Discrete X-Wave Formation in Nonlinear Waveguide Arrays

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We present experimental evidence for the spontaneous formation of discrete X waves in AlGaAs waveguide arrays. This new family of optical waves has been excited, for the first time, by using the interplay between discrete diffraction and normal temporal dispersion, in the presence of Kerr nonlinearity. Our experimental results are in good agreement with theoretical predictions.

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The study of nonlinear dynamics in periodic or discrete systems has recently led to the prediction and observation of many new effects. In several cases, these nonlinear phenomena are unique to discrete systems, a direct consequence of the unusual properties of the discrete diffraction itself [1]. In the field of optics, nonlinear arrays of weakly coupled waveguides provide a fertile ground on which such discrete nonlinear dynamics can be theoretically and experimentally investigated [1]. Among the most intriguing aspects associated with the rich phenomenology of nonlinear discrete lattices is the possibility of self-localized states, better known as "discrete solitons" [2,3]. These nonlinear self-trapped waves result from the collective excitation of the nonlinear lattice and they are the outcome of the balance between discrete diffraction and the nonlinear decoupling process. Following the first prediction [2], the existence of this family of optical waves was successfully confirmed in a series of experimental studies carried out in highly nonlinear AlGaAs waveguide arrays [3]. Lately, discrete solitons have been observed in two-dimensional settings such as optically induced photorefractive nonlinear lattices [4–6], as well as in media with other types of nonlinearity [7]. By exploiting the linear properties of discrete systems, one can manipulate their diffraction properties in ways not possible in continuous media [8]. This in turn can lead to unusual nonlinear effects such as, for example, the existence of dark solitons in self-focusing lattices [9], or the possibility of discrete modulational instability [10].

We would like to note that so far, optical discrete dynamics have been primarily explored in the spatial domain. Very recently the temporal evolution of optical pulses in normally dispersive nonlinear array structures has also been theoretically considered [11]. One of the interesting outcomes of this latter work was the possibility of exciting two-dimensional X waves in coupled waveguide arrays. More specifically, it was shown that X waves may be spontaneously induced in such periodic structures at moderate power levels as a result of bidispersion [11]. In general, X waves represent stationary X shaped field patterns that can propagate free of spatial diffraction and/or temporal dispersion. Three-dimensional X-wave solutions were first identified in the field of ultrasonics [12] and were later intensely investigated in other fields [13–23]. In optics, such wave structures can only be supported in bidispersive systems, e.g., physical arrangements that exhibit both normal and anomalous dispersive or diffractive behavior in different spatial or spatiotemporal coordinates [23]. The generation of nonlinearly induced X waves has been considered in both quadratic and Kerr normally dispersive media [13,14], and spontaneously generated X light bullets have been experimentally observed in lithium triborate crystals [13]. Nonlinear X-wave formation by femtosecond filamentation in Kerr media has also been investigated [15–17]. The generalization of the X-wave families in 2D and 3D configurations was also recently suggested [23].

In this Letter we report the first experimental observation of discrete X waves in normally dispersive AlGaAs waveguide arrays. Discrete X waves in array structures exhibit features that are unique to this arrangement. More specifically, these waves are by nature 2D entities and as a result their X-wave signature is quite strong. This is in contrast to those observed in previous 3D experimental configurations where the energy of the X-wave component was distributed in all three dimensions, hence making their detection a rather demanding task. In addition, the periodicity or discreteness of the array structure significantly

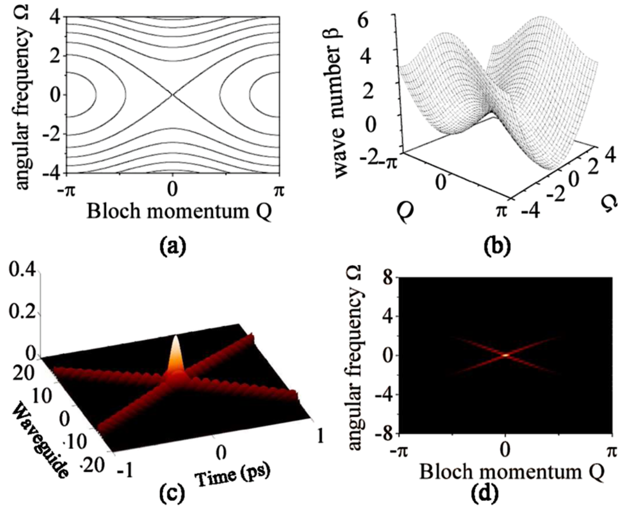

FIG. 1 (color online). Dispersion diagram for a normally dispersive waveguide array: (a) contour plot, (b) three-dimensional depiction, (c) the intensity profile of a discrete X-wave structure when the weight functions are Gaussian, (d) the power spectrum for this X wave.

alters the "hyperbolic" character of the dispersion curves thus leading to new X-wave behavior. We describe the physical mechanism leading to spontaneous X-wave formation and we experimentally characterize the temporal intensity features of the output wave when the array has been excited by a high intensity femtosecond pulse. In all cases our experimental results are in good agreement with theoretical predictions.

To gain insight into the nature of discrete X waves, let us first consider the linear properties of dispersive waveguide arrays. As previously discussed [11], the spatiotemporal evolution of optical pulses in such array structures can be described by a discrete Schrödinger-like equation:

$$i\frac{\partial U_n}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 U_n}{\partial t^2} + C(U_{n+1} + U_{n-1}) = 0 \qquad (1)$$

where β_2 is the group velocity dispersion of each waveguide and C represents the linear coupling between channels. The dispersion relation can then be obtained by substituting the Bloch mode solution $U_n(z, t) = U_o \exp[i(\beta z + Qn + \Omega t)]$ into Eq. (1). This yields the linear dispersion relation

$$\Omega_\pm = \pm\sqrt{\frac{2}{\beta_2}(\beta - 2C\cos Q)} \qquad (2)$$

where Ω is the angular frequency, β is the longitudinal wave vector and $Q = k_x D$ represents the Bloch momentum with k_x being the transverse spatial wave vector and D the spatial period of the system. The $\pm$ signs in Eq. (2) simply express the existence of two symmetrical branches for every β. We note that the periodicity in the transverse spatial coordinate x imposes a periodicity to the spectral coordinate Q as well, thus bounding the Bloch momentum in the Brillouin zone $-\pi \leq Q \leq \pi$, in contrast to what happens in the bulk. When $\beta_2 > 0$, i.e., for normally dispersive systems like AlGaAs, it is evident from Eq. (2) that the dispersion relation acquires a hyperbolic-like form, as shown in Figs. 1(a) and 1(b). As previously discussed in [13,14] for the case of bulk systems the linear X-wave formation is inherently connected to the X-like shape of the spectral space. A linear X-wave packet propagates undistorted, provided that all spectral components travel at the same group velocity [24], i.e., the propagation constant β depends linearly on Q and Ω. Consequently, any superposition of Q-Ω spectral components obtained from the same section of the dispersion diagram with the plane $C_1 = C_2 Q + C_3\Omega$ in (β, Q, Ω) space should be free of diffraction and dispersion. For a constant wave number, i.e., for $C_2 = C_3 = 0$, the section corresponds to isocontour branches of the dispersion diagram. Hence, for a given wave number β and with appropriate weight functions $F_+(Q)$, $F_-(Q)$ (here the sign corresponds to the appropriate branch), such a linear X-wave packet can be synthesized as follows:

$$U_n(z, t) = e^{i\beta z}\int_{-\pi}^{\pi} F_\pm(Q)\exp[i(Qn + \Omega_\pm(Q)t)]dQ, \qquad (3)$$

where the integration is carried out along both branches for a given wave number β. Figure 1(c), depicts the intensity profile of such a discrete X-wave structure when the weight functions are Gaussian-like. The power spectrum of this X wave in $Q - \Omega$ is also shown in Fig. 1(d). In the experimental work we present here, we took advantage of the strong nonlinearity exhibited by AlGaAs (about 500 times higher than conventional SiO_2 fibers), and show that a discrete X wave can be excited spontaneously with just a narrow high intensity beam without the need of rather involved input beam-shaping techniques [24,25]. As it will be shown in our study, the resulting spectrum and spatiotemporal light distribution resembles that of Fig. 1, thus revealing a clear X-wave signature.

The experimental setup is shown in Fig. 2(a). Our laser source is an optical parametric oscillator pumped by a Ti:sapphire laser, producing 170 fs FWHM pulses at a repetition rate of 80 MHz at a wavelength of 1530 nm. A schematic drawing of the sample used in our experiments is also shown. This waveguide array is fabricated in an AlGaAs planar waveguide by way of conventional photolithography techniques. The waveguides are 4 μm wide, and the array period is $D = 9$ μm. The lateral effective refractive-index step is 0.007, which results in a calculated coupling constant of 125 m^{-1} between waveguides. The sample length is 5 mm, and the nonlinear Kerr coefficient for the TE polarization is taken to be $\hat{n}_2 = 1.5 \times 10^{-13}$ cm^2/W. A beam splitter is used to direct some of the laser output to the reference arm of a cross correlator. The input power to the sample is adjusted by a variable

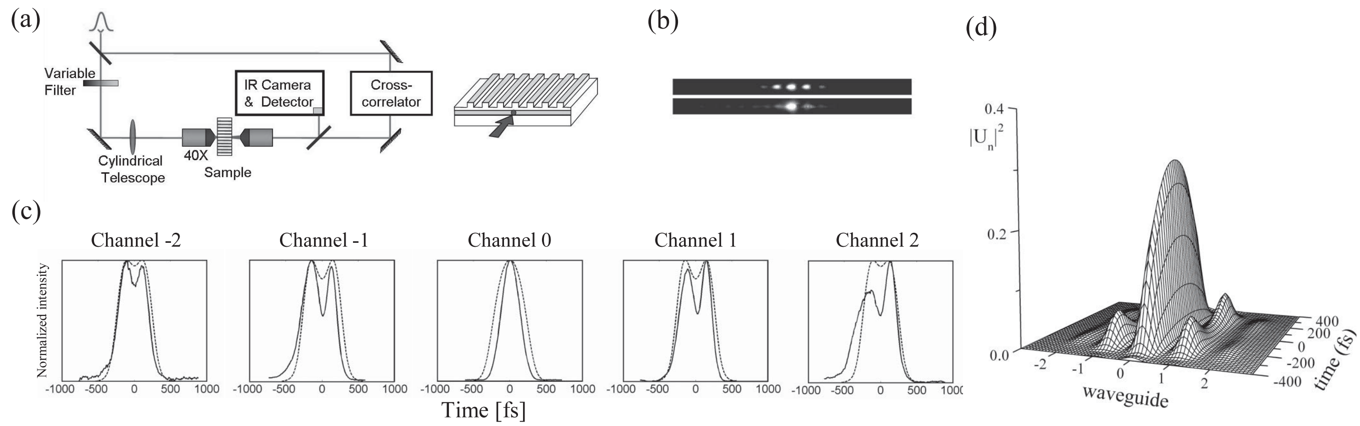

FIG. 2. (a) The experimental setup and a schematic view of the sample and the coupling geometry. (b) Experimental measurements of the linear discrete diffraction (top panel) and collapse to the input waveguide at peak power of 730 W (bottom panel). At this power the measurements presented in (c) where taken. (c) High power experimental (solid) and numerical results (dotted) for cross correlations of the pulses emerging from each waveguide, where channel 0 is the input waveguide. The numerical results where convolved with the laser input pulse, to simulate the cross-correlation experiment. (d) The unconvolved output of the simulated array, shown in three-dimensional plot.

filter. Light is coupled into the waveguide array, collected from the output facet, and directed to the signal arm of the cross correlator. The signal pulses were cross-correlated with the reference pulses in a noncollinear second-harmonic configuration. By moving the output objective across the sample we could measure the temporal envelopes of the pulses at different positions at the output facet. In this specific experimental configuration, light was coupled directly into the central waveguide of the array. Typically, as light propagates in the sample, it exhibits discrete diffraction as it couples to adjacent waveguides. By increasing the input power, we observed a collapse of the diffraction pattern to the input waveguide at an output average power of 3.4 mW, corresponding to a peak power of 730 W in the sample [see Fig. 2(b)]. At this power level we took cross-correlation measurements at the exit of the input waveguide and of two adjacent waveguides on each side. The results are presented in Fig. 2(c), and clearly show the single-humped temporal shape of the pulse emerging from the input waveguide, and the double humped temporal shape of the pulses emerging from the adjacent waveguides. This is a clear indication of X-wave formation in the array.

To simulate these results numerically, we have included nonlinear effects in Eq. (1). In doing so we obtain a discrete nonlinear Schrödinger equation (DNLSE) with a dispersion term added [1,2,11,18]. The modal field amplitude U_n in the nth waveguide evolves according to

$$i\frac{\partial U_n}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 U_n}{\partial t^2} + C(U_{n+1} + U_{n-1}) + k_0 n_2 |U_n|^2 U_n = 0, \tag{4}$$

where $k_0 = 2\pi/\lambda_0$ is the free-space wave number and $n_2 = (\hat{n}_2 n)/(2\eta_0)$ stands for the nonlinear coefficient. In this last expression $\hat{n}_2 = 1.5 \times 10^{-13}$ $\mathrm{cm^2/W}$ is the AlGaAs self-focusing Kerr coefficient, n is the linear refractive index of AlGaAs, and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$. The dispersion in the AlGaAs array is approximately $\beta_2 = 1.35$ $\mathrm{ps^2/m}$. In this experiment, 2- and 3-photon absorption effects were negligible compared to the Kerr effect and were consequently not included. The coefficients of Eq. (4) as well as the input pulse parameters were set to the values used in our experiment. Equation (4) was then solved numerically by means of a symmetrized split-step Fourier scheme. The experimental and theoretical results are compared in Fig. 2(c). To fit the cross-correlation results, we have convolved the numerical data in each channel with the laser pulse, as done in our experiment. As the results show, the X-wave features observed in the experiments are successfully reproduced by the DNLSE simulations. The unconvolved output of the simulated array is shown in Fig. 2(d) as a three-dimensional plot.

The X-wave formation in this experiment owes its shape to the interplay between diffraction, dispersion and nonlinearity [11]. The initial pulse is relatively short (170 fs FWHM) and hence the high normal dispersion of AlGaAs combined with the high nonlinearity leads to the observed pulse broadening. As the pulse broadens, its peak power drops. During this process the main part of the pulse remains confined within the central waveguide due to the intense nonlinearity, while the leading and trailing edge (that behave linearly) undergo discrete diffraction. This process gives rise to the double humped pattern observed at the output of the side waveguides [see Figs. 2(c) and 2(d)]. Furthermore, because of normal dispersion the leading and trailing edge of the pulse that undergo discrete diffraction contain the high and low temporal frequencies, while the intermediate frequencies are confined in the

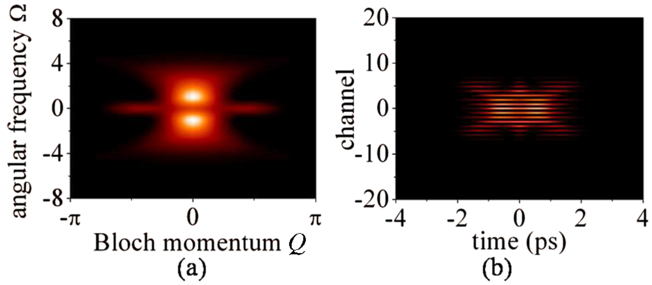

FIG. 3 (color online). (a) Spectrum of the pulse at $z = 30$ mm. The signature of the X-wave spectrum is clearly seen. (b) Simulated spatiotemporal profile of the pulse at $z = 30$ mm showing the formation of a cascaded X wave under the chosen experimental conditions.

central part of the pulse (i.e., around the peak). It is also expected that the injection of light into a single waveguide results in the full excitation of the Bloch momentum spectrum. However, this is only true for the linear part of the pulse (i.e., the edges), while the central part (the peak) is highly nonlinear and is therefore self-trapped by the structure, resulting in a limited excitation of the associated Bloch momentum. Therefore, a symmetric X shaped spectrum similar to that of Fig. 1 would be reasonably expected.

To determine the spatiotemporal and spectral profiles of the generated X wave after a long propagation distance where the nonlinear process has fully relaxed, we simulated this process under the experimental conditions up approximately 5–6 times our sample length. At this distance, the peak power asymptotically diminishes, the spectrum settles down and the pulse evolves linearly over its entire extent. The spectrum and spatiotemporal light distribution for this case are presented in Fig. 3. The similarity of the spectrum in Fig. 3(a) to the theoretically calculated linear spectrum for our system as predicted by Eq. (2) and presented in Fig. 1(a), clearly suggest the X like nature of the propagating pulse. We note that the results presented in Fig. 3 show that under the experimental conditions, long propagation will result in temporal pulse splitting and formation of two X waves, or a cascaded X wave [11]. This phenomenon is reminiscent to formation of two X waves in nonlinear bulk media by light filaments reported recently [16,17].

In conclusion, we have observed the formation of nonlinearly induced discrete X waves in an AlGaAs waveguide array. We have shown that these waves originate from the interplay between discrete diffraction, normal dispersion, and Kerr nonlinearity. This work demonstrates that beyond the well investigated spatial effects, this interplay also changes significantly the temporal shape of short pulses propagating in a periodic environment. We note that discrete diffraction can be tailored, i.e., suppressed or even reversed in sign by choosing a suitable array design or by controlling the beam propagation angle [8]. This flexibility opens further possibilities for the study of new spatiotemporal effects on short pulses propagating in nonlinear optical lattices and can lead to novel effects, such as ultrafast all-optical pulse shaping techniques in an integrated geometry.

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