Parabolic Version of Extended Categories $\mathcal{O}$ for Root-Reductive Lie Algebras

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Abstract

For a root-reductive Lie algebra $\mathfrak{g}$ over the field of complex numbers, let $\mathfrak{h}$ be a splitting Cartan subalgebra contained in a Dynkin Borel subalgebra $\mathfrak{b}$. For a parabolic subalgebra $\mathfrak{p}$ containing $\mathfrak{b}$, let $\mathfrak{l}$ and $\mathfrak{u}$ denote the Levi subalgebra and the nilradical of $\mathfrak{p}$, respectively. Write $\overline{\mathcal{O}}_{\mathfrak{p}}^{\mathfrak{g}}$ for the full-subcategory of the category of $\mathfrak{g}$-modules M such that

(i) M is an $\mathfrak{h}$-weight module with finite-dimensional weight spaces

(ii) M is both locally $[\mathfrak{b},\mathfrak{b}]$-finite and locally $\mathfrak{u}$-finite, and

(iii) M is an integrable $\mathfrak{l}$-module.

In this talk, we shall discuss properties of the category $\overline{O}_{\mathfrak{p}}^{\mathfrak{g}}$. For example, we shall determine when an object in $\overline{O}_{\mathfrak{p}}^{\mathfrak{g}}$ is simple, when $\overline{\mathcal{O}}_{\mathfrak{p}}^{\mathfrak{g}}$ has enough projectives, and whether $\overline{O}_{\mathfrak{p}}^{\mathfrak{g}}$ has an analogue of BGG Reciprocity.