Distributed Graph Algorithms	**Spring 2021**

Exercise 3: June 07

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Exercise 1 (List Coloring via Symmetric LLL). In the problem of *list coloring*, every vertex v is associated with a list (or palette) of colors $Pal(v)$. It is required to compute a legal vertex coloring where the color of each vertex v is taken from $Pal(v)$. (1a) Show that if for every v it holds that: (i) $|Pal(v)| \geq \ell$ (ii) each color $c \in Pal(v)$ appears in at *most* $\ell/8$ of its neighbors, then there is a legal coloring (i.e., solution to the list coloring instance).
(1b) We consider a weighted variant of the list coloring problem. Given is a graph G with maximum degree Δ where every vertex v has a palette of colors $Pal(v)$. Each color $c \in Pal(v)$ has a weight $w_v(c)$ such that $\sum_{c \in Pal(v)} w_v(c) = 1$. Prove that if for every edge (u, v) we have

$$\sum_{c \in Pal(u) \cap Pal(v)} w_v(c) \cdot w_u(c) \leq 1/(8\Delta)$$

then G has a legal coloring.
(1c) Use the LLL algorithms shown in class (as a black box) to devise distributed algorithms for computing the coloring in (1a) and (1b).

Exercise 2 (MST and Connectivity). (2a) Show that one can compute an MST on the clique graph within $O(\log n)$ rounds w.h.p.
(2b) Let G be an n-vertex D-diameter graph and let H be a subgraph of G given in a distributed manner. I.e., each vertex v knows its incident edges in H. In the *Connectivity Identification* task, it is required for each vertex v to output the largest vertex identifier in its connected component in H. Show a distributed algorithm for this problem that runs in $\widetilde{O}(D + \sqrt{n})$ rounds w.h.p.