Questions from Past Exams
Distributed Algorithms

Question 1

The distributed Depth-First Search algorithm studied in class had time complexity $O(|E|)$ on a graph $G(V, E)$ (in a model allowing $O(\log n)$ bit messages). Modify the algorithm to make it faster (in the same model), and describe the resulting algorithm in detail. (This new algorithm must still be based on a single traveller touring the entire graph.) Analyze the time complexity of the modified algorithm and prove your bound.

Question 2

A ring of processors is said to enjoy the *consistent orientation* property if the edges of the ring are oriented in a consistent manner, that is, each vertex identifies its two edges as going "left" and "right", and the edge $e = (v, w)$ is marked as going "left" by one of its endpoints and as going "right" by the other.

We have seen in class an algorithm that given an MIS on the ring, colors the vertices with 3 colors in a single round.

(a) Show that this algorithm might fail if the ring does not enjoy a consistent orientation.

(b) Prove that on an anonymous ring without a consistent orientation, it is impossible to deterministically 3-color the vertices, even given an MIS.

(c) Prove that on a non-anonymous ring without a consistent orientation, it is still possible to deterministically 3-color the vertices in a constant number of rounds given an MIS. (Try to use the smallest number of rounds.)

(d) What lower bound can be proved for the number of rounds required for computing MIS on a ring without a consistent orientation? Specify precise constants, not only asymptotic lower bound.

(e) What is the smallest n for which the lower bound you got in the previous question is greater than 1?

What is the smallest n for which the lower bound on 3-coloring is greater than 1?

Question 3

Consider the model KT_k of "known topology" to radius k, and the variant of the flooding algorithm studied for that model (based on disconnecting short cycles).

Prove or disprove: On a graph of diameter D, the time complexity of this algorithm in the synchronous model is $O(kD)$.

Question 4

Consider a path graph G with vertex ID's taken to be a random permutation of $\{1, \ldots, n\}$. Prove that in this case $T(MIS_{rank}) \leq c \log n$ for a constant $c > 0$ with probability at least $1 - 1/n$.

Question 5

Consider m distinct data items taken from an ordered set. With each item is associated its rank in the set, so that the items are given in the form of pairs (i, a_i) for $1 \leq i \leq m$. Suppose that the items are initially stored at some of the vertices of the tree T. Items can be replicated, namely, each item is stored in one or more vertices (and each vertex may store zero or more items). The goal is to end up with the root knowing the values of all the items.

Denote the depth of the subtree T_v rooted at a vertex v in T by $Depth(T_v)$. The data items are gathered to the root using the following policy:

- At round $Depth(T_v) + i$ (for $i \geq 1$), the vertex v will forward to its parent the ith item, (i, a_i), if this item is currently stored at v. (Otherwise, v will do nothing at this round.)

Prove that by time $Depth(T) + m$, all items are collected at the root.

Question 6

Consider algorithm MIS_{rank} on an n-vertex, D-diameter graph G. Is its time complexity $\Theta(n)$ or $\Theta(D)$? Prove your answer.

(To establish an answer of $\Theta(X)$, you have to prove that the algorithm terminates in $O(X)$ time for every graph and ID assignment, and show an example graph and ID assignment that cause the algorithm to take $\Omega(X)$ steps.)

Question 7

The *acyclic orientation problem* requires the vertices of an anonymous undirected network G to agree on an *orientation* (i.e., a direction) for each of the edges, such that the resulting directed graph $\tilde{G}$ is acyclic (i.e., there is no directed cycle).

Consider the following *Monte-Carlo* randomized algorithm for the problem.

1. Each vertex draws uniformly at random a label from the range $\{1, 2, \ldots, n^3\}$.
2. Each edge (u, v) is directed from the endpoint with the larger label to the one with the smaller label. (In case of equality, the edge is oriented arbitrarily.)

(a) Prove that with probability at least $1 - 1/n$, all of the drawn labels are distinct.

(b) Prove that if the drawn labels are all distinct, then the algorithm produces an acyclic orientation.

(c) Describe how the algorithm can be modified in order to turn it into a *Las Vegas* algorithm, namely, one that always halts with a correct solution. What is the expected running time of this algorithm?

Question 8

Prove that the communication complexity of the distributed Bellman-Ford algorithm on a *complete* (unweighted) n-vertex graph in the asynchronous model is $\Theta(n^3)$ (namely, that it is $O(n^3)$ under any execution scenario, and that there exists an execution scenario that causes it to be $\Omega(n^3)$).