Questions from Past Exams
Computational Geometry

Question 1
We are given a collection $S = \{s_1, \ldots, s_n\}$, of $n \geq 3$ segments in the plane, with each segment $s_i = (p_i, q_i)$ given by the coordinates of its two endpoints. It is required to check if the segments form a closed polygon. Design an efficient algorithm for the problem and analyze its time complexity.

Question 2
We are given two nonintersecting segments $S_1 = (p_1, q_1)$ and $S_2 = (p_2, q_2)$ in the plane, represented by the coordinates of their endpoints, with $x_{p_i} \leq x_{q_i}$ for $i = 1, 2$, and an $x$-coordinate $x_0$ common to both segments, i.e., such that the line $\ell : x = x_0$ intersects both. Describe a procedure for deciding, in a constant number of operations, whether the intersection of $\ell$ with $S_1$ is higher or lower than its intersection with $S_2$. The procedure is not allowed to solve the equations and compute the coordinates of the intersection points. (The following figure illustrates one of the possible configurations for the problem.)

![Diagram of two segments intersecting at $x_0$](image)

Question 3
Describe an efficient algorithm that given two sets $A, B$ each containing $n$ disjoint points in the plane, computes the shortest distance between a point in $A$ and a point in $B$, i.e., $\min\{\text{dist}(p, q) \mid p \in A, q \in B\}$.

Question 4
Two sets of points $S_1$ and $S_2$ are known to have the same Voronoi diagram, i.e., $\text{Vor}(S_1) = \text{Vor}(S_2)$. Prove or disprove each of the following claims:

1. $S_1 = S_2$.
2. $|S_1| = |S_2|$. 