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NONTRIVIAL RELAXATION OF A NARROW WAVE PACKET DURING THREE-WAVE INTERACTION

G. E. Fal'kovich

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We consider the evolution, as a result of three-wave processes, of a narrow wave packet with a number of waves which is much larger than the number of thermal waves which interact with it. It is shown that the system of kinetic equations which describe the simultaneous evolution of a nonlinear wave packet and the thermal waves which interact with it, has a Hamiltonian structure which is not violated by the introduction of linear sources or damping decrements of the waves. We prove that the steady state, not coincident with equilibrium, is stable. We describe the consequences of the quasidynamical character of the kinetics: anomalously large times to reach the steady state, and the possibility of oscillations of the occupation numbers.

Of considerable interest in wave kinetics is the damping of a wave, or a wave packet, in a nonlinear medium. In many physical situations (for example, for sound in crystals [1], spin waves in magnetodielectrics [2], Langmuir waves in nonisothermal plasma [3]), the main processes which determine the evolution of a nonlinear wave, are the three-wave interactions. The medium here plays the role of a reservoir of thermal waves, and the interaction with the thermal waves ensures damping of the original wave or a wave packet. If the amplitude of the nonlinear wave or the number of quanta in the wave packet is small, the distortion of the equilibrium distribution of the thermal waves can be neglected. In this case, as it was done, for example, in [1], the calculation of the damping decrement requires the linearization of the collision term in the kinetic equation for the waves on the background of the Planck distribution.

However, as was noted in [2], the laws of energy and momentum conservation for the three-wave coalescence

$$\omega_{\mathbf{k}} + \omega_{\mathbf{k}'} = \omega_{\mathbf{k}+\mathbf{k}'} \tag{1}$$

and decay

$$\omega_{\mathbf{k}} = \omega_{\mathbf{k}'} + \omega_{\mathbf{k}-\mathbf{k}'} \tag{2}$$

allow, in the relaxation of a wave with frequency $\omega_{\mathbf{k}}$, the participation of thermal waves with wave vectors $\mathbf{k}'$ and $\mathbf{k} \pm \mathbf{k}'$, which lie on the surface (1) or (2) in the $\mathbf{k}$ space. Consequently, only a small part of the total reservoir which lies near the surfaces (1) and (2), takes part in the relaxation of a narrow packet with $\Delta\omega_{\mathbf{k}} \ll \omega_{\mathbf{k}}$. Therefore, even for a small number of waves in the original wave packet $N_{\mathbf{k}}$, the occupation numbers of the thermal waves $n_{\mathbf{k}'}$ which interact with it, can deviate considerably from the equlibrium values. The damping decrement then begins to depend on its amplitude, and the relaxation becomes nonlinear.

The description of the nonlinear kinetics depends considerably on the relationship between the three characteristic times of the problems: the time of randomization of the phases in the wave packet $\tau_p^{-1} \simeq \Delta\omega_{\mathbf{k}}$, the time of nonlinear interaction $\tau^{-1} \simeq V^2 N(\Delta\omega_{\mathbf{k}})^{-1}$ (V is the three-wave matrix element), and the time of linear damping of waves as a result of the three-

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wave interaction with the thermal background $\tau_l^{-1} = \gamma_k \simeq V^2 n_k k^3 \omega_k^{-1}$. The case of weak nonlinearity $\tau^{-1} \ll \gamma_k$, when the distortions of the equilibrium distribution are small, was considered recently in [4] using the example of spin waves in magnets. However, for experimental studies, for example, of the parametric excitation of spin waves, it is more interesting to consider the case of strong nonlinearity, since when the pumping power exceeds the threshold by only 10% $\tau^{-1} \gg \gamma_k$ (see the estimate in [4]). The case of strong nonlinearity was considered in [2] in the approximation $\Delta\omega_k \ll \gamma_k$ (damping of a nonlinear monochromatic wave). However, experiments show [5] that even for excitation of the waves by a coherent pumping, the wave packet is broadened considerably in the **k** space ($\Delta\omega_k \gtrsim \gamma_k$) even for a small excess over the threshold. Therefore, we study the relaxation of a wave packet in the intermediate case

$$\omega_k \gg \Delta\omega_k \gg \tau^{-1} \gg \gamma_k. \tag{3}$$

The study of relaxation of nonlinear packets with $\Delta\omega_k \gg \gamma_k$ is also of interest for the interpretation of experiments on the parametric excitation of spin-waves by incoherent pumping [7], phonons by laser radiation [6], and of Langmuir waves by a beam of charged particles with sufficiently large scatter of velocities [9].

The condition $\Delta\omega_k \gg \tau^{-1}$ makes it possible to describe the interaction of nonlinear waves with the thermal waves in the weak-turbulence approximation, i.e., using kinetic equations. In these equations, the terms which are quadratic with respect to the occupation numbers of thermal waves, are smaller by the parameter $(\gamma_k\tau)^{-1}$ than the terms which contain the amplitude of the nonlinear packet. These small terms describe the thermal noise. In the zero order in $\gamma_k\tau$, they can be neglected, and in the first order, they are assumed time independent. As shown in the present work, the system of kinetic equations in the zero order in $\gamma_k\tau$ has a Hamiltonian structure. This fact is sufficient for drawing concrete conclusions about the relaxation of the packet, without solving these nonlinear intergral equations. Indeed, the Hamiltonian system cannot have asymptotically stable steady-state solutions with a finite relaxation time. Consequently, the steady state is approached after times which are determined by the thermal noise (which violates the Hamiltonian structure). In Sec. 1 we consider the relaxation of a wave packet in the case when the coalescent processes are dominant, and it is shown that the decay time $t_{dom} \simeq \gamma_k^{-1}(\gamma_k\tau)^{-1}$, i.e., it is much longer than the decay time of a linear wave. In Sec. 2 we consider the case when both processes of the type (1) and (2) are allowed. It is shown that the presence of linear sources or damping decrements of waves does not violate the Hamiltonian structure of the system (it is only necessary to redefine the Hamiltonian). This makes it possible to study the approach to a steady state which does not coincide with equilibrium. It is proved that, if the steady state satisfies (3), any initial distribution which also satisfies (3) evolves towards the steady state. This evolution consists of slowly damped oscillations of the occupation numbers near the steady-state values.

We note that the obtained results are sufficiently universal, i.e., are independent of the concrete form of the matrix elements and dispersion laws (only conditions (3) must be satisfied, and the corresponding processes must be allowed by the conservation laws). It is also assumed that the wave vector of the original wave is such that the three-wave interaction of the packet, which leads to frequency doubling, is absent: $2\omega_k \neq \omega_{2k}$.

1. We consider first the case when the dispersion laws of the wave are such that, for the original wave, the decay processes are forbidden, and its relaxation is determined only by coalescence. This situation is typical, for example, for the majority of experiments on parametric excitation of spin waves (see, for example, [5]). We shall elucidate what is required for the wave packet, which satisfies (3), to relax after the pump is switched off. It is clear that if $\Delta\omega_k \gg \gamma_k$, the anomalous correlators $\langle a_k a_{-k}\rangle$ decay rapidly (during time τ_p), which was also observed experimentally [5]. The evolution of the packet, and of the thermal waves which interact with it, will be described by kinetic equations for the normal correlators $N_k = \langle a_k a_k^*\rangle$, $n_{k'} = \langle b_{k'} b_{k'}^*\rangle$:

$$\frac{\partial N_k}{\partial t} = -N_k \int |V_{k+k'kk'}|^2 \delta(\omega_k + \omega_{k'} - \omega_{k+k'}) [n_{k'}^{(2)} - n_{k'+k}^{(3)}] dk' +$$

$$+ \int |V_{k+k'kk'}|^2 \delta(\omega_k + \omega_{k'} - \omega_{k+k'}) n_{k'}^{(2)} n_{k'+k}^{(2)} dk', \quad \frac{\partial n_{k'}^{(2)}}{\partial t} = \tag{4}$$

$$= -\int |V_{k_1+k'k_1k'}|^2 \delta(\omega_{k_1} + \omega_{k'} - \omega_{k_1+k'}) \left[\left(n_{k'}^{(2)} - n_{k'+k_1}^{(3)}\right) N_{k_1} +\right.$$

$+ n^{(2)}_{k'} n^{(3)}_{k'+k_1}] \, d\mathbf{k}_1 + \Phi_{k'}, \quad \frac{\partial n^{(3)}_{k''}}{\partial t} = \int |V_{k''k_1k''-k_1}|^2 \, \delta(\omega_{k_1} + \omega_{k''-k_1} - \omega_{k''}) \quad [(n^{(2)}_{k''-k_1} - n^{(3)}_{k''}) N_{k_1} - n^{(2)}_{k''-k_1} n^{(3)}_{k''}] \, d\mathbf{k}_1 + \Phi_{k''}.$

The nonlinear wave with the wave vector $\mathbf{k}$ coalesces with the waves whose wave vectors $\mathbf{k}'$ lie on the surface $\omega_k + \omega_{k'} = \omega_{k+k'}$ (their occupation numbers are denoted by $n^{(2)}_{k'}$). As a result of coalescence, waves with wave vectors k" are formed on the surface $\omega_k + \omega_{k''-k} = \omega_{k''}$ (occupation numbers $n^{(3)}_{k''}$). The terms $\Phi_{k'}$, $\Phi_{k''}$ describe the three-wave interaction of the waves $n^{(2)}_k$ and $n^{(3)}_{k''}$ with the remaining thermostat. In equilibrium, $\Phi_{k'} = \Phi_{k''} = 0$, for small deviations $\Phi_k = \gamma_k(n_k - n^0_k)$, and for deviations of order n^0_k, $\Phi_k \simeq \gamma_k n^0_k$. Here, γ_k, n^0_k are the damping, and the equilibrium occupation numbers. It is seen from (4) that the interaction is constructed in such a way that the difference $n^{(2)}_{k'} - n^{(3)}_{k'+k}$ decreases while the sum $n^{(2)}_{k'} + n^{(3)}_{k'+k}$ remains constant. This indicates that the quantities $n^{(2)}_{k'}$, $n^{(3)}_{k''}$ change by quantities of the order of $n^0_{k'}$, and $\Phi_{k'} \simeq \gamma_k \cdot n^0_{k'}$. As long as the amplitudes of the nonlinear packet satisfy condition (3), the quadratic terms in $n_{k'}$, $n_{k''}$ and also $\Phi_{k'}$, $\Phi_{k''}$, are small in comparison with terms which contain the amplitude of the nonlinear packet. Neglecting the thermal noise, the system (4) takes the form

$$\frac{\partial N_k}{\partial t} = - N_k \int |V_{k+k'kk'}|^2 \, \delta(\omega_{k+k'} - \omega_k - \omega_{k'}) \, n_{k'} \, d\mathbf{k}',$$
$$\frac{\partial n_{k'}}{\partial t} = - 2n_{k'} \int |V_{k_1+k'k_1k'}|^2 \, \delta(\omega_{k_1+k'} - \omega_{k_1} - \omega_{k'}) \, N_{k_1} \, d\mathbf{k}_1. \tag{5}$$

Here, assuming that the nonlinear packet is narrow, we put $n^{(2)}_{k'} - n^{(3)}_{k'+k_1} \approx n^{(2)}_{k'} - n^{(3)}_{k'+k} \equiv n_{k'}$. The system (5) can be rewritten in terms of $z_k = \ln 2N_k$, $y_{k'} = \ln n_{k'}$:

$$\frac{\partial z_k}{\partial t} = \int |V_{k+k'kk'}|^2 \, \delta(\omega_{k+k'} - \omega_k - \omega_{k'}) \, \frac{\delta H}{\delta y_{k'}} \, d\mathbf{k}',$$
$$\frac{\partial y_{k'}}{\partial t} = - \int |V_{k_1+k'k_1k'}|^2 \, \delta(\omega_{k_1+k'} - \omega_{k_1} - \omega_{k'}) \, \frac{\delta H}{\delta z_{k_1}} \, d\mathbf{k}_1,$$
$$H = \int e^{z_k} d\mathbf{k} - \int e^{y_{k'}} d\mathbf{k}' = 2\int N_k \, d\mathbf{k} - \int n_{k'} \, d\mathbf{k}',$$

Hence, its Hamiltonian structure is apparent. The Hamiltonian H is the Menly—Row integral. The decay of the nonlinear packet, i.e., the approach to equilibrium, is caused by the weak interaction of the waves $n_{k'}$ with the thermostat

$$\frac{\partial H}{\partial t} = - \Delta\omega_k \int \Phi_{k'} \, \delta(\omega_{k+k'} - \omega_k - \omega_{k'}) \, d\mathbf{k}'.$$

Hence, one can obtain an approximate estimate for the decay time

$$t_{\text{dam}} \simeq \gamma_{k'}^{-1} (\gamma_k \tau)^{-1}. \tag{6}$$

This time is much longer than the characteristic time of nonlinear interaction. We also note that the evolution, in the framework of Eq. (5), conserves the difference $2N - \int n_{k'} \, d\mathbf{k}'$, i.e., Eq. (3) remains in force.

To illustrate the above discussion, we solve the system of equations in the model case $V_{k_1k_2k_3} = \text{const} = V$. Naturally, we shall not specify here the details of the distribution of the secondary waves $n^{(2)}_{k'}$, $n^{(3)}_{k''}$, but focus attention on the integrated quantities.

We introduce the notation $N = 2\int N_k \, d\mathbf{k}$, $n = \Delta\omega_k \int n_{k'} \, \delta(\omega_{k+k'} - \omega_k - \omega_{k'}) \, d\mathbf{k}'$, $\Phi = \Delta\omega_k \int \Phi_{k'} \, \delta(\omega_{k+k'} - \omega_k - \omega_{k'}) \, d\mathbf{k}'$, write

$$\frac{\partial N}{\partial t} = - \frac{1}{\Delta\omega_k} N V^2 n, \quad \frac{\partial n}{\partial t} = - \frac{1}{\Delta\omega_k} N V^2 n + \Phi. \tag{7}$$

At times $t \simeq \tau \simeq \Delta\omega_k / V^2 H \ll H/\Phi$, the solution has the form

$$N(t) = \frac{N_0 H \exp(t/\tau)}{N_0 \exp(t/\tau) - n_0}, \quad n(t) = N(t) - H = \frac{n_0 H}{N_0 \exp(t/\tau) - n_0}.$$

For $H/\Phi \gg t \gg \tau$,

$$N(t) = H + H(n_0/N_0)\exp(-t/\tau) - \Phi t,$$

where $H = N_0 - n_0$.

It is seen that the packet evolves as follows. During the time of nonlinear interaction τ, $n(t)$ becomes a small quantity Φ/H determined by the thermal noise. During this time, the amplitude of the packet N changes negligibly, and $N_0 \to N_0 - n_0$. Subsequently, the packet is damped slowly since $n(t)$ is finite. The characteristic decay time is $t_{dam} \simeq H/\Phi$. Thus, the decay time of the nonlinear packet in a medium with coalescence increases linearly with increasing amplitude, $t_{dam} \sim \sqrt{N}$, in contrast to the radical principle $t_{dam} \sim \sqrt{N}$ for a nonlinear monochromatic wave [2]. The decay time of the normal correlator can be measured, for example, using the experimental method described in [5].

2. We now turn attention to the case when the original nonlinear wave can decay into secondary waves whose wave vectors k' and $k - k'$ lie on the surface $\omega_k = \omega_{k'} + \omega_{k-k'}$. Their occupation numbers will be denoted by $n_{k'}^{(1)} = n_{k'} + n_{k-k'}$. Assuming that the condition (3) holds, the kinetic equations take the form (we change the notation $N_k \to 2N_k$):

$$\frac{\partial N_k}{\partial t} = -N_k\left[\int |V_{kk'k-k'}|^2\,\delta(\omega_k - \omega_{k'} - \omega_{k-k'})\,n_{k'}^{(1)}\,dk' + \int |V_{k+k''kk''}|^2\,\delta(\omega_{k+k''} - \omega_k - \omega_{k''})\,n_{k''}\,dk'' + \Gamma\right], \quad \frac{\partial n_{k''}}{\partial t} =$$

$$= -n_{k''}\left[\int |V_{k_1+k''k_1k''}|^2\,\delta(\omega_{k_1+k''} - \omega_{k_1} - \omega_{k''})\,N_{k_1}\,dk_1 + \gamma\right] + \Phi_{k''}, \tag{8}$$

$$\frac{\partial n_{k'}^{(1)}}{\partial t} = n_{k'}^{(1)}\left[\int |V_{kk'k-k'}|^2\,\delta(\omega_k - \omega_{k'} - \omega_{k-k'})\,N_k\,dk - \gamma^{(1)}\right] + \Phi_{k'}^{(1)}.$$

Here, we introduced the linear growth increment Γ, which models a source and the linear damping decrements of the secondary waves γ, $\gamma^{(1)}$. These decrements can be caused, for example, by four-wave processes. The essential point is that a four-wave interaction with the given nonlinear wave involves thermal waves from a finite region of the k space. Overheating of these waves and the associated nonlinear effects occur when the amplitude of the packet is considerably in excess of that which is discussed in the present work.

It will be seen below that the introduction of these terms does not violate the Hamiltonian structure of the system (8) in the absence of small noise terms $\Phi_{k''}$ and $\Phi_{k'}^{(1)}$. However, one must note the following fact. In contrast with the system (5) which remains applicable in times smaller than t_{dam} (but much larger than τ) Eqs. (8), in the absence of Γ, γ, $\gamma^{(1)}$, cease to be valid already at times of the order τ. During this time, the original packet is reduced by a considerable factor, and the secondary wave $n_{k'}^{(1)}$ increases to its level. This is due to the fact that, in the decay processes, there is a direct transfer from the nonlinear packet to thermal waves (the sum $2N + \int n_{k'}^{(1)}\,dk'$ is conserved, but the difference decreases). However, in the presence of growth of the original packet Γ and damping of the secondary waves $\gamma^{(1)}$, one can have a steady state of the system which does not coincide with the thermodynamic equilibrium. The steady-state values of the numbers of waves $\bar{N}_k$, $\bar{n}_{k'}^{(1)}$, $\bar{n}_{k''}$ must obey the equations

$$\int |V_{kk'k-k'}|^2\,\delta(\omega_k - \omega_{k'} - \omega_{k-k'})\,\bar{n}_{k'}^{(1)}\,dk' + \int |V_{k+k''kk''}|^2\,\delta(\omega_{k+k''} - \omega_k - \omega_{k''})\,\bar{n}_{k''}\,dk'' = \Gamma,$$

$$\int |V_{k+k''kk''}|^2\,\delta(\omega_{k+k''} - \omega_k - \omega_{k''})\,\bar{N}_k\,dk = -\gamma + \frac{\Phi_{k''}}{\bar{n}_{k''}}, \tag{9}$$

$$\int |V_{kk'k-k'}|^2\,\delta(\omega_k - \omega_{k'} - \omega_{k-k'})\,\bar{N}_k\,dk = \gamma^{(1)} - \frac{\Phi_{k'}^{(1)}}{\bar{n}_{k'}^{(1)}}.$$

For $\gamma^{(1)} \gg \Gamma$, for states of the system close to the steady state, condition (3) is satisfied, and in Eqs. (8), $\Phi_{k''}$, $\Phi_k^{(1)}$ are small terms which, in the zero order, can be neglected and in the first order, can be assumed independent of N_k, $n_{k'}^{(1)}$, $n_{k''}$.

We shall now show that any initial condition which satisfies (3), relaxes to the steady state (9). During a time of the order τ, the difference of occupation numbers of the secondary waves which participate in the coalescences $n_{k''}$, decreases to a small quantity determined by the thermal noise. Therefore, we below neglect the effect of the coalescence processes on the evolution of the packet, since this leads only to a renormalization of Γ by a small quantity. Expressing Γ and $\gamma^{(1)}$ from (9), we write the system (8) in the form

$$\frac{\partial N_k}{\partial t} = -N_k \int |V_{kk'k-k'}|^2 \delta(\omega_k - \omega_{k'} - \omega_{k-k'})(n_{k'}^{(1)} - \bar{n}_{k'}^{(1)})\, d\boldsymbol{k}',$$

$$\frac{\partial n_{k'}^{(1)}}{\partial t} = n_{k'}^{(1)} \int |V_{kk'k-k'}|^2 \delta(\omega_k - \omega_{k'} - \omega_{k-k'})(N_k - \bar{N}_k)\, d\boldsymbol{k} + \Phi_{k'}^{(1)}\left(1 - n_{k'}^{(1)}/\bar{n}_{k'}^{(1)}\right). \tag{10}$$

We consider the functional

$$H(N_k, n_k^{(1)}) = \int (N_k - \bar{N}_k \ln N_k)\, d\boldsymbol{k} + \int (n_{k'}^{(1)} - \bar{n}_{k'}^{(1)} \ln n_{k'}^{(1)})\, d\boldsymbol{k}'.$$

It is easily seen that H is bounded from below, and the minimum is reached for the steady state

$$H(N_k, n_{k'}^{(1)}) \geqslant H(\bar{N}_k, \bar{n}_{k'}^{(1)}) = \bar{H}.$$

Calculating its time derivative, we obtain

$$\frac{\partial H}{\partial t} = -\int \frac{\Phi_{k'}^{(1)}(n_{k'}^{(1)} - \bar{n}_{k'}^{(1)})^2}{n_{k'}^{(1)}\bar{n}_{k'}^{(1)}}\, d\boldsymbol{k}'. \tag{11}$$

Thus, in the absence of noise, H is conserved (by writing (10) in terms of $z_k = \ln N_k$, $x_k = \ln n_k^{(1)}$ one can verify that H is the Hamiltonian of the system (10)). The presence of thermal noise leads to the fact that an arbitrary initial distribution which satisfies the criterion (3) evolves, during a finite time, to the steady state (9). This time can be estimated from (11):

$$\frac{1}{t} \simeq \frac{1}{H - \bar{H}} \int \frac{\Phi_{k'}^{(1)}(n_{k'}^{(1)} - \bar{n}_{k'}^{(1)})^2}{n_{k'}^{(1)}\bar{n}_{k'}^{(1)}}\, d\boldsymbol{k}'. \tag{12}$$

A search for a minimum of (12) shows that the final stage of the evolution when H is close to $\bar{H}$, has the longest duration. Then

$$H - \bar{H} \approx \int \frac{(N_k - N_k)^2}{\bar{N}_k}\, d\boldsymbol{k} + \int \frac{(n_{k'}^{(1)} - \bar{n}_{k'}^{(1)})^2}{\bar{n}_{k'}^{(1)}}\, d\boldsymbol{k}'.$$

(We note that in all integrals which contain $n_{k'}^{(1)}$, the integration is over the surface $\omega_k = \omega_{k'} + \omega_{k-k'}$.)

Linearizing (10) near the steady state we obtain, for the integral quantities δN and δn, an equation for harmonic oscillations:

$$(\partial^2/\partial t^2)\,\delta N = -\omega^2 \delta N.$$

Here

$$\omega^2 = \frac{\bar{N}}{\Delta\omega_k} \int |V_{kk'k-k'}|^2 \delta(\omega_k - \omega_{k'} - \omega_{k-k'})\, \bar{n}_{k'}^{(1)}\, d\boldsymbol{k}' \simeq \gamma^{(1)}\Gamma.$$

The same equation is obtained for the integral quantity $\delta n = \Delta\omega_k \int (n_{k'}^{(1)} - n_{k'}^{(1)})\, \delta(\omega_k - \omega_{k'} - \omega_{k-k'})\, dk'$. Averaging over the period of oscillations $T \simeq 2\pi/\sqrt{\delta^{(1)}\Gamma}$, we obtain the relation $\langle(\delta N)^2\rangle/\bar{N} = \langle(\delta n)^2\rangle/\bar{n}$, which is an analog of the virial theorem. Returning to the estimate (12), we find that

$$t \simeq \max(\bar{n}_{k'}/\Phi_{k'}) \simeq \gamma_{k'}^{-1}\Gamma/\gamma_{k'}. \tag{13}$$

Thus, the evolution of the nonlinear packet caused by the decay, in a medium with strong damping of secondary waves, consists of oscillations with frequency $\sqrt{\gamma^{(1)}\Gamma}$ which decay to the steady state (9) during time (13). These oscillations can be observed by switching on a sufficiently powerful source ($\Gamma \gg \tau^{-1}$) (for example, an incoherent parametric pump) over time not less than τ. Subsequently, this source is switched off while retaining the weak excitation ($\Gamma \ll \gamma^{(1)}$), and registering the absorbed power. We note that, by measuring the oscillation frequency, we obtain a method for the determination of relaxation time of the waves which is not directly associated with the pump.

In conclusion, we make a remark of historical nature. The system (8) which describes the evolution of a narrow nonlinear packet as a result of three-wave processes, is very similar to the kinetic equation which specifies the evolution of the spectrum of waves n_k induced by scattering from particles [8]:

$$\partial n_k/\partial t = n_k \left(\gamma_k + \int T_{kk'} n_{k'} dk'\right). \tag{14}$$

Because of the antisymmetry of the kernel $T_{kk'} = -T_{k'k}$, this equation is of the Hamiltonian form, and this causes the nontrivial evolution of the spectrum described in [8]: anomalously large times needed to reach the steady state (determined by the thermal noise which is not included in (14)), the possibility of oscillations, etc.

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BACKSCATTERING OF WAVES BY A SURFACE WITH DUAL SCALE ROUGHNESS WITH CONSIDERATION OF REREFLECTION

V. U. Zavorotnyi

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The problem of wave scattering by a surface containing roughness both large and small in comparison to the wavelength is considered. An attempt is made to refine Kuryanov's method by considering rereflection. An expression is obtained for the mean intensity of the diffuse component of the scattered field for arbitrary configuration of the coarse scale surface component. In the case of backscattering coherent superposition of certain field components occurs, due to the presence of rereflection from the surface. This effect has much in common with the well-known effect of intensification of backscattering from a body located in a random inhomogeneous medium.

Scattering of waves by a set of planes coated with microroughness was considered in [1]. Consideration of mirror rereflections between the planes permitted examination of a backscattering intensification effect, analogous to the backscattering intensification which occurs on a body located in a volume random inhomogeneous medium or near the random boundary between two media [2-4]. It will be shown in the present study that this effect occurs in the more general case of a two-scale surface, where the larger scale corresponds to a roughness of arbitrary configuration.

We will consider the scalar wave equation

$$(\Delta + k^2) G(r, r_0) = \delta(r - r_0) \tag{1}$$

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