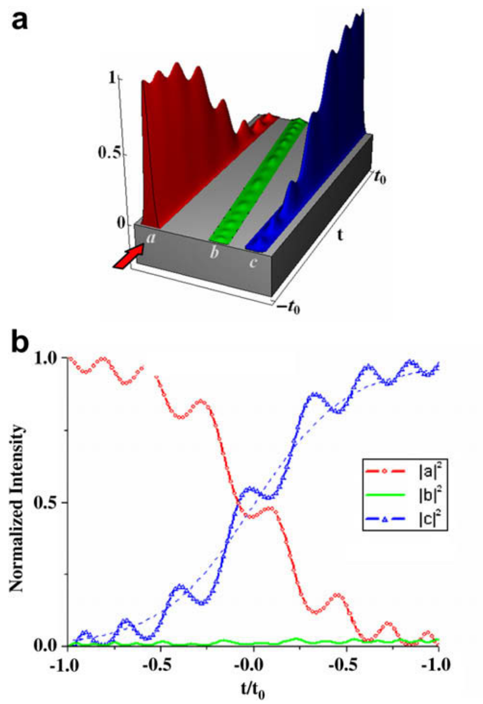

Fig. 2. (a) Normalized power evolution in the adiabatic coupler under a [1,0,0] excitation. (b) Normalized power in the three channels. The dashed line represents the average monotonic increase of power in the waveguide ***c***.

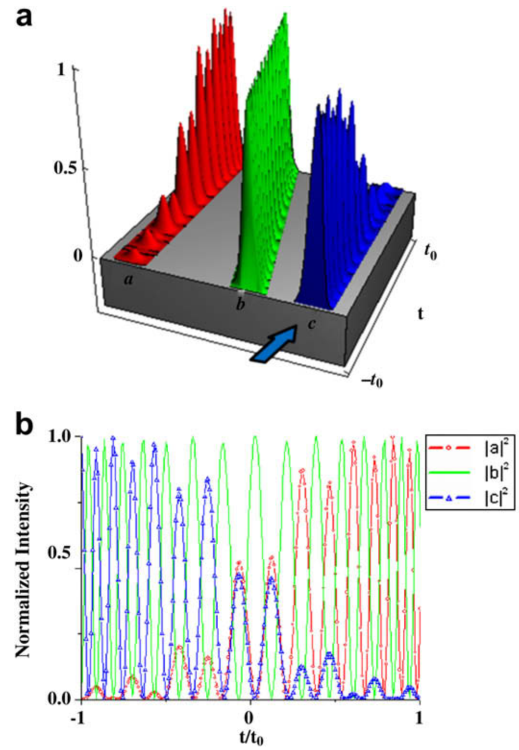

Fig. 3. (a) Normalized power evolution in the adiabatic three-core waveguide coupler under a [0,0,1] excitation. (b) Normalized power in the three channels.

device length (provided the structure is long enough). In view of these properties this structure may find applications as a device coupler that happens to be robust to mechanical and thermal perturbations. In addition, its performance is expected to be broadband (insensitive to wavelength) since any variations in the coupling are not as crucial. The analytical solution found can also provide an accurate estimate of the rate at which the energy flows from ***a*** to ***c*** which is monotonically increasing as a function of distance. This monotonic increase can be obtained from the first term of Eq. (7), that is $|c(t)|^2 \approx C_1^2[1+\exp(4t_0)][1+\exp(-4t)]^{-1}$, as also shown by the dashed line in Fig. 2b. The small decaying oscillations occurring during this process are determined from the second term in (7). In all occasions the degree of adiabaticity is determined by the dimensionless perturbation parameter γ. As indicated above, our approach can handle adiabatic parameters up to $\gamma \approx 0.2$, with errors as low as 3%. This due to the fact that a second-order perturbation theory was used in our study.

Similarly this structure can be analyzed for the second set of initial conditions, i.e., $a_{in} = b_{in} = 0,\ c_{in} = 1$, for which case one can show that $a(-t_0) = 0,\ \dot{a}(-t_0) = 0,\ \ddot{a}(-t_0) = -1/\gamma^2$. Under these excitation conditions, the optical field in waveguide ***a*** can be directly obtained from Eq. (6) where now the constants involved are given by Eq. (9) and are equal to: $A_j = f_j(-t_0, -\gamma)$ and $\varphi_a = f_3(-t_0, -\gamma)$. From here $b(t), c(t)$ can be computed using Eqs. (1). Fig. 3 depicts the evolution of the intensity in these three waveguides when (as in the previous example) $\gamma \cong 0.1$ and $t_0 = 1.15$. As the figure indicates, the power in waveguide ***c***, after a damped oscillation, is adiabatically transferred to both channels ***a*** and ***b***. As opposed to the previous case, where only a small fraction of power was coupled to ***b***, in this regime the energy is eventually shared (via oscillations) between ***a*** and ***b***. This is because the two local approximate supermodes $[0, 1, \pm 1]$ are both excited and adiabatically switched over in space at the output. Again, the process is non-periodic quasi-monotonic, e.g. the energy never returns back to channel ***c***.

3. Conclusion

In conclusion we have theoretically analyzed a new family of adiabatic three-core couplers. By using a multiple-scale WKB approach we provided closed-form expressions describing the adiabatic energy exchange process. Some of the peculiar features of these structures have been highlighted and explained. Before closing we would like to note that our procedure is general and thus can be used to systematically study other classes of adiabatic coupled-wave devices.

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