Routing Scheme

**Exercise 1.** Describe an efficient routing scheme for the unweighted $\sqrt{n} \times \sqrt{n}$ 2-dimensional grid. The labels and the routing tables should be of size $O(\log n)$ bits. Bonus: extend it to the $d$-dimensional $n$-vertex hypercube for $n = 2^d$.

Labeling Scheme

**Exercise 2.** Show that any labeling scheme for adjacency in general graphs has a label size of at least $\Omega(n)$ bits.

**Exercise 3.** We saw in class an $(2k - 1)$ approximate labeling scheme for general graphs with $\tilde{O}(n^{1/k})$ bits. In this exercise, we are required to design exact distance labels but only for nodes that are sufficiently far away from each other in $G$. Consider an unweighted graph $G$ and define a $\delta$-distance labeling scheme by $(L,D)$ such that given $L(u)$ and $L(v)$, the decoding function should return the exact distance only for vertices $u, v$ at distance at least $\delta$ in $G$. That is, it is required that $D(L(u), L(v)) = \text{dist}(u, v, G)$ for every $u, v \in G$ satisfying that $\text{dist}(u, v, G) \geq \delta$. Show an $\delta$-distance labeling scheme with label size of $O(n \cdot \log^2 n/\delta)$. You are allowed to have a randomized construction and in such a case, guarantee that the decoding function returns the right answer with high probability.