“Size” of a representation of a finite group controls the size of its character values

Abstract:

Many problems about finite groups (e.g., convergence of random walks, properties of word maps, spectrum of Cayley graphs, etc.) can be approached in terms of sums of group characters. More precisely, what intervenes in such sums are the character ratios:

\[ \frac{X_r(g)}{\dim(r)}, \quad g \in G, \]

where \( r \) is an irreducible representation of \( G \), and \( X_r \) is its character. This leads to the quest for good estimates on the character ratios.

In this talk I will introduce a precise notion of “size” for representations of finite classical groups and show that it tends to put together those with character ratios of the same order of magnitude.

As an application I will show how one might generalize to classical groups the following result of Diaconis-Shahshahani (for \( k=2 \)) and Berestycki-Schramm-Zeitouni (for general \( k \)): The mixing time for the random walk on the group \( G=S_n \) using the cycles of length \( k \) is \( (1/k) n \log(n) \). The talk should be accessible for beginning graduate students, and is part from our joint project with Roger Howe (Yale and Texas A&M).